

Unpacking Mathematical Habits of Mind: What's in your suitcase?



Susan Oesterle, PhD
Douglas College

Changing the Culture Conference
May 13, 2016--SFU Harbour Centre

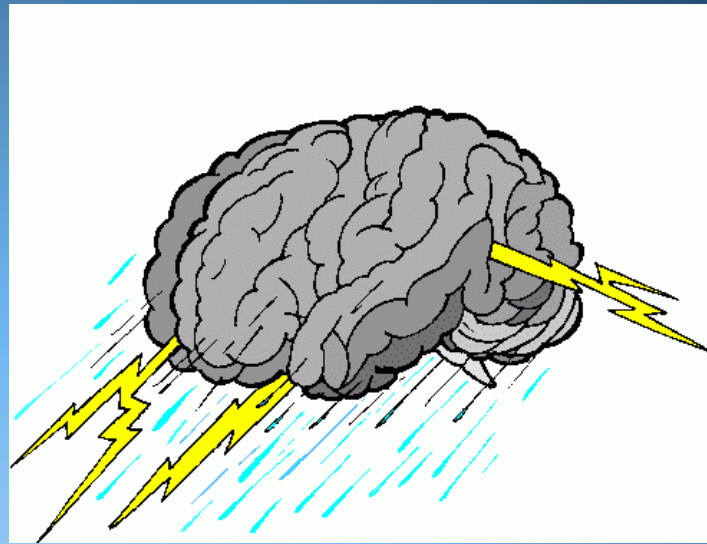
Overview

- Why are we talking about Mathematical Habits of Mind?
- What are Mathematical Habits of Mind?
- Are you aware of your own MHoM?
- How can we foster MHoM in our students—no matter what level we teach?



Question 1

Take 1 minute:



What do you think of when someone refers to:

Mathematical Habits of Mind?

My Journey with MHoM

- PMENA 2011 – Working Group ...
- Workshops at BCAMT, CtC, myPITA
- 2014 Working Group at CMESG in Edmonton
- Book Chapter



Why are we talking about MHOM?

- It's not new!
- Front & centre in the New BC Mathematics Curriculum

How did it get there?

Cuoco, Goldenberg, & Mark (1996)

“Given the uncertain needs of the next generation of high school graduates, how do we decide what mathematics to teach?”

They question the appropriateness of a content driven curriculum.

They propose organising the curriculum around Mathematical Habits of Mind...

Cuoco, Goldenberg, & Mark (1996)

*A curriculum organised around habits of mind tries to close the gap between what the users and makers of mathematics **do** and what they **say**. ...[It] lets students in on the process of creating, inventing, conjecturing and experimenting ... It is a curriculum that encourages false starts, calculations, experiments, and special cases. (p. 376)*

Where did this lead?

- Many others have tried to pin down or elaborate on particular “mathematical habits of mind” (see reference list)
- A number of advisory boards (in the US) have advocated MHOM as important
 - The NCTM Principles and Standards (2000)
 - National Research Council Report: Adding It Up (2001)
 - The Conference Board of the Mathematical Sciences (2012)

The NCTM Principles and Standards (2000)

PROCESS STANDARDS

- **Problem-solving**
 - By solving mathematical problems, students acquire ways of thinking, **habits of persistence and curiosity**, and confidence in unfamiliar situations that serve them well outside the mathematics classroom.
- **Reasoning and proof**
 - People who reason and think analytically **tend to note patterns, structure, or regularities** in both real-world and mathematical situations. They **ask if** those patterns are accidental or if they occur for a reason. **They make and investigate mathematical conjectures**. They **develop and evaluate mathematical arguments and proofs**.

The NCTM Principles and Standards (2000)

PROCESS STANDARDS

- **Communication**

- When students are challenged to communicate the results of their thinking to others orally or in writing, they learn to **be clear, convincing, and precise in their use of mathematical language.**

- **Representations**

- When students **gain access to mathematical representations** and the ideas they express and when they can create representations to capture mathematical concepts or relationships, they **acquire a set of tools** that significantly **expand their capacity to model and interpret** physical, social, and mathematical phenomena.

Adding it Up(2001)

Strands of Mathematical Proficiency

- Conceptual understanding
- Procedural fluency
- Strategic competence (i.e. problem solving)
- Adaptive reasoning (i.e. logic, reflection, explanation, justification)
- Productive disposition: **habitual inclination** to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

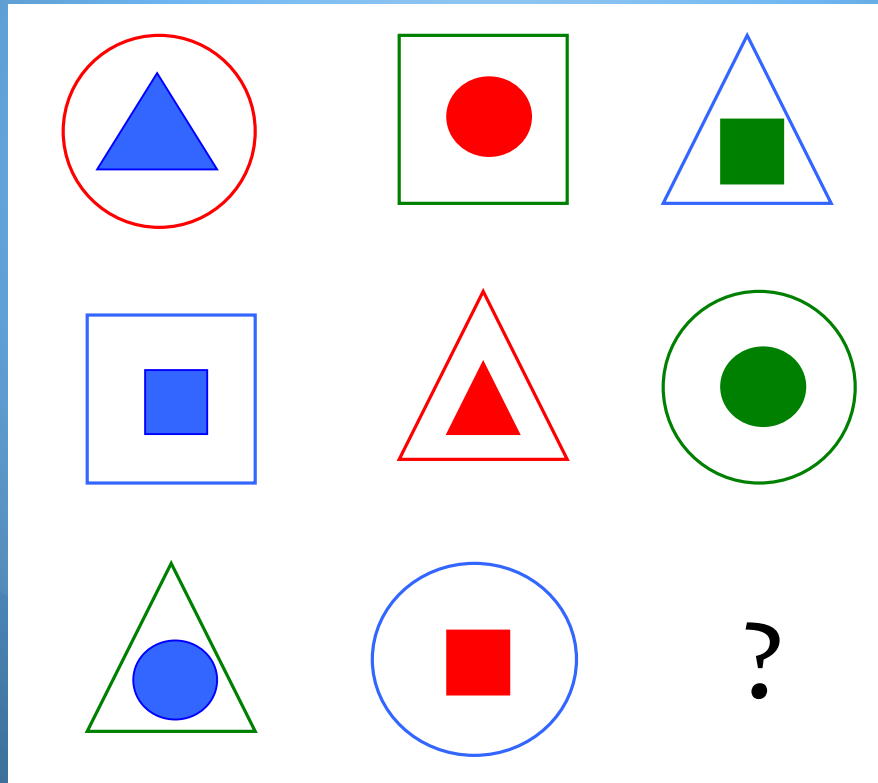
Kilpatrick, Swafford , & Findell (2001)

Conference Board of the Mathematical Sciences (2012)

*All courses and professional development experiences for mathematics teachers should develop **the habits of mind of a mathematical thinker and problem-solver**, such as reasoning and explaining, modeling, seeing structure, and generalizing. (p. 19)*

Question 2

Pay attention to your thinking as you determine what is missing:



Our Former BC (WNCP) Mathematics Curriculum

Mathematical Processes

- Communication
- Connections
- Mental Mathematics and Estimation
- Problem Solving
- Reasoning
- Technology
- Visualisation

Our New K-9 Curriculum

- From **What's New?:**

*a focus on developing **mathematical habits of mind** and encouraging students to wonder how mathematicians think and work*

- From **What is the Essence?:**

*helping students appreciate a uniquely mathematical perspective: how embodying **mathematical ways of thinking and acting** changes how one interprets the world around them.*

Introduction to the New Curriculum

Students who have developed mathematics habits of mind exhibit expertise in:

- persevering and using mathematics to solve problems in everyday life
- recognizing there are multiple ways to solve a problem
- demonstrating respect for diversity in approaches to solving problems
- choosing and using appropriate strategies and tools
- pursuing accuracy in problem solving.

What are Mathematical Habits of Mind?

In its broadest interpretation...

thinking about mathematics (and the world) the way that mathematicians do



The thing about habits is...

- They are often automatic
- They can be learned and unlearned (though it's not always easy!)
- They can be good or bad



Question 3

What is the product when the following algebraic expression is expanded:

$$(x - a) \cdot (x - b) \cdot (x - c) \cdot \dots \cdot (x - z)$$

(The answer is on the back of the page.)

Bad Habits

- Tendency to look for a quick answer
- Lack of persistence when the answer isn't obvious
- Preference for memorisation over understanding
- Try to guess or recall **what is expected** rather than engage in the problem

A Tighter Definition

We demonstrate MHoM when we **habitually** choose **actions** and **strategies**, pose **questions** and display **attitudes** that are PRODUCTIVE in a mathematical context.

They help us understand the math, solve problems and maybe even help us create mathematics.



Question 4

How many factors does the number

$$2^5 \cdot 3^6$$

have?

Question 5: Quick and Toasty

Three slices of bread are to be toasted under a grill. The grill can hold two slices at once but only one side is toasted at a time. It takes 30 seconds to toast one side of a piece of bread, 5 seconds to put a piece in or take a piece out and 3 seconds to turn a piece over. What is the shortest time in which the three slices can be toasted?



From Mason, Burton, & Stacey (2010), Thinking Mathematically , 2nd Ed.

Good Habits—Attitudes

- Curiosity
- Willingness to play with numbers
- Willingness to take risks
- Perseverance, tenacity, determination
- Confidence (?)
- Sense of wonder

Good Habits—Actions

- Noticing shapes, patterns
- Making comparisons—noticing variance & invariance
- Visualising
- Organising information efficiently
- Taking care
- Checking/verifying

Good Habits—Strategies

- Starting with a smaller number or simpler case (**Specialising**—see Mason (2010))
- Looking for patterns and making conjectures (**Generalising**—see Mason (2010))

Etc., etc....See Polya!

- Switching perspectives when reaching a “dead end”

Good Habits—Questions

- What am I given? What do I know?
- What do I need to find out?
- Have I seen something like this before?

And later:

- Why is this true? What is really going on here?
- Could I have done this differently?
- What if...?

Bonus Round—The Game “15”

For two players

1	2	3	4	5	6	7	8	9

Choose top or bottom row.

Take turns choosing a number and recording it in your row.

The winner is the first to obtain a sum of 15.

A few thoughts/questions

- Hard to argue that MHoM aren't useful
- Can they be taught?
- What do we need to attend to in our classrooms to foster MHoM?

Looking at the World Through Mathematical Lenses

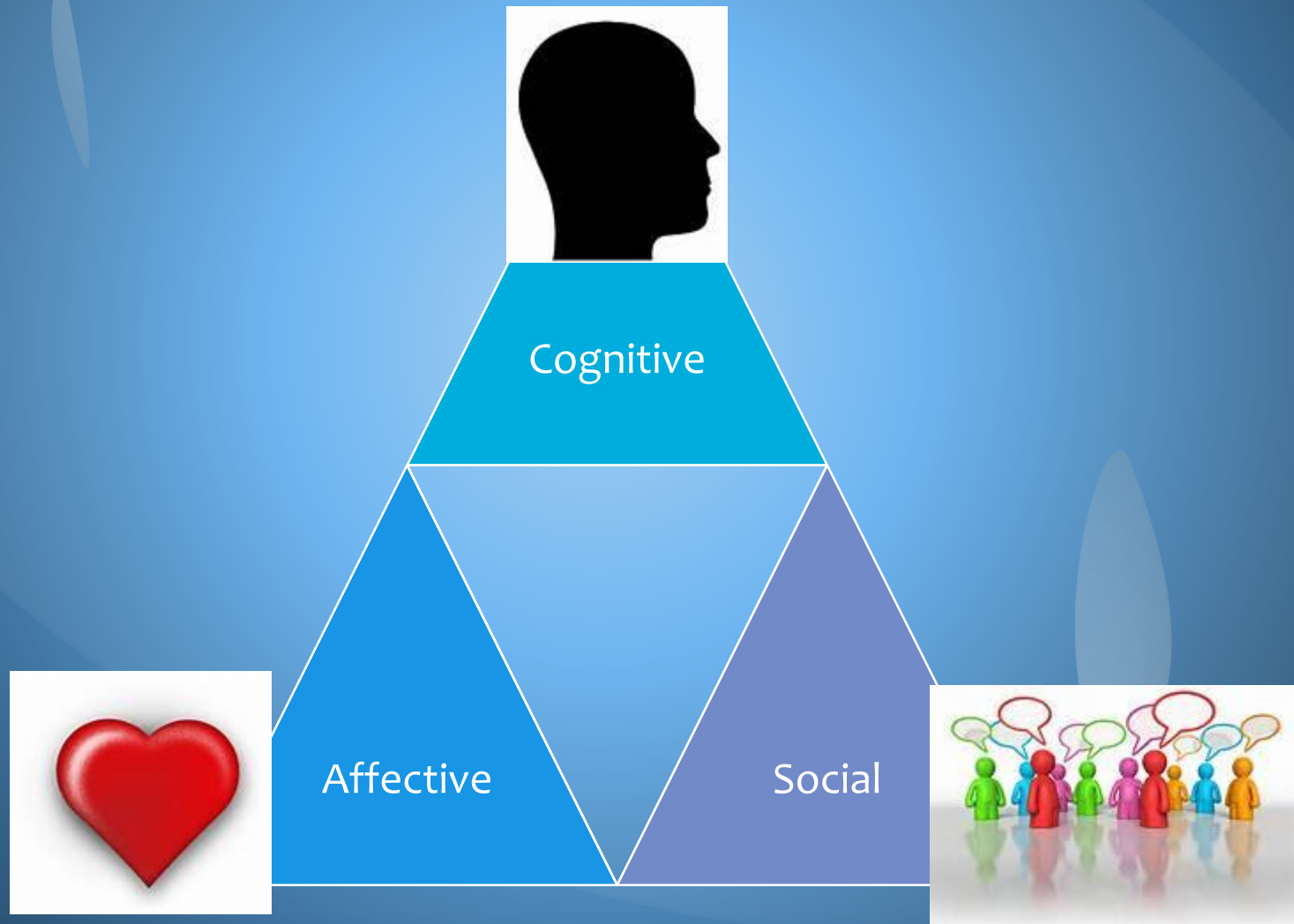


- Noticing quantity and shape
- Making comparisons
- Finding patterns
- Asking mathematical questions

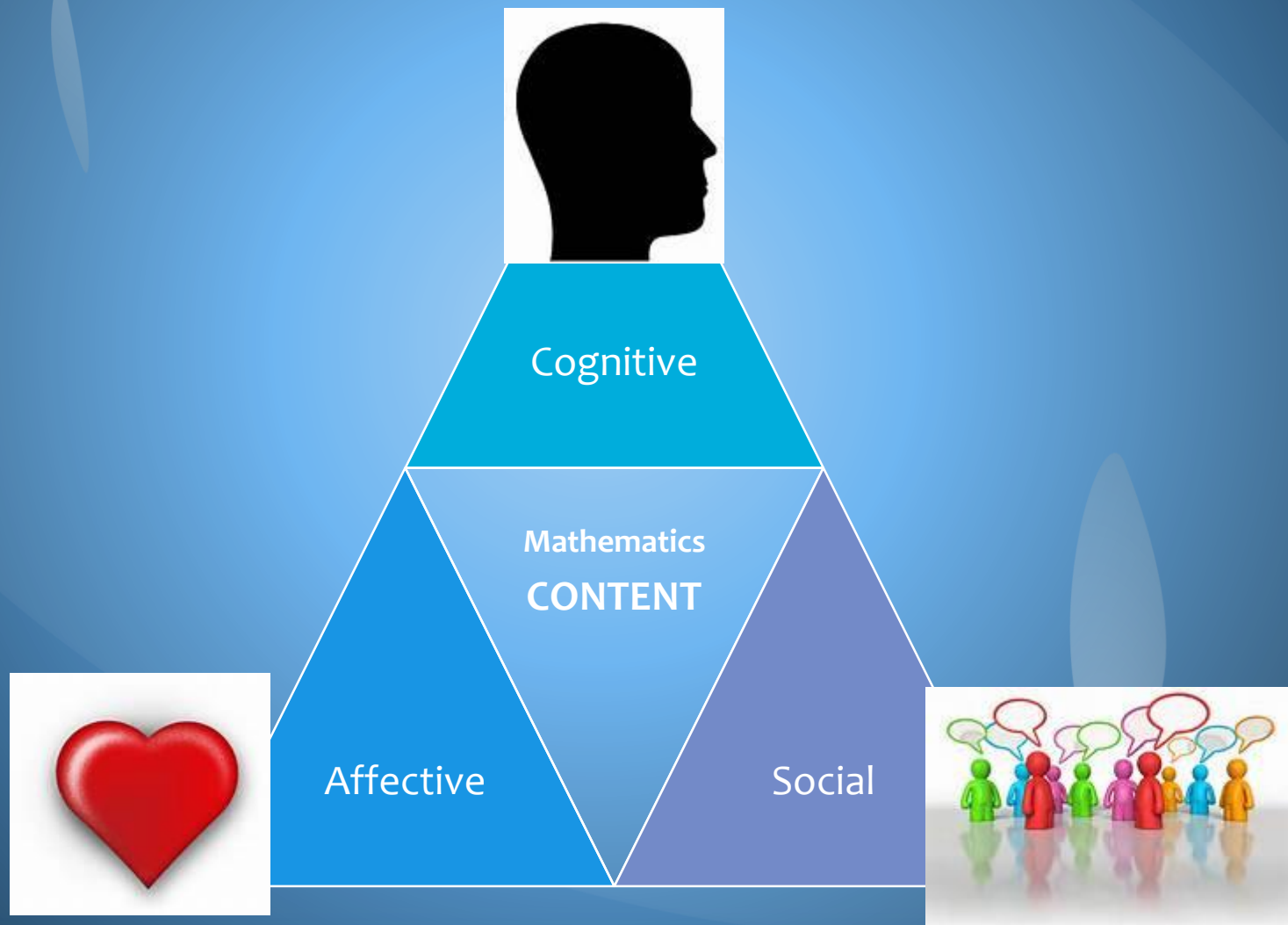
From the Ministry documentation

Extensive research indicates that for students to develop mathematical habits of mind they must encounter and interact in intentional learning settings. Classroom design combined with active participation strategies will enhance student learning, increase achievement, and factor in the development of the well-educated citizen.

Three Dimensions to Consider



Three **Four** Dimensions to Consider



What can we do in our classrooms?

- Become aware: know what to look for
- Recognise and reward mathematical thinking and habits of mind
- Model and help students develop metacognition
- Teach MHoM explicitly
- Create opportunities

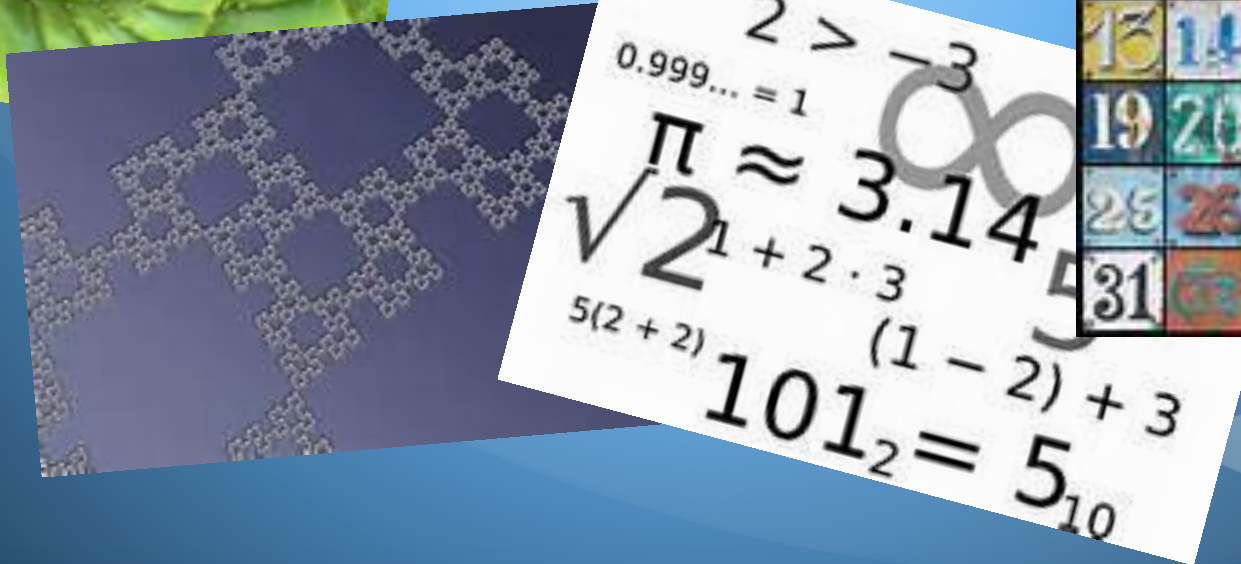
Authentic Mathematical Experiences

Students will need opportunities to:

- Discover properties of numbers, shapes, functions, ...
- Support their reasoning verbally and in writing to their peers as well as their teachers
- Explore and play, take risks
- Reflect on **Actions, Attitudes, Strategies** and **Questions** that are productive in mathematics



Thank you!



Ultimate Tic-Tac-Toe

