"PIMS" MARSDEN MEMORIAL LECTURE
10 Juin 2015, Centre Bernoulli, EPF-Lausanne

FROM EULER TO BORN AND INFELD, FLUIDS AND ELECTROMAGNETISM

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WARNING!

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BUT WILL BE TWICE AS LONG AS THE OTHER ONES :-((

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WHERE $(q, p, v) \in \mathbb{R}^{1+1+3}$ (keeping Euler's notations) ARE THE DENSITY, PRESSURE AND VELOCITY FIELDS AND THE PRESSURE p IS A GIVEN FUNCTION OF q ONLY.

XXI. Nous n'avons donc qu'à égaler ces forces accélératrices avec les accélerations actuelles que nous venons de trouver, & nous obtiendrons les trois équations suivantes :

$$P - \frac{1}{q}\left(\frac{dp}{dx}\right) = \left(\frac{du}{dt}\right) + u\left(\frac{du}{dx}\right) + v\left(\frac{du}{dy}\right) + w\left(\frac{du}{dz}\right)$$

$$Q - \frac{1}{q}\left(\frac{dp}{dy}\right) = \left(\frac{dv}{dt}\right) + u\left(\frac{dv}{dx}\right) + v\left(\frac{dv}{dy}\right) + w\left(\frac{dv}{dz}\right)$$

$$R - \frac{1}{q}\left(\frac{dp}{dz}\right) = \left(\frac{dw}{dt}\right) + u\left(\frac{dw}{dx}\right) + v\left(\frac{dw}{dy}\right) + w\left(\frac{dw}{dz}\right)$$

Si nous ajoutons à ces trois équations premièrement celle, que nous a fournie la confidération de la continuité du fluide :

$$\left(\frac{dq}{dt}\right)+\left(\frac{d.qu}{dx}\right)+\left(\frac{d.qv}{dy}\right)+\left(\frac{d.qw}{dz}\right)=0.$$

Si le fluide n'étoit pas compreſſible, la denſité q ſeroit la même en Z, & en Z′, & pour ce cas on auroit cette équation :

$$\left(\frac{du}{dx}\right)+\left(\frac{dv}{dy}\right)+\left(\frac{dw}{dz}\right)=0.$$

qui eſt auſſi celle ſur laquelle j'ai établi mon Mémoire latin allégué ci-deſſus.

A QUADRATIC CHANGE OF TIME IN THE EULER MODEL

$$\tau(t) = t^2/2, \quad (\tilde{q}, \tilde{p}, \tilde{v})(t, x) = (q(\tau(t), x), p(\tau(t), x), \tau'(t)v(\tau(t), x))$$

(so that $\tilde{v}(t, x)dt = v(\tau, x)d\tau$)

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For small times, $(\tau')^2 = t^2 = 2\tau << 1$, while $\tau'' = 1$,
we get an ASYMPTOTIC EQUATION after withdrawing the red terms.

THE RESULTING "ASYMPTOTIC" EQUATION

$$\boxed{\partial_\tau q + \mathrm{div}(qv) = 0, \quad qv = -\mathrm{grad} p,}$$

IS NOTHING BUT THE HEAT EQUATION, in the case of an "isothermal" fluid (i.e. as p is proportional to q),

$$\boxed{\partial_\tau q = \kappa \triangle q}$$

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$$\boxed{\int (\sqrt{-\mathrm{det} g} - \sqrt{-\mathrm{det}(g + d\mathcal{A})})}$$

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$$\boxed{\int (\sqrt{-\mathrm{det} g} - \sqrt{-\mathrm{det}(g + d\mathcal{A})})}$$

We limit ourself to the usual 3+1 dimensional Minkowski space (as Max Born and Leopold Infeld did in 1934).

Max BORN (1882-1970) 1954 Nobel Prize in Physics


THIS CHRISTMAS
JOHN TRAVOLTA &
OLIVIA NEWTON-JOHN

????

????...Max Born's grand-daughter!

THE BORN-INFELD THEORY IN TRADITIONAL NOTATIONS

After tedious but simple calculations, Born and Infeld got

$$\boxed{\partial_t B + \mathrm{curl}\left(\frac{B \times (D \times B) + D}{\sqrt{1 + D^2 + B^2 + (D \times B)^2}}\right) = 0, \quad \mathrm{div} B = 0}$$

$$\boxed{\partial_t D + \mathrm{curl}\left(\frac{D \times (D \times B) - B}{\sqrt{1 + D^2 + B^2 + (D \times B)^2}}\right) = 0, \quad \mathrm{div} D = 0}$$

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We recover (through the terms in black) the vacuum Maxwell equations whenever the electromagnetic field B, D, is of weak amplitude.

Four extra conservation laws come out from Emmy Noether's theorem

$$\boxed{\partial_t q + \mathrm{div}(qv) = 0, \quad \partial_t(qv) + \mathrm{div}(qv \otimes v - \frac{B \otimes B - D \otimes D}{q}) = \mathrm{grad}(q^{-1})}$$

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Observe the (electro-magneto-)hydrodynamic style of these conservation laws (q and v standing for the density and velocity fields of some "fluid"). Nothing similar would occur for the Maxwell equations!

THE AUGMENTED BORN-INFELD (ABI) SYSTEM

Following Y.B. Arma 2004, it is consistent (and much simpler) to ignore the algebraic constraints

$$v = \frac{D \times B}{q}, \quad q = (1 + D^2 + B^2 + (D \times B)^2)^{1/2}$$

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The augmented BI systems describe the interaction of an electromagnetic field (B, D) with some "matter" (q, v) and enjoys the Galilean invariance of classical mechanics:

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It also admits a convex energy $\mathcal{E} = \mathcal{E}(q, B, D, P = qv) \ = \ q^{-1}(1 + D^2 + B^2 + P^2)$.

PART 3: A MHD-TYPE DIFFUSION EQUATION

Performing a quadratic change of time in the augmented BI system,

$$\boxed{t \rightarrow \tau = t^2/2, \quad (q, B, D, v)(t, x) \rightarrow (q(\tau, x), B(\tau, x), \tau' D(\tau, x), \tau' v(\tau, x))}$$

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(as we did to get the heat equation out of the Euler model), we obtain

$$\boxed{\partial_\tau q + \mathrm{div}(qv) = 0, \quad qv = \mathrm{div}(\eta B \otimes B) - \mathrm{grad} p}$$

$$\boxed{\partial_\tau B + \mathrm{curl}(B \times v) + \mathrm{curl}(\mu\, \mathrm{curl}(\nu B)) = 0}$$

where $(q, p, v, B) \in \mathbb{R}^{1+1+3+3}$ are the density, pressure, velocity and magnetic fields and $\mu = \nu = \eta = q^{-1} = -p$.

THE "INCOMPRESSIBLE" VERSION

$$\boxed{v = \mathrm{div}(B \otimes B) - \mathrm{grad}p, \quad \mathrm{div}v = 0, \quad \partial_\tau B + \mathrm{curl}(B \times v) = -\mathrm{curl}(\mu \ \mathrm{curl}B)}$$

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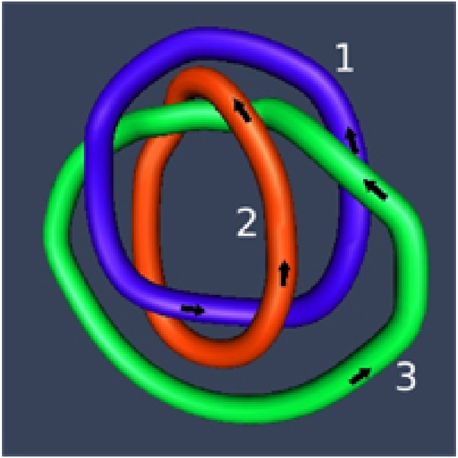
Then, we recover one of the models of "magnetic relaxation" proposed by Moffatt to get, as $\tau \to \infty$ and $v \to 0$, some stationary solutions B_∞ to $\boxed{\mathrm{div}(B_\infty \otimes B_\infty) = \mathrm{grad}p_\infty, \quad \mathrm{div}B_\infty = 0}$ of prescribed knot topology.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ANALYSIS OF THE INCOMPRESSIBLE DIFFUSION EQUATION

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In any case, the analysis of the large time behavior seems widely open.

FINAL COMMENTS

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The Born-Infeld model of Electromagnetism is very geometric and has known a strong revival in high energy physics (string theory) in the 90s. Once set up in the framework of special relativity and properly augmented by Noether's extra conservation laws, it can be expressed as a Galilean system very much in the style of Euler's hydrodynamics. Furthermore, some diffusion equations, apparently very remote from "first principles", can be (formally) derived from the (augmented) BI equations in just one step.

FEW REFERENCES

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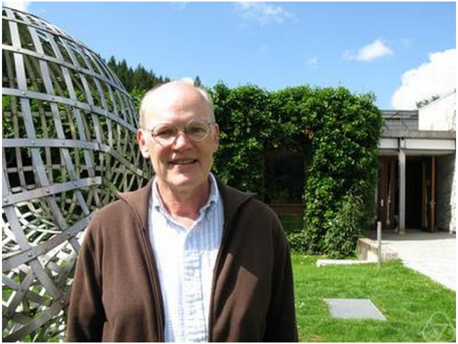

Jerry MARSDEN, Oberwolfach 2008