Schedule

Sunday, May 29

All events on Sunday evening are located in the Senate Room at the Hotel Alma.

4:15 *Registration opens*

5:15 **Colin Weir**
For graduate students: What the L?

6:30 *Welcome reception*

Monday, May 30

Except where otherwise noted, all events (starting on Monday) will take place in room ICT 122 of the Information & Communication Technologies Building.

8:15 *Coffee and registration*

9:00 *Welcoming remarks*

9:15 **Ram Murty**
Introduction to Artin L-series

10:15 *Coffee and refreshments*

10:45 **Frank Thorne**
Secondary terms in counting functions for cubic fields

11:15 **Habiba Kadiri**
Zeros of Dedekind zeta functions

12:00 *Lunch*

2:00 **Brian Conrey**
Introduction to random matrix theory

3:00 *Coffee and refreshments*

3:30 **Hugh Montgomery**
Geometric properties of the zeta function

4:15 **Michael Rubinstein**
Fun with the explicit formula

5:15 **Amir Akbary**
For graduate students: Rankin–Selberg convolutions

7:00 *Pub night—Last Defence Lounge (Grad Lounge), in the student centre building (MacEwen Hall)*

Tuesday, May 31

8:15 *Coffee and refreshments*

9:00 **Brian Conrey**
The recipe

10:00 *Coffee and refreshments*

10:30 **Steve Gonek**
Zeros of a family of approximations of the Riemann zeta-function

11:15 **David Farmer**
The explicit formula and the approximate functional equation

12:00 *Lunch*

2:00 **Ram Murty**
Artin's holomorphy conjecture and recent progress

3:00 *Coffee and refreshments*

3:30 **Youness Lamzouri**
Prime number races

4:00 *Problem session*

5:15 **Greg Martin**
For graduate students: Size matters

Wednesday, June 1

8:15 *Coffee and refreshments*

9:00 **Ram Murty**
Special values of Artin L-series

10:00 *Coffee and refreshments*

10:30 **Dimitris Koukoulopoulos**
When is a multiplicative function small on average?

11:00 **Brian Conrey**
Ranks of elliptic curves

1:00 *Bow River rafting—departing from Hotel Alma*

Thursday, June 2

8:15 *Coffee and refreshments*

9:00 **K. Soundararajan**
Distribution of values of zeta and L-functions

10:00 *Coffee and refreshments*

10:30 **Chantal David**
Elliptic curves with a given number of points modulo p

11:15 **Kumar Murty**
An omega theorem for the logarithmic deriviative of a real Dirichlet L-function

12:00 *Lunch*

2:00 **K. Soundararajan**
Moments of zeta and L-functions on the critical line, I

3:00 *Coffee and refreshments*

3:30 **Micah Milinovich**
Simple zeros of L-functions of modular forms

4:00 **Himadri Ganguli**
On the behaviour of the Liouville function on polynomials with integer coefficients

4:30 **Tim Trudgian**
In the middle of the Pólya and Turán "conjectures"

5:15 **Matt Greenberg**
For graduate students: Special values of L-functions

Friday, June 3

8:15 *Coffee and refreshments*

9:00 **K. Soundararajan**
Moments of zeta and L-functions on the critical line, II

10:00 *Coffee and refreshments*

10:30 **Nathan Ng**
Non-zero values of Dirichlet L-functions

11:15 **Alina Bucur**
Statistics for points on curves over finite fields

Index of Speakers and Locations of Abstracts

Speakers, Titles, and Abstracts

Sunday, May 29	*evening*

All events on Sunday evening are located in the Senate Room at the Hotel Alma.

4:15 PM: *Registration opens*

5:15–6:00 PM

Colin Weir (University of Calgary)
For graduate students: What the L?

When asked what is an L-function, someone once said "You know it when you see it." Perhaps not surprisingly, Selberg seemed unsatisfied with this answer and in 1991 gave an axiomatic definition of what he believed an L-function should be. We will recall the L-functions of Riemann, Dirichlet, Dedekind, and Hecke and see examples of L-functions coming from elliptic curves and modular forms. Noting the extensive list of similarities between these functions, we will try to infer the motivation behind Selberg's definition.

Questions from graduate students in the audience are encouraged throughout the talk.

6:30-8:00 PM: *Welcome reception*

Monday, May 30	*morning*

Except where otherwise noted, all events (starting on Monday) will take place in room ICT 122 of the Information & Communication Technologies Building.

8:15–9:00 AM: *Morning coffee and registration*

9:00–9:15 AM: *Welcoming remarks*

9:15–10:05 AM

Ram Murty (Queen's University)
Introduction to Artin L-series

After defining Artin L-series, we will discuss the Chebotarev density theorem and its applications.

10:15–10:45 AM: *Coffee and refreshments*

10:45–11:05 AM

Frank Thorne (Stanford University)
Secondary terms in counting functions for cubic fields

We will discuss our proof of secondary terms of order $X^{5/6}$ in the Davenport–Heilbronn theorems on cubic fields and 3-torsion in class groups of quadratic fields. For cubic fields this confirms a conjecture of Datskovsky–Wright and Roberts. We also will describe some generalizations, in particular to arithmetic progressions, where we discover a curious bias in the secondary term.

Roberts' conjecture has also been proved independently by Bhargava, Shankar, and Tsimerman. Their proof uses the geometry of numbers, while our proof uses the analytic theory of Shintani zeta functions. We will also discuss a combined approach which yields further improved error terms.

This is joint work with Takashi Taniguchi.

11:15–11:45 AM

Habiba Kadiri (University of Lethbridge)
Zeros of Dedekind zeta functions

In this talk, I will discuss various results concerning zero free-regions, the Deuring-Heilbronn phenomenon, and the density of zeros for Dedekind zeta functions. The arguments are based on McCurley's work on zeros of Dirichlet L-functions, Heath–Brown's work on Linnik's constant, and Lagarias, Montgomery, and Odlyzko's work on the Chebotarev density theorem. I will also mention an application to prime ideals in number fields.

Monday, May 30	*afternoon*

2:00–2:50 PM

Brian Conrey (American Institute for Mathematics)
Introduction to random matrix theory

We show how to calculate some basic statistics such as n-correlation of the eigenvalues of unitary matrices.

3:00–3:30 PM: *Coffee and refreshments*

3:30–4:00 PM

Hugh Montgomery (University of Michigan)
Geometric properties of the zeta function

We describe the nature of the level sets $|\zeta(s)| = c$ for various values of c. This gives rise to several interesting questions concerning the value of $|\zeta(s)|$ at critical points. A solution to one such question will be outlined.

4:15–4:45 PM

Michael Rubinstein (University of Waterloo)
Fun with the explicit formula

The explicit formula relates the zeros of an L-function to its logarithmic derivative's Dirichlet coefficients. I'll discuss some amusing experiments that have been carried out with the formula.

5:15–6:00 PM

Amir Akbary (University of Lethbridge)
For graduate students: Rankin–Selberg convolutions

We describe how the study of the analytic properties of convolutions of Dirichlet series (L-functions) leads to the results on the size of coefficients of Dirichlet series and non-vanishing of Dirichlet series on the line $\sigma = 1$.

Questions from graduate students in the audience are encouraged throughout the talk.

7:00-9:00 PM: *Pub night—Last Defence Lounge (Grad Lounge), in the student centre building (MacEwen Hall)*

This pub night has been organized for the benefit of graduate students and postdocs. All are welcome to attend of course, but we request that faculty buy their own beer.

Tuesday, May 31	*morning*

8:15–9:00 AM: *Coffee and refreshments*

9:00–9:50 AM

Brian Conrey (American Institute for Mathematics)
The recipe

We show how to make conjectures for averages over a family of products of L-functions and ratios of products of L-functions, and indicate some applications.

10:00–10:30 AM: *Coffee and refreshments*

10:30–11:00 AM

Steve Gonek (University of Rochester)
Zeros of a family of approximations of the Riemann zeta-function

Let $F_N(s) = \sum_{n \le N} n^{-s}$ be a partial sum of the Riemann zeta-function $\zeta(s)$, where $s = \sigma + it$ is a complex variable. Also, set $\zeta_N(s) = F_N(s) + \chi(s)F_N(1-s)$, where $\chi(s)$ is the factor from the functional equation for $\zeta(s)$. Then $\zeta_N(s)$ satisfies the same functional equation as $\zeta(s)$, and the approximate functional equation tells us that $\zeta_N(s)$ is a good approximation of $\zeta(s)$ when $N = \sqrt{|t|/2\pi}$. We are interested in the distribution of the zeros of $\zeta_N(s)$ when N is not a function of t. We determine a "critical" strip for $\zeta_N(s)$, count the number of zeros $\zeta_N(s)$ has in rectangles, find lower bounds for the number of zeros on the critical line $\sigma = 1/2$, and relate this to the number of zeros of $F_N(s)$ in $\sigma > 1/2$. In particular, we show that if $N \ge 1$ is fixed and T is sufficiently large, then 100% of the zeros with ordinates in $(T,\, 2T]$ lie on the critical line and are simple. This is joint work with Hugh Montgomery.

11:15–11:45 AM

David Farmer (American Institute for Mathematics)
The explicit formula and the approximate functional equation

I will compare the explicit formula and the approximate functional equation as tools in analytic number theory. Several recent results, both computational and theoretical, will be described.

Tuesday, May 31	*afternoon*

2:00 PM

Ram Murty (Queen's University)
Artin's holomorphy conjecture and recent progress

Artin conjectured that each of his non-abelian L-series extends to an entire function if the associated Galois representation is nontrivial and irreducible. We will discuss the status of this conjecture and discuss briefly its relation to the Langlands program.

3:00–3:30 PM: *Coffee and refreshments*

3:30–3:50 PM

Youness Lamzouri (University of Illinois)
Prime number races

Assuming the Generalized Riemann Hypothesis and the Grand Simplicity Hypothesis M. Rubinstein and P. Sarnak (1994) proved that the set of real numbers x such that $\pi(x;q,a_1) > \cdots > \pi(x;q,a_r)$ has a positive logarithmic density $\delta_{q;a_1,\ldots,a_r}$. In this talk, I will present how an asymptotic formula for the densities $\delta_{q;a_1,\ldots,a_r}$ when $r \geq 3$ is used to derive some surprising consequences on prime number races with three or more competitors.

I will also describe a recent joint work with K. Ford and S. Konyagin concerning a construction of certain hypothetical zeros of Dirichlet L-functions off the critical line, called *barriers*, that would imply $\delta_{q;a_1,a_2} = 0$ for some races $\{q; a_1, a_2\}$.

4:00–5:00 PM: *Problem session*

The problem session will be moderated by Brian Conrey.

5:15–6:00 PM

Greg Martin (University of British Columbia)
For graduate students: Size matters

Many techniques in analytic number theory require us to know how quickly L-functions grow on vertical lines; as you might expect, the answer is almost trivial outside the critical strip but mysterious inside the critical strip. We will describe the basic bounds ("convexity" bounds) for the size of the Riemann ζ-function and Dirichlet L-functions inside the critical strip, and then mention some improvements of these bounds ("subconvexity" results), as well as the conjectured "Lindelöf Hypothesis".

Questions from graduate students in the audience are encouraged throughout the talk.

8:15–9:00 AM: *Coffee and refreshments*

9:00–9:50 AM

Ram Murty (Queen's University)
Special values of Artin L-series

Dirichlet's class number formula has a nice conjectural generalization in the form of Stark's conjectures. These conjectures pertain to the value of Artin L-series at $s = 1$. However, the special values at other integer points also are interesting and in this context, there is a famous conjecture of Zagier. We will give a brief outline of this and display some recent results.

10:00–10:30 AM: *Coffee and refreshments*

10:30–10:50 AM

Dimitris Koukoulopoulos (Centre de recherches mathmatiques/Université de Montréal)
When is a multiplicative function small on average?

Let f be a multiplicative function. The main problem we will be concerned with in this talk is understanding when f is small on average. Halasz showed that, unless f 'pretends to be' n^{it} for some small t, this is true and gave quantitative estimates on the rate of decay of the partial sums of f. The estimate provided by Halasz's theorem is in general tight but there are functions f for which it is far from the truth. A natural question that arises is to classify the functions f whose partial sums are significantly smaller than what one might predict by Halasz's theorem. More precisely, if $\sum_{n\leq x} f(n) \ll x(\log x)^{-A}$ for some big constant A, then what can we say about f? We show that if this is the case, then either f pretends to be $\mu(n)n^{it}$ for some small t or $\sum_{p\leq x} f(p) \ll x(\log x)^{-(A-2)/3}$. Also, we prove a partial converse to the above statement. Finally, we show how these methods can be used to give a new proof of the prime number theorem in arithmetic progressions.

11:00–11:50 AM

Brian Conrey (American Institute for Mathematics)
Ranks of elliptic curves

We show how to use conjectures for moments of L-functions to get insight into the frequency of rank 2 elliptic curves within a family of quadratic twists.

Wednesday, June 1	*afternoon*

1:00–5:30 PM: *Bow River rafting*

Bow River rafting is a Calgary tradition that involves chilling out in inflatable rafts (paddling *not* required) and chatting amicably with co-rafters while the current of the Bow River carries you through the city. These are proper river rafts, and lifejackets will be provided. All conference participants have been signed up for this activity by default at no charge.

A bus will pick us up at Hotel Alma at 1:00 to take us to the launch site. We will raft on the river about 2–3 hours (going from Bowness Park to Eau Claire Park). The bus will pick us up at the end and bring us back to Hotel Alma. Bring your sunscreen, hat, and so on.

Thursday, June 2	*morning*

8:15–9:00 AM: *Coffee and refreshments*

9:00–9:50 AM

K. Soundararajan (Stanford University)
Distribution of values of zeta and L-functions

I will discuss the distribution of values of zeta and L-functions when restricted to the right of the critical line. Here the values are well understood by probabilistic models involving "random Euler products". This fails on the critical line, and the L-values here have a different flavor here with Selberg's theorem on log normality being a representative result.

10:00–10:30 AM: *Coffee and refreshments*

10:30–11:00 AM

Chantal David (Université Concordia)
Elliptic curves with a given number of points modulo p

Let E be an elliptic curve over $\mathbb{Q}$, and N a positive integer. We consider the problem of counting the number of reductions of E modulo p with exactly N points over $\mathbb{F}_p$. The Hasse bound implies the trivial bound $\sqrt{N}/\log N$ for the number of such reductions, but no other bound is known. On average over the set of all elliptic curves over $\mathbb{Q}$, we can obtain bounds which are significantly better, and under some hypothesis for the short interval distribution of primes in arithmetic progressions, we can show an asymptotic formula for the average number of reductions with N points.

This is joint work with E. Smith.

11:15–11:45 AM

Kumar Murty (University of Toronto)
An omega theorem for the logarithmic deriviative of a real Dirichlet L-function

We prove an omega theorem for the value at 1 of the logarithmic deriviative of a Dirichlet L-function associated to a real character. Our result is an analogue of Chowla's theorem that gives an omega result for the L-function value itself. This is joint work with Mariam Mourtada.

2:00–2:50 PM

K. Soundararajan (Stanford University)
Moments of zeta and L-functions on the critical line, I

I will discuss techniques to get upper and lower bounds for moments of zeta and L-functions. The lower bounds are unconditional and the upper bounds in general rely on the Riemann Hypothesis. In several cases of low moments, one can obtain asymptotics, and I may discuss a couple of such recent cases.

3:00–3:30 PM: *Coffee and refreshments*

3:30–3:50 PM

Micah Milinovich (University of Mississippi)
Simple zeros of L-functions of modular forms

Let $L(s, f)$ be the L-function associated to a holomorphic cuspidal newform f of weight k and level q. In this talk, I will describe ongoing joint work with Nathan Ng where we are interested in counting the number of simple zeros of $L(s, f)$ with ordinates in interval $(0, T]$.

4:00–4:20 PM

Himadri Ganguli (Simon Fraser University)
On the behaviour of the Liouville function on polynomials with integer coefficients

Let $\lambda(n)$ denote the Liouville function. Complementary to the prime number theorem, Chowla conjectured that $\sum_{n\leq x}\lambda(f(n)) = o(x)$ for any polynomial $f(x)$ with integer coefficients, not in the form of $bg(x)^2$.

Chowla's conjecture is proved for linear functions, but for degree greater than 1 the conjecture seems to be extremely hard and still remains wide open. One can consider a weaker form of Chowla's conjecture, given by Cassaigne *et al.*: *If $f(x) \in \mathbb{Z}[x]$ and is not in the form of $bg^2(x)$ for some $g(x) \in \mathbb{Z}[x]$, then $\lambda(f(n))$ changes sign infinitely often.*

Although it is weaker, this conjecture of Cassaigne *et al.* is still wide open for polynomials of degree greater than 1. In this talk, I will describe some recent progress made while studying Conjecture 1 for the quadratic polynomials. This is joint work with Peter Borwein and Stephen Choi.

4:30–4:50 PM

Tim Trudgian (University of Lethbridge)
In the middle of the Pólya and Turán "conjectures"

Liouville's function $\lambda(n)$ is $+1$ when n has an even number of prime factors (counted with multiplicity), and -1 otherwise. This talk discusses the possible constancy in sign of $L_\alpha(x) = \sum_{n=1}^{x}\lambda(n)/n^\alpha$. The case $\alpha = 0$ relates to a conjecture of Pólya; the case $\alpha = 1$ relates to a conjecture of Turán; the case $\alpha = 1/2$ is of particular interest. This is joint work with Michael Mossinghoff.

5:15–6:00 PM

Matt Greenberg (University of Calgary)
For graduate students: Special values of L-functions

Special values of ζ and L-functions are arithmetic in nature. The prototypical result illustrating this paradigm is Dirichlet's class number formula. In this talk, we will discuss this formula and its relatives, e.g., the conjecture of Birch and Swinnerton–Dyer. In addition, I will try to highlight the role of analytic methods in the study of special values.

Questions from graduate students in the audience are encouraged throughout the talk.

Friday, June 3	*morning*

8:15–9:00 AM: *Coffee and refreshments*

9:00–9:50 AM

K. Soundararajan (Stanford University)
Moments of zeta and L-functions on the critical line, II

I will discuss techniques to get upper and lower bounds for moments of zeta and L-functions. The lower bounds are unconditional and the upper bounds in general rely on the Riemann Hypothesis. In several cases of low moments, one can obtain asymptotics, and I may discuss a couple of such recent cases.

10:00–10:30 AM: *Coffee and refreshments*

10:30–11:00 AM

Nathan Ng (University of Lethbridge)
Non-zero values of Dirichlet L-functions

In this talk we consider certain subsets S of the complex plane and investigate whether a fixed Dirichlet L-function is non-vanishing for many members of S. The two cases we consider are S equal to a vertical arithmetic progression and S equal to a certain set of linear combination of zeros of Dirichlet L-functions. This is joint work with Greg Martin.

11:15–11:35 AM

Alina Bucur (University of California, San Diego)
Statistics for points on curves over finite fields

A curve is a one dimensional space cut out by polynomial equations. In particular, one can consider curves over finite fields, which means the polynomial equations should have coefficients in some finite field and that points on the curve are given by values of the variables in the finite field that satisfy the given polynomials. A basic question is how many points such a curve has, and for a family of curves one can study the distribution of this statistic. We will give concrete examples of families in which this distribution is known or predicted, and give a sense of the different kinds of mathematics that are used to study different families.