Coarse Grained Stochastic Birth-Death Processes for Tropical Convection and Climate

B. Khouider

University of Victoria

Summer School on stochastic and probabilistic methods, UVic
14-18, 2008

OUTLINE

- Part I:
  - Introduction, unresolved features (CAPE and CIN)
  - Stochastic spin/flip model and coarse-graining
  - Deterministic model and coupling
  - Walker cell simulations: stochastic effect on climate
  - Summary
- Part II: (Aquaplanet setup)
  - Mean field/Stochastic RCE's
  - Selecting stochastic regimes and RCEs
  - Intermittency in single column
  - Effect of CIN on deep convective activity and convectively coupled waves
- Conclusion

Relevant papers

- Atmospheric science:

  Majda, Franzke, & Khouider, (2008), Phyl. Trans.: *Applied math perspective on stochastic modeling for climate*

  Khouider, Majda, & Katsoulakis (2003), PNAS: *Coarse grained stochastic models for tropical convection and climate.*

  Majda & Khouider (2002), PNAS: *Stochastic and mesoscopic models for tropical convections.*

- Theory

  Katsoulakis, Majda, & Vlachos , JCP(2003) and PNAS (2003), (material science, coarse graining)

  Katsoulakis, Majda, & Sopasakis (2004, 2005a, 2005b): Multiscale Coupling, Intermittency, metastability, and phase transition

Introduction

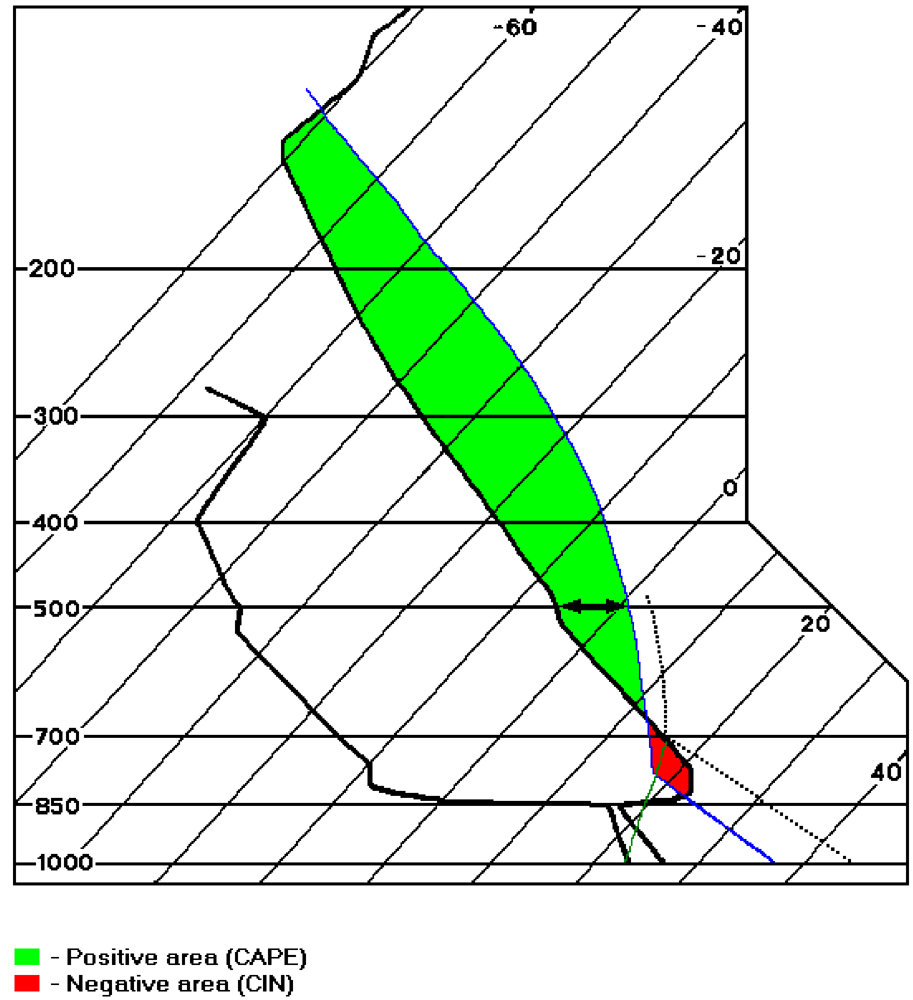
- Moist convection: Transport of latent heat.
- Source of energy for local and large scale circulation.
- Generates and maintains tropical waves and storms.
- Organized tropical convection ranges from mesoscale individual clouds (1-10 km) to large scale superclusters (1000-10,000 km).
- Poorly represented by GCM's despite Today's supercomputers.
- Major contemporary problem: How large-scale circulation supplies energy and maintains deep convection?
- Convective Inhibition (CIN): Energy Barrier for spontaneous convection

Motivation

- Can Stochastic parametrizations alter tropical Climatology?
- Can they increase the wave fluctuations?
- Lin & Neelin: suggest plausible influence of stochastic convective parametrizations on the variability in GCM's.

Static stability of Lifted parcel

- Sounding (black)
- Lifted parcel (blue line) cools by expansion,
- LCL: parcel warming by latent heat release of condensation
- LFC: level of free convection
- Red area: Convective Inhibition (CIN); Energy Barrier for spontaneous convection.
- Green area: CAPE, convetively available potential energy



“ This is a typical loaded gun sounding”. From NOAA’s weather glossary–www

- (Thermal) Buoyancy of a lifted parcel:

$$B = g\frac{\theta_{e,p} - \theta_{e,a}}{\theta_{e,a}}$$

θ_e = temperature + moisture content $\times$ latent heat

- Potential energy of lifted parcel

$$\begin{aligned} E_p &= \int_0^{LNB} g\frac{\theta_{e,p} - \theta_{e,a}}{\theta_{e,a}} dz \\ &= \int_0^{LFC} \text{---}\, dz + \int_{LFC}^{LNB} \text{---}\, dz \\ &= -\mathrm{CIN} + \mathrm{CAPE} \end{aligned}$$

- When (how) parcel has (could have) enough energy to overcome CIN and reach LFC?

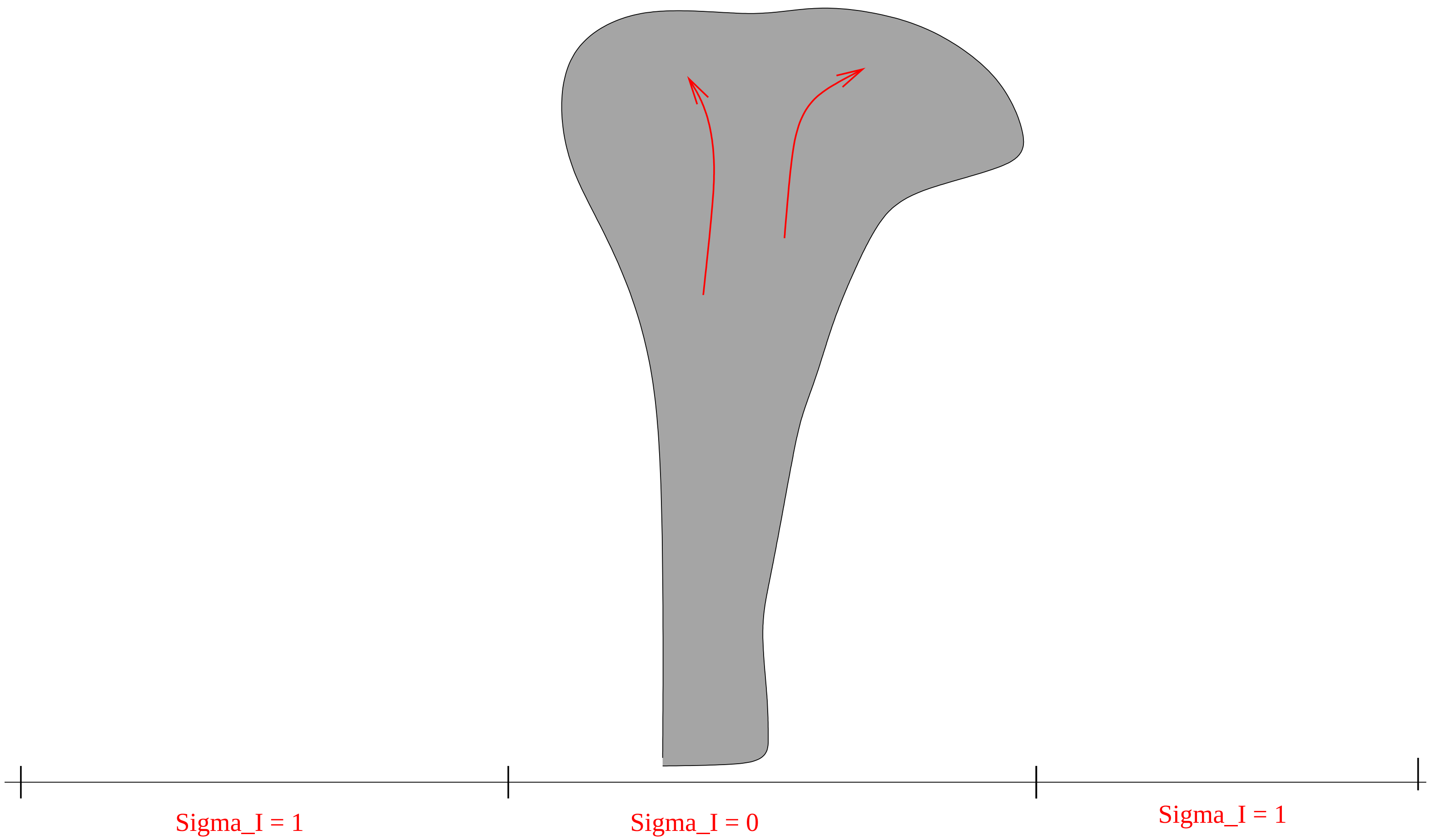
Microscopic stochastic Model for CIN

- CIN: Energy Barrier for spontaneous convection
- Observationally, factors for CIN complex:
  gust fronts, gravity waves, density currents, turbulent fluctuations in boundary layer equivalent potential temperature, etc.
- Too complex to model in detail; instead, borrow ideas from statistical physics and material science of representing these effects by an order parameter, σ_I

- Order parameter, σ_I, sites (1-10 km apart)

  $\sigma_I = 1$ if deep convection is inhibited: a CIN site

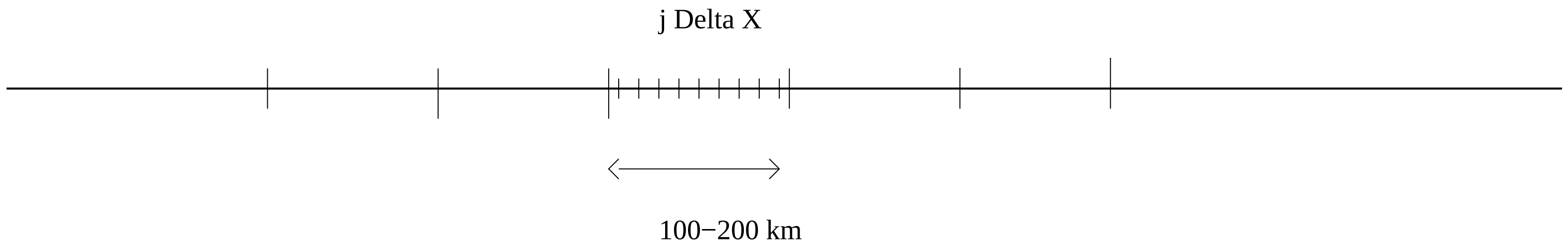
  $\sigma_I = 0$ if there is potential for deep convection: a PAC site



- Coarse Mesh: Average CIN

  $j\Delta x, \Delta x = O(100, 200 \text{ km})$ (mesoscopic scale)

$$\bar{\sigma}_I(j\Delta x, t) = \frac{1}{\Delta x}\int_{(j-1/2)\Delta x}^{(j+1/2)\Delta x} \sigma_I(x,t)\,dx$$



Intuitive Stochastic Rules for Interaction of Order Parameter, σ_I

A) If CIN site is surrounded mostly by CIN sites, should remain so with high probability

B) If PAC site is surrounded by CIN sites, should have high probability to switch to CIN site

C) The external large scale mesoscopic mesh values, $\vec{u}_j$, should supply external potential, $h(\vec{u}_j)$, which modifies dynamics in A) and B) according to whether external conditions favor CIN or PAC

Stochastic Model

- View boundary layer as heat bath with external Potential:
  Ising model (magnetization and phase transition)
  Materials science: Souganidis, Katsoulakis, etc.

- Microscopic energy for CIN:

$$H_h(\sigma_I) = \sum_{x \neq y} J(\frac{|x-y|}{L})\sigma_I(x)\sigma_I(y) + h\sum_x \sigma_I(x)$$

  – J: microscopic interaction potential: (Currie-Weiss)

$$J(r) = \begin{cases} U_0, & r < r_0 \\ 0, & r > r_0 \end{cases}$$

  – h: external potential. $H_h \nearrow h$

- Invariant Gibbs measure: $G = (Z_\Lambda)^{-1} \exp[\beta H_h(\sigma)]d\sigma$
  Z_Λ : partition function.

- Spin flip rule: $\sigma_I^x(y) = \begin{cases} 1 - \sigma_I(x), & y = x \\ \sigma_I(y), & y \neq x \end{cases}$

- Arrhenius dynamics:

  Rate: $c(x, \sigma_I) = \begin{cases} \tau^{-1} \exp[-\beta V(x)], & x = 1 \\ \tau^{-1}, & x = 0 \end{cases}$

  $V(x) \equiv \Delta H = \sum_{z \neq x} J(x - z)\sigma_I + h(x)$ (detailed balance)

  τ_I : CIN characteristic time O(days)

- With $U_0 > 0$: if a CIN site is mostly surrounded by CIN sites, then it needs to overcome a larger energy barrier.

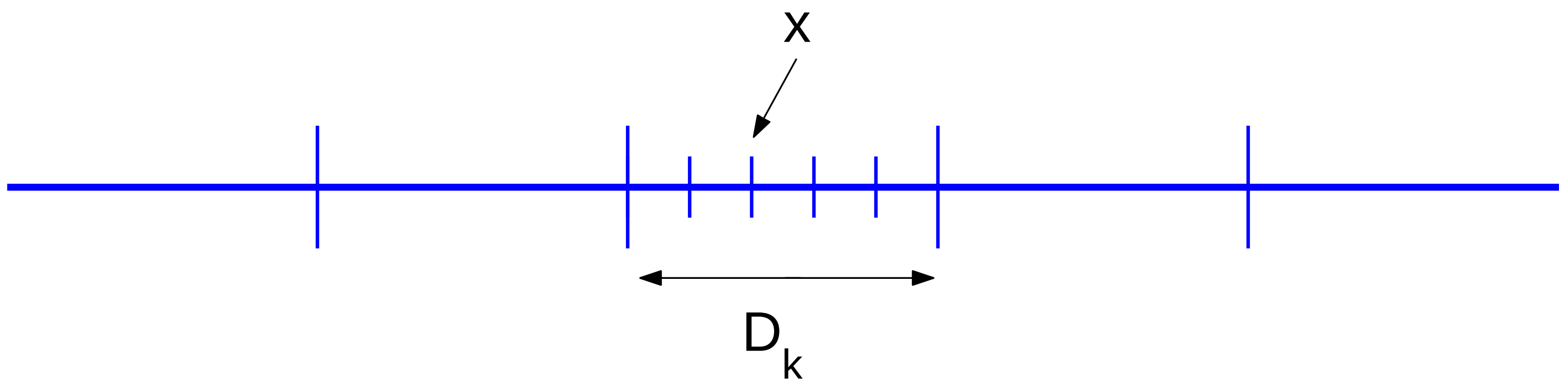
- External field also builds/destroys energy for CIN: $H_h \nearrow h$

Coarse-Graining

- Coarse-grained stochastic process:

$$\eta_t(k) = \sum_{x \in D_k} \sigma_{I,t}(x); \qquad \eta(k) \in \{0, 1, \cdots, q\}$$

average CIN on coarse-mesh: $\bar{\sigma}_I[D_k] = \frac{1}{q}\eta(k)$



Features of coarse-grained process

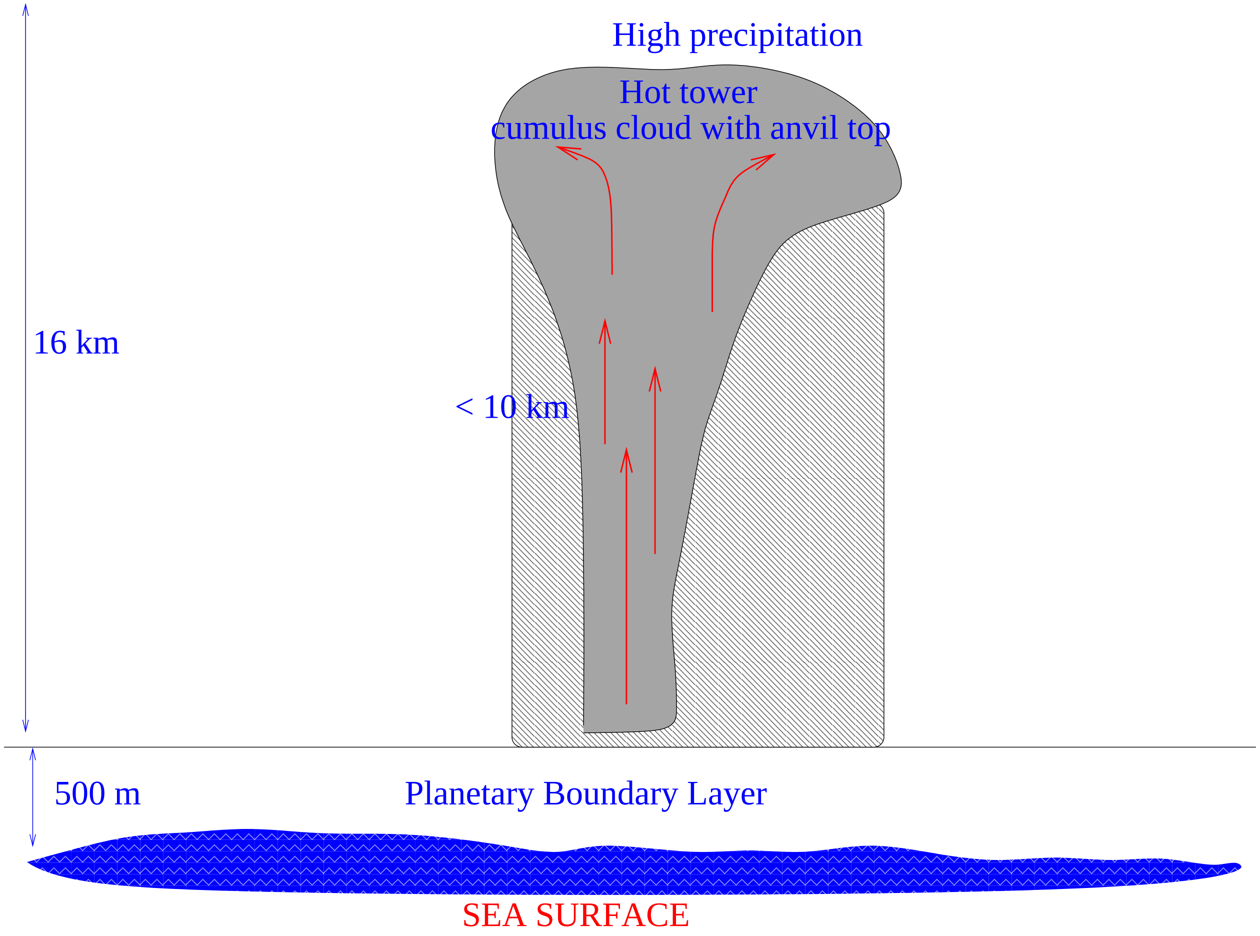
- Canonical invariant Gibbs measure:
  $G_{m,q,\beta}(\eta) = \frac{1}{Z_{m,q,\beta}} e^{\beta \bar{H}(\eta)} P_{m,q}(d\eta)$
- Coarse grained Hamiltonian
  $\bar{H}(\eta) = \frac{U_0}{q-1} \sum_{l \in \Lambda_c} \eta(l)\big(\eta(l) - 1\big) + h \sum_{l \in \Lambda_c} \eta(l)$
- Arrhenius Dynamics lead to birth/death process with Adsorption/Desorption rates:

$$C_a(k,n) = \frac{1}{\tau_I}[q - \eta(k)]$$

$$C_d(k,n) = \frac{1}{\tau_I}\eta(k) e^{-\beta \bar{V}(k)}$$

$$\text{with } \bar{V}(\eta) = \Delta \bar{H}(\eta) = \frac{2U_0}{q-1}\big(\eta(k) - 1\big) + h$$

Stochastic model for CIN coupled into a one-and-half layer model convective parametrization (toy GCM)



The Deterministic Model (Toy GCM): model convective parametrization

- Prognostic Eqns: One vertical baroclinic mode, no rotation

$$\frac{\partial u}{\partial t} - \bar{\bar{\alpha}} \frac{\partial \theta}{\partial x} = -Friction \qquad \frac{\partial \theta}{\partial t} - \bar{\alpha} \frac{\partial u}{\partial x} = Q_c - Q_R$$

$$h \frac{\partial \theta_{eb}}{\partial t} = -D + E \qquad H \frac{\partial \theta_{em}}{\partial t} = D - Q_R$$

Convective heating:

$$Q_c = M \sigma_c \big((\mathrm{CAPE})^+\big)^{1/2}$$

$\mathrm{CAPE} \propto \theta_{eb} - \gamma\theta.$

σ_c called *area fraction of deep convection*, plays key role in linear stability.

Coupling Stochastic model into toy GCM

- Order parameter modifies CAPE flux:

$$\sigma_c = (1 - \overline{\sigma}_I)\sigma_c^+; \;\; \sigma_c^+ = .002$$

- External potential depends on large scale dynamics and thermodynamics: (Good guess)

$$h \propto m_-,$$

Downward mass flux $\propto$ Convective mass flux

- convective events build CIN by downdraft cooling of boundary layer and/or convective heating of middle troposphere (stabilization)

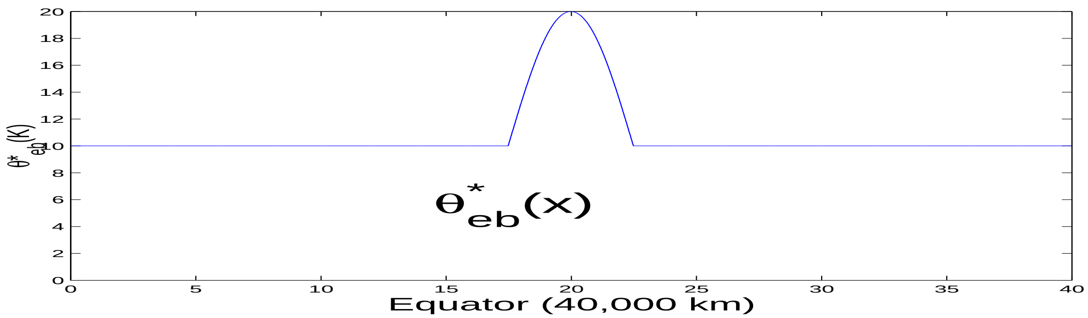
- Other choices of h are also considered (Part 2).

Nonlinear Simulations with Toy GCM

Walker circulation set-up:

mimicking the Indian Ocean/Western Pacific warm pool

$$\frac{\theta^*_{eb}(x)}{\theta^{*,0}_{eb}} = 1 + A_0 \cos\left(\frac{\pi(x-x_0)}{L_0}\right), \quad |x - x_0| < \frac{L_0}{2}$$



- Periodic geometry, $\Delta x = 80$ km
- Initial data: RCE + small random perturbation
- Integrate to statistical equilibrium
- Effects of stochastic model on waves and climate?
- Vary stochastic parameters, βU_0, τ_I, and A_0

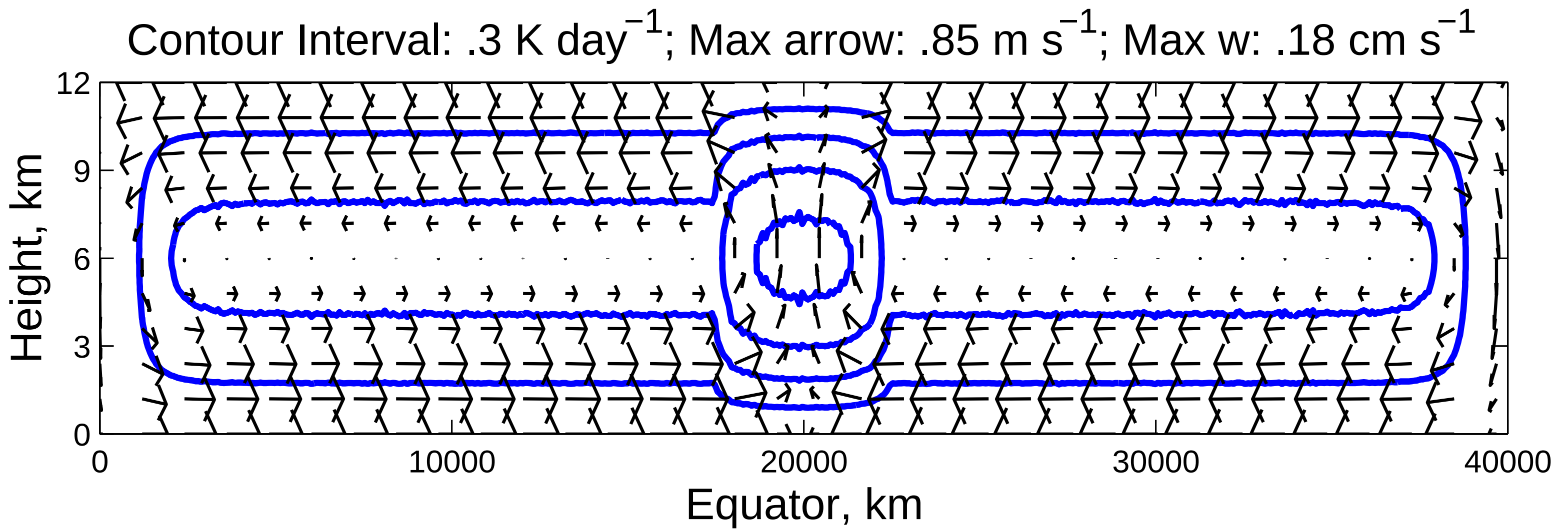
Table 1: Effect of stochastic parameters on climatology and fluctuations, with heating strength $A_0 = .5$.

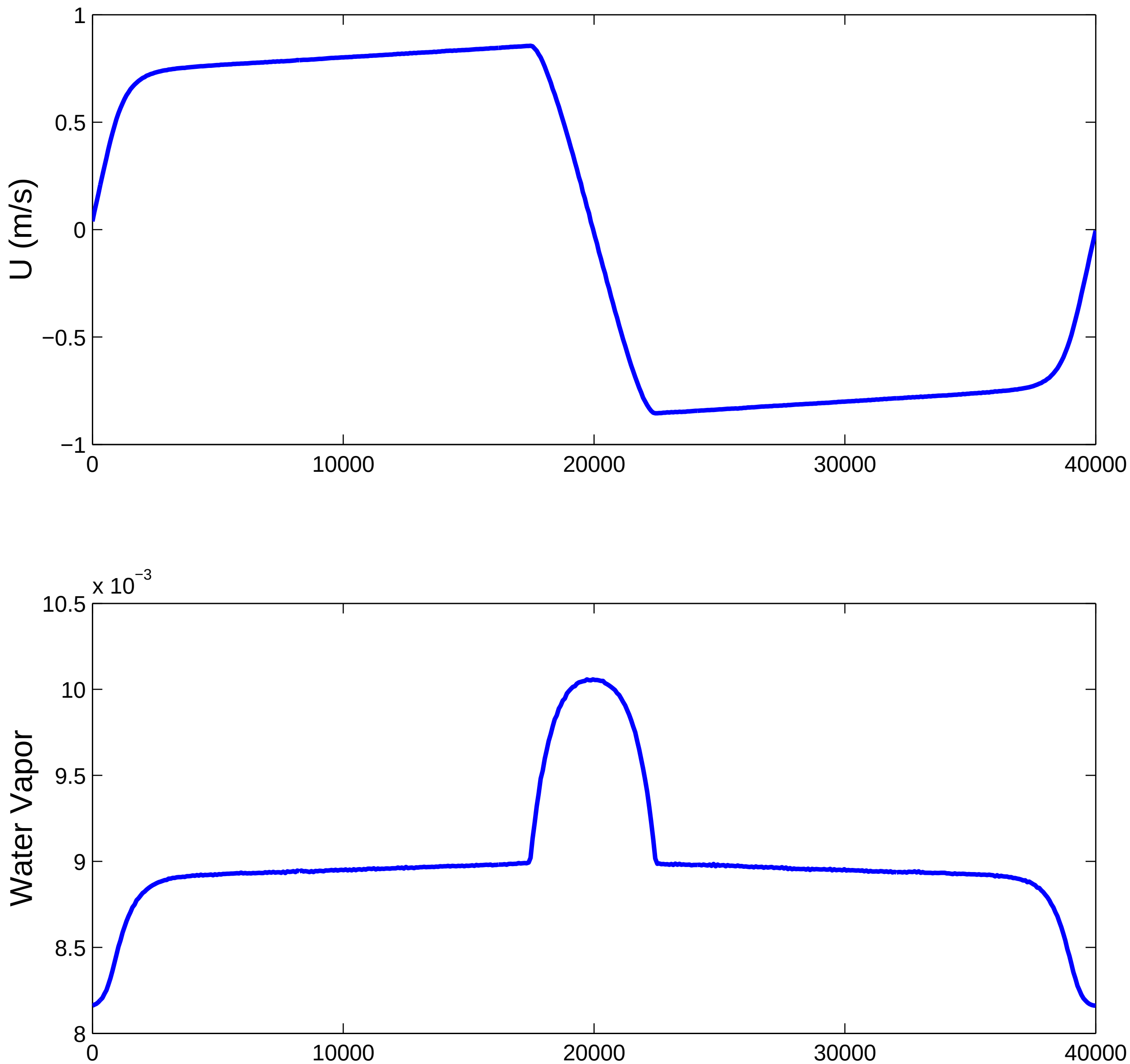
Interac. pot. βU_0	τ_I (days)	$\bar{u}_-$	$\bar{u}_+$	u'_-	(m s^{-1}) u'_+	$\bar{\sigma}_c$	Std. Dev.
1	5	−.856	.855	−.207	.214	4.55E−04	3.00E−04
1	20	−.855	.856	−.214	.208	4.55E−04	2.96E−04
.01	5	−1.047	1.046	−.508	.486	9.96E−04	3.18E−04
.01	20	−1.048	1.040	−.804	.676	9.96E−04	3.15E−04
−.01	5	−1.047	1.049	−.603	.572	1.00E−03	3.15E−04
−.01	10	−.923	.920	−4.497	4.429	1.00E−03	3.14E−04
−.1	5	−.816	.867	−4.820	4.727	1.04E−03	3.11E−04
−.1	10	−.824	.877	−4.861	4.737	1.04E−03	3.12E−04

Table 2: Same as in Table 1, except for $A_0 = 1$.

Interac. pot. βU_0	τ_I (days)	$\bar{u}_-$	$\bar{u}_+$	u'_-	u'_+ (m s^{-1})	$\bar{\sigma}_c$	Std. Dev.
1	5	−1.417	1.417	−.536	.436	4.56E−04	3.00E−04
1	20	−1.415	1.417	−.330	.546	4.56E−04	3.00E−04
.01	5	−1.692	1.691	−1.196	1.603	9.96E−04	3.17E−04
.01	20	−1.692	1.691	−1.180	1.266	9.96E−04	3.17E−04
−.01	5	−1.693	1.693	−1.421	1.470	1.00E−03	3.15E−04
−.01	10	−1.693	1.693	−1.277	1.243	1.00E−03	3.16E−04
−.1	5	−1.700	1.699	−.990	1.092	1.04E−03	3.10E−04
−.1	10	−1.700	1.700	−1.447	1.269	1.04E−03	3.07E−04

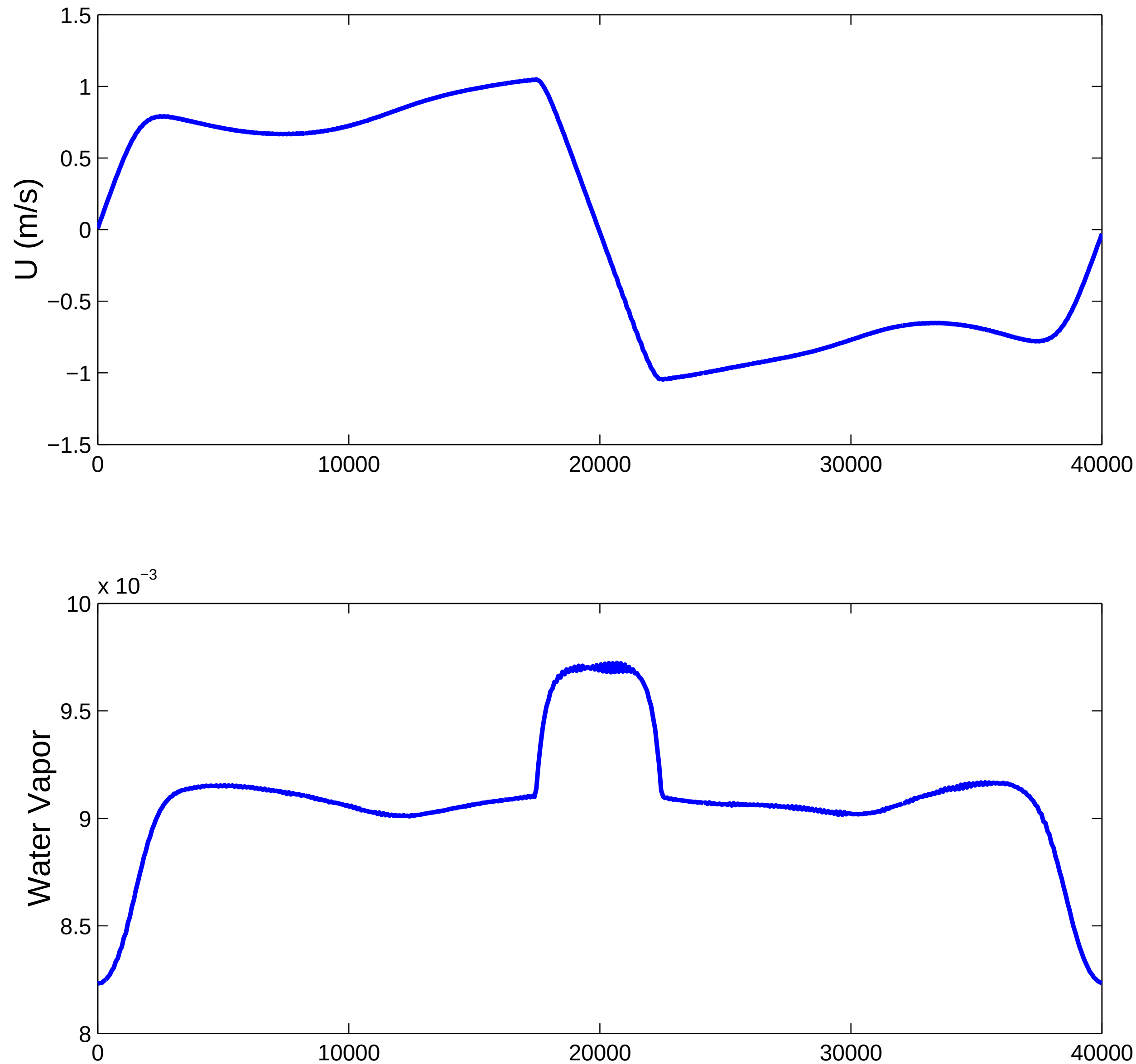
Typical case: $\beta U_0 = 1, \tau_I = 20$ days, $A_0 = .5$



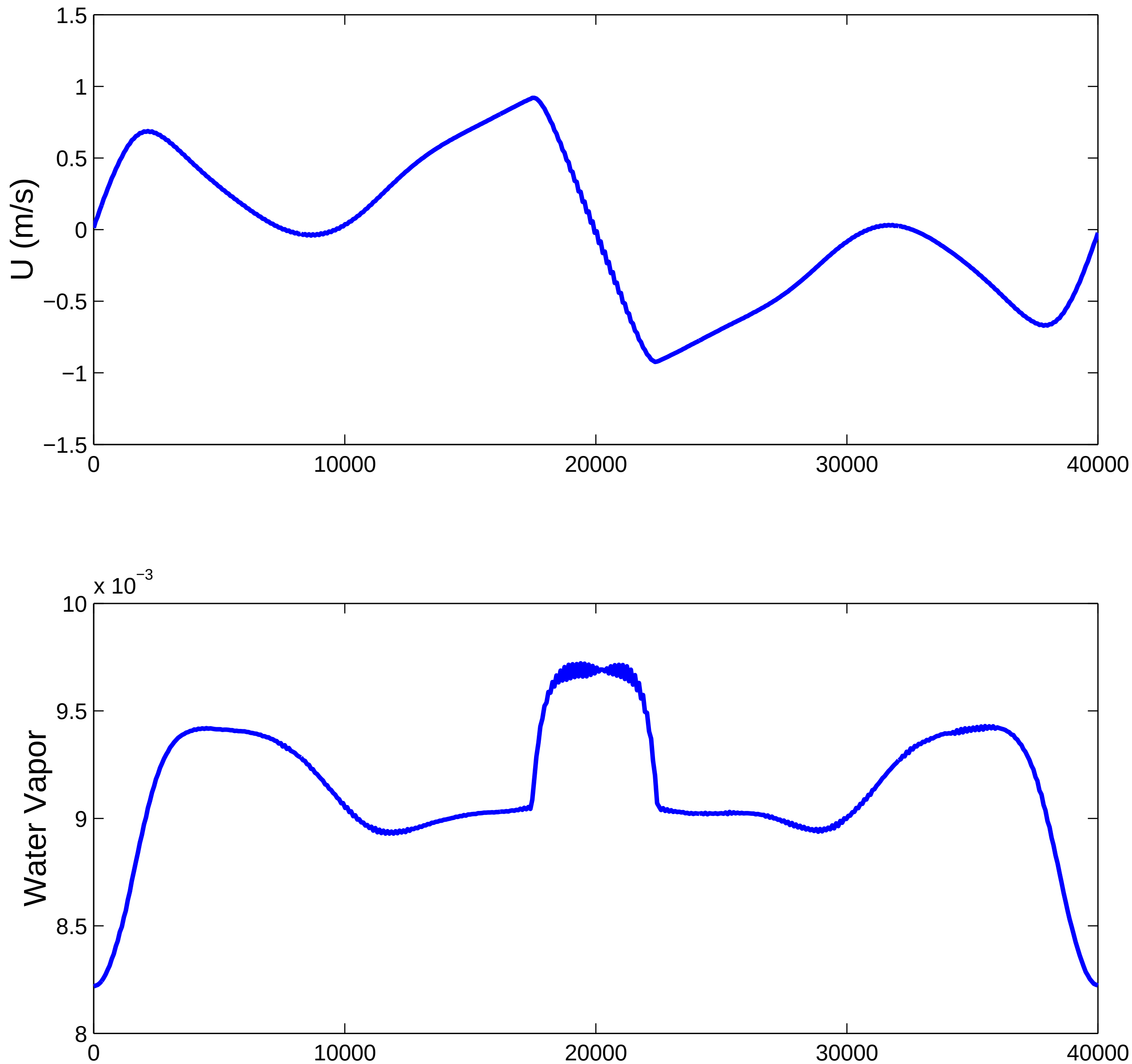


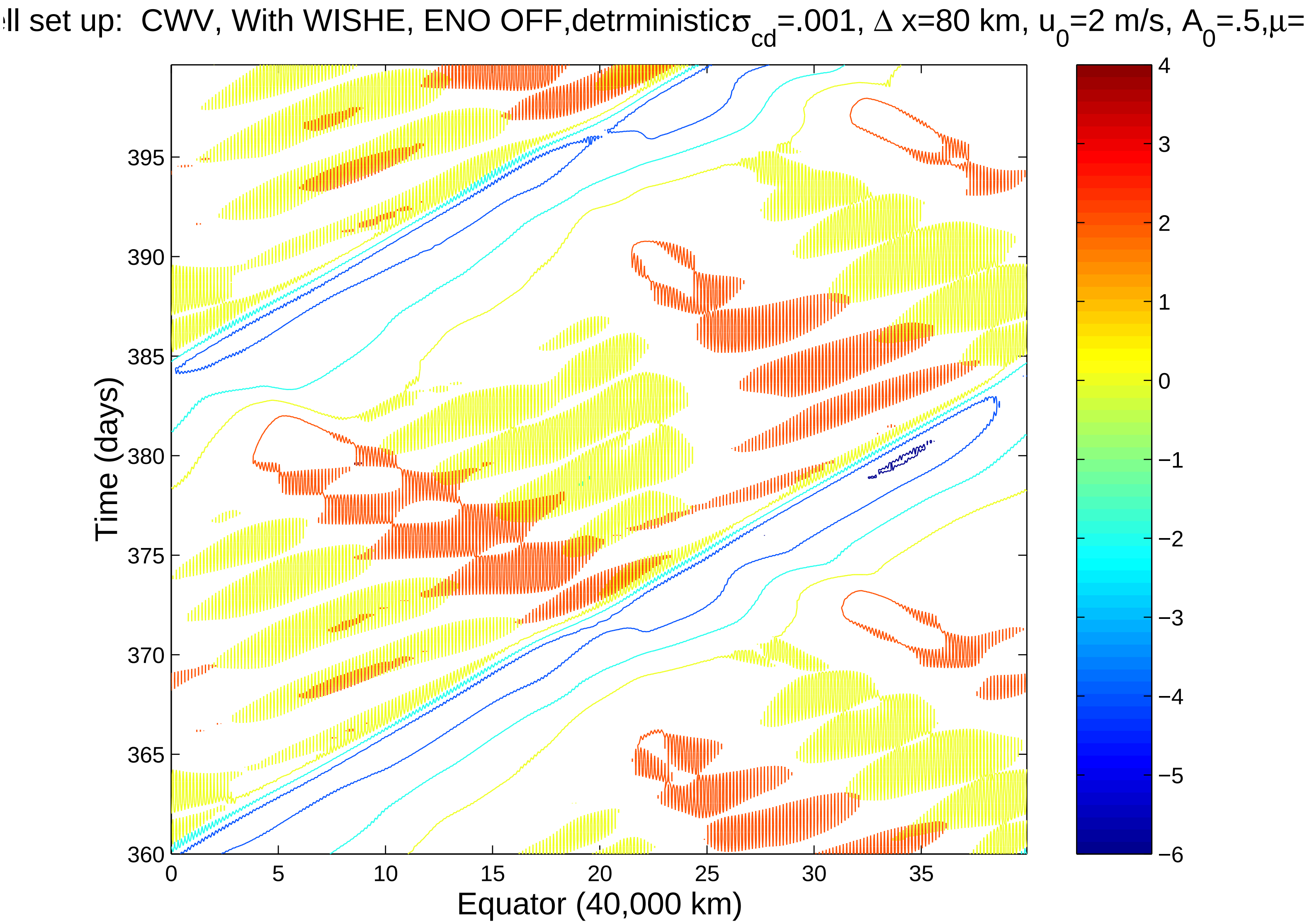
U (m/s)
1
0.5
0
−0.5
−1
0
10000
20000
30000
40000
Water Vapor
x 10^{-3}
10.5
10
9.5
9
8.5
8
0
10000
20000
30000
40000

PAC favoring but small CIN time: $\beta U_0 = -.01$, $\tau_I = 5$ days, $A_0 = .5$

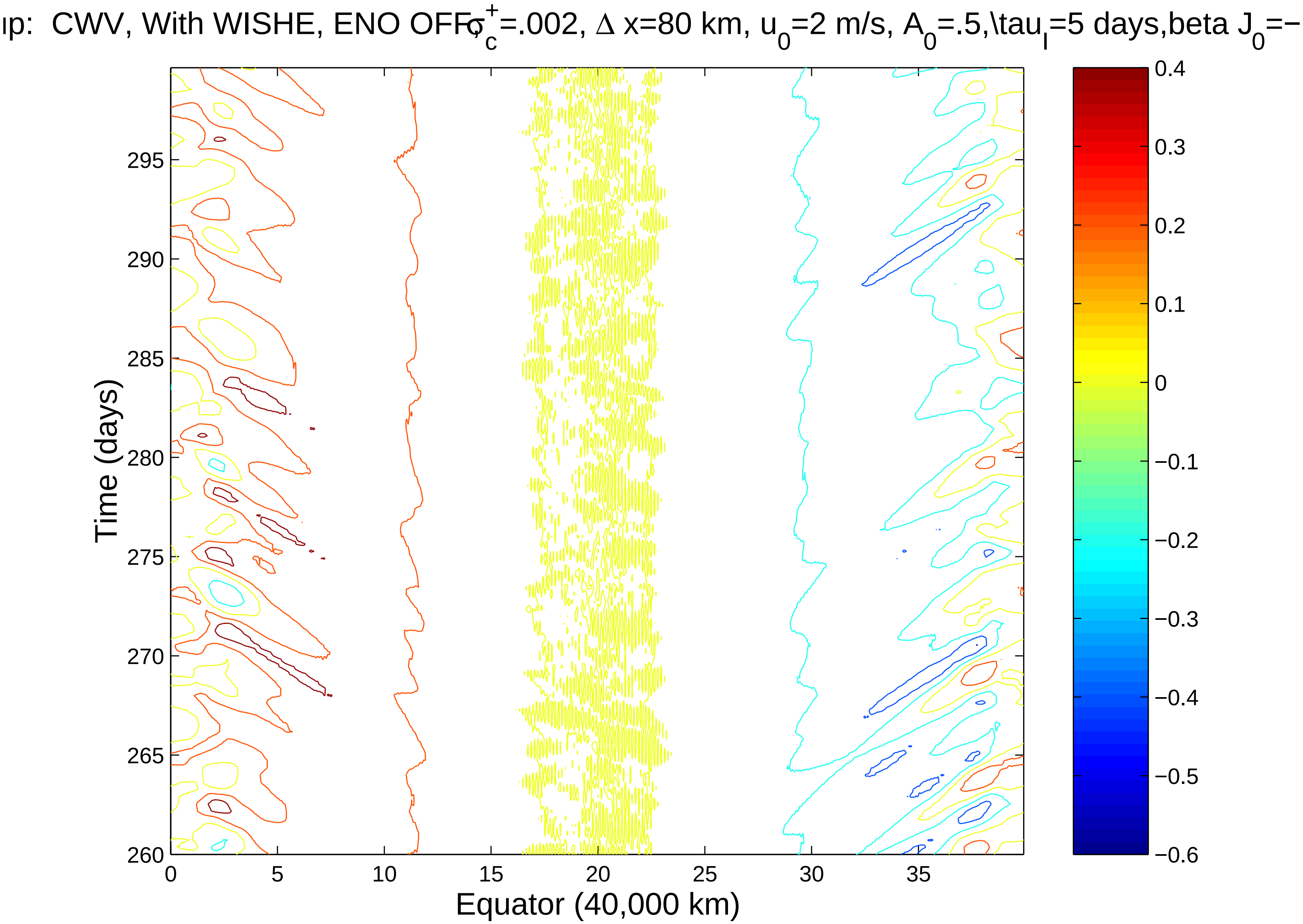


PAC favoring with larger CIN time: $\beta U_0 = -.01$, $\tau_I = 10$ days, $A_0 = .5$

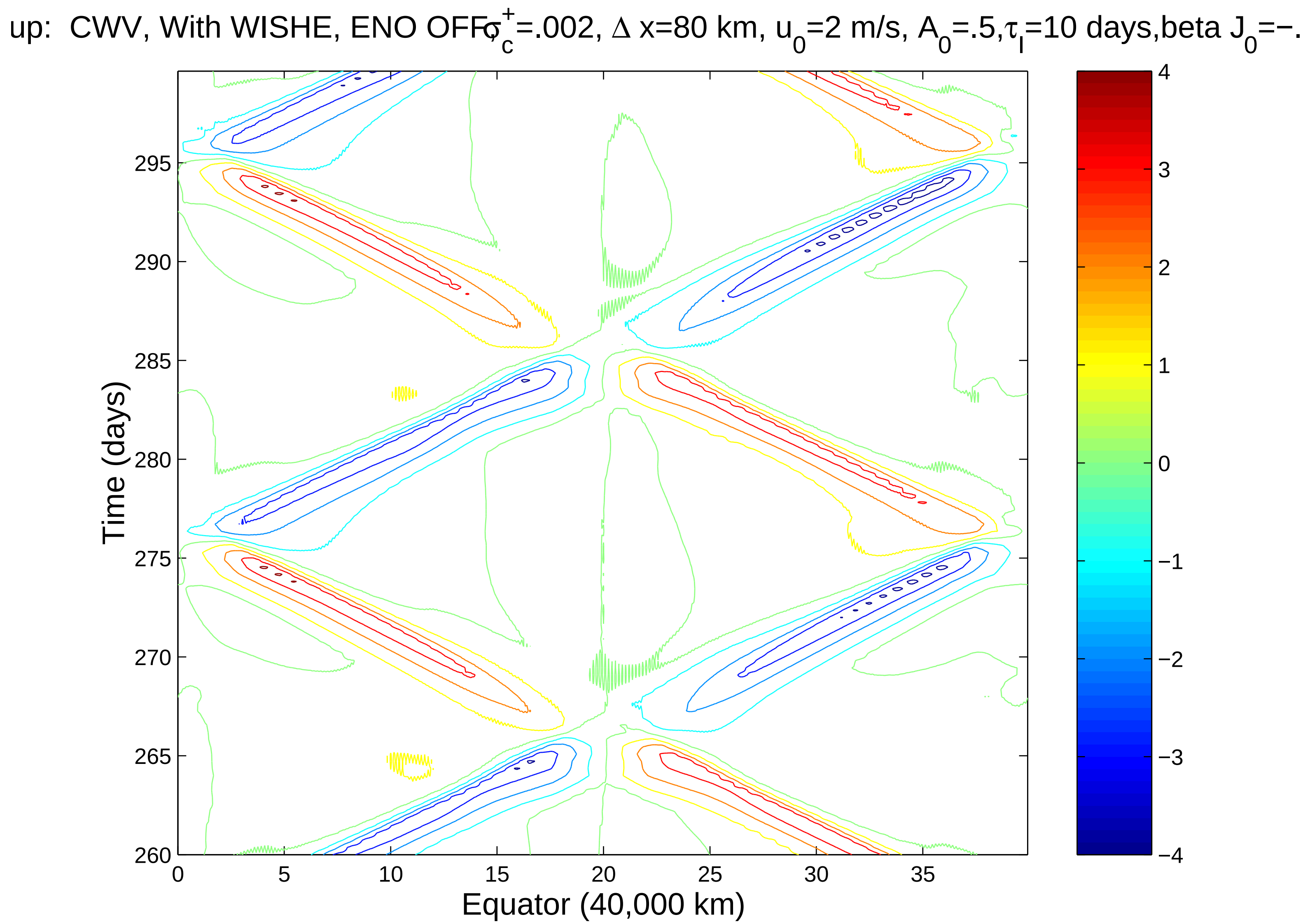




Constant area fraction: $\sigma_c = .001$, $A_0 = .5$



$$\beta U_0 = -.01, \tau_I = 5 \text{ days}, \ A_0 = .5 \ (\bar{\sigma}_c = .001)$$



$$\beta U_0 = -.01, \tau_I = 10 \text{ days}, \ A_0 = .5 \ (\bar{\sigma}_c = .001)$$

- Strong forcing ($A_0 = 1$): Walker cell climatology + weak gravity waves (except for deterministic case)
- Moderate forcing ($A_0 = .5$):
  - Deterministic case: one large scale wave propagating around globe, no Walker cell
  - CIN favoring interaction potential ($\beta U_0 > 0$): Walker cell forms + moderate small scale squall line-like waves
  - As interaction potential decreases strength and length scale of convective waves increases
  - Also sensitive to CIN charac. time (τ_I)
  - PAC favoring int. pot. ($\beta U_0 < 0$): Walker cell destroyed and two symmetric waves propagating far from source
- Stochastic (noise) creates and maintains Walker cell and
- Affects wavelength and strength of waves

Mean field/Stochastic RCE's

- Large scale variable equations

$$\frac{\partial v}{\partial t} - \frac{\partial \theta}{\partial x} = -\frac{C_d(u_0)}{h_b} v - \frac{1}{\tau_D} v$$

$$\frac{\partial \theta}{\partial t} - \frac{\partial v}{\partial x} = Q_c - Q_R^0 - \frac{1}{\tau_R} \theta$$

$$\frac{\partial \theta_{eb}}{\partial t} = -\frac{1}{\tau_{eb}} D(\theta_{eb} - \theta_{em}) + \left[\frac{1}{\tau_e} + \delta_{wishe} \frac{C_\theta}{h_b}(|v|)\right](\theta_{eb}^* - \theta_{eb})$$

$$\frac{\partial \theta_{em}}{\partial t} = \frac{1}{\tau_{em}} D(\theta_{eb} - \theta_{em}) - Q_R^0 - \frac{1}{\tau_R} \theta \qquad (1)$$

$$Q_c = (1 - \sigma_I)\sqrt{R(\theta_{eb} - \gamma\theta)^+}.$$

$$D = \epsilon_p \left(Q_c + \tfrac{\partial v}{\partial x}\right)^+ + (1 - \epsilon_p) Q_c$$

- Dominant time scales: $\tau_e = 8$ hours, $\tau_{eb} = 45$ mn, $\tau_{em} = 12$ hr.

- Stochastic Birth-death process

$$\sigma_I = \eta_t/q; \quad 0 \leq \eta_t \leq q; \quad q = 10\text{–}100$$

$$Prob\{\eta_{t+\Delta t} = k+1/\eta_t = k\} = C_a \Delta t + O(\Delta t)$$

$$Prob\{\eta_{t+\Delta t} = k-1/\eta_t = k\} = C_d \Delta t + O(\Delta t)$$

$$C_a(t) = \frac{1}{\tau_I}(q - \eta_t) \quad C_d(t) = \frac{1}{\tau_I}\eta_t \exp(-\bar{J}_0(\eta_t - 1) - h) \qquad (2)$$

where $\bar{J}_0 = 2\beta J_0/(q-1)$ and $h = -\tilde{\gamma}\theta_{eb}$

- Mean field Equation

$$\frac{\partial \sigma_I}{\partial t} = \frac{1}{\tau_I}(1 - \sigma_I) - \frac{1}{\tau_I}\sigma_I \exp(-\beta J_0 \sigma_I - h)$$

- External potential $h = -\tilde{\gamma}\theta_{eb}$
- Time scale: $\tau_I = 2$ hours

Mean field RCE's

$$\bar{v} = 0,$$

$$(\bar{\theta}, \bar{\theta}_{eb}, \bar{\theta}_{em}, \bar{\sigma}_I) \text{ solve}$$

$$(1-\bar{\sigma}_I)\sqrt{R(\bar{\theta}_{eb}-\gamma\bar{\theta})} - Q_R^0 - \frac{1}{\tau_R}\bar{\theta} = 0$$

$$-\frac{1}{\tau_{eb}}(1-\bar{\sigma}_I)\sqrt{R(\bar{\theta}_{eb}-\gamma\bar{\theta})}(\bar{\theta}_{eb}-\bar{\theta}_{em}) + \frac{1}{\tau_e}(\theta^*_{eb}-\bar{\theta}_{eb}) = 0$$

$$\frac{1}{\tau_{em}}(1-\bar{\sigma}_I)\sqrt{R(\bar{\theta}_{eb}-\gamma\bar{\theta})}(\bar{\theta}_{eb}-\bar{\theta}_{em}) - Q_R^0 - \frac{1}{\tau_R}\bar{\theta} = 0$$

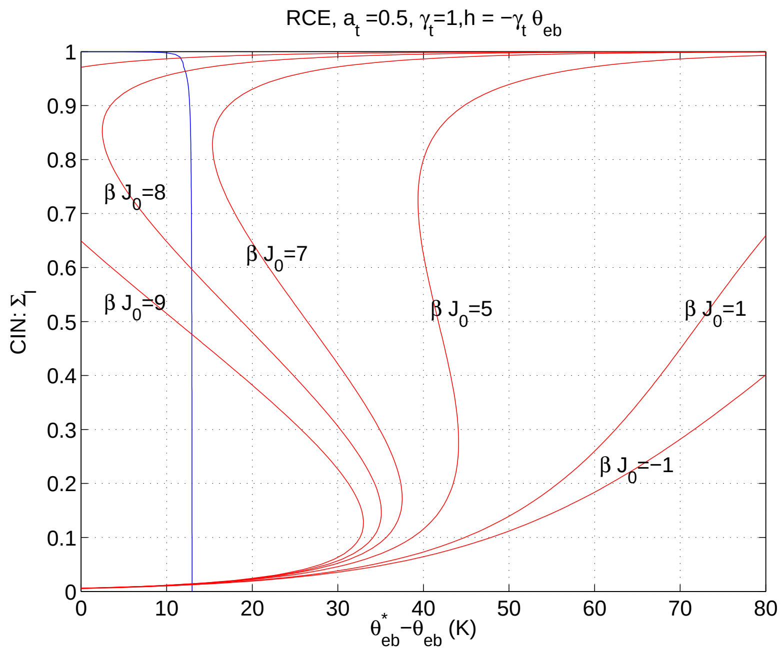
$$\frac{1}{\tau_I}(1-\bar{\sigma}_I) - \frac{1}{\tau_I}\bar{\sigma}_I \exp(-\beta J_0 \bar{\sigma}_I + \tilde{\gamma}\bar{\theta}_{eb}) = 0$$

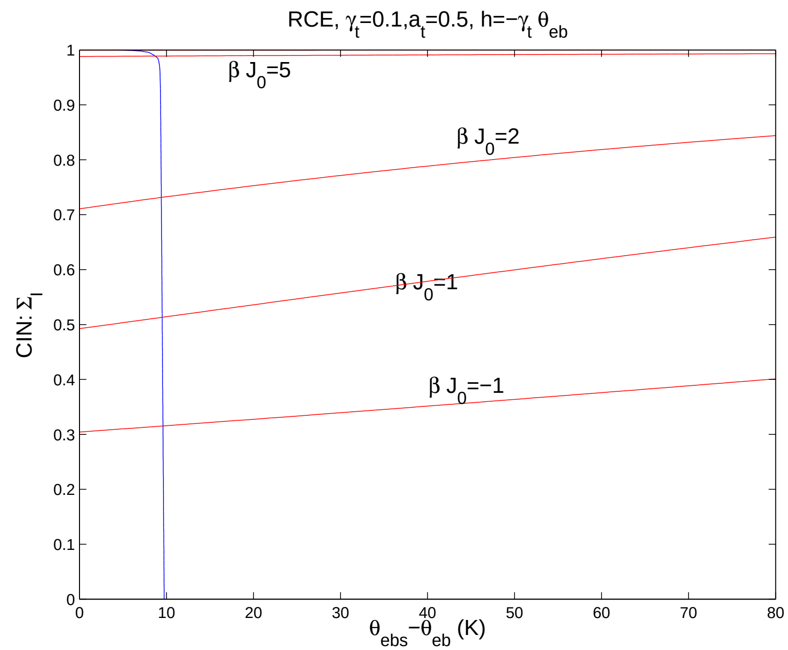
$$\theta = \frac{\tau_R}{\tau_e}\frac{\tau_{eb}}{\tau_{em}}(\theta_{eb}^* - \theta_{eb}) - \tau_R Q_R^0$$

$$F(\theta_{eb}, \sigma_I) = (1 - \sigma_I)\sqrt{R(\theta_{eb} - \gamma\theta)^+} - \frac{\tau_{eb}}{\tau_{em}}\frac{1}{\tau_e}(\theta_{eb}^* - \theta_{eb}) = 0$$

$$G(\sigma_I, \theta_{eb}) = 1 - \sigma_I - \sigma_I \exp(-\beta J_0 \sigma_I + \tilde{\gamma}\theta_{eb}) = 0$$

$F(\theta_{eb}, \sigma_I) = 0$ and $G(\sigma_I, \theta_{eb}) = 0$





$\tilde{\gamma} = 1$

Single (PAC) RCE for $\beta J_0 \leq 7$

Three RCE's $\beta J_0 \geq 7$

$\tilde{\gamma} = 0.1$

Single RCE

PAC ——-> CIN, $\beta J_0 \nearrow$

Stability of mean-field RCE's.

NO WISHE:

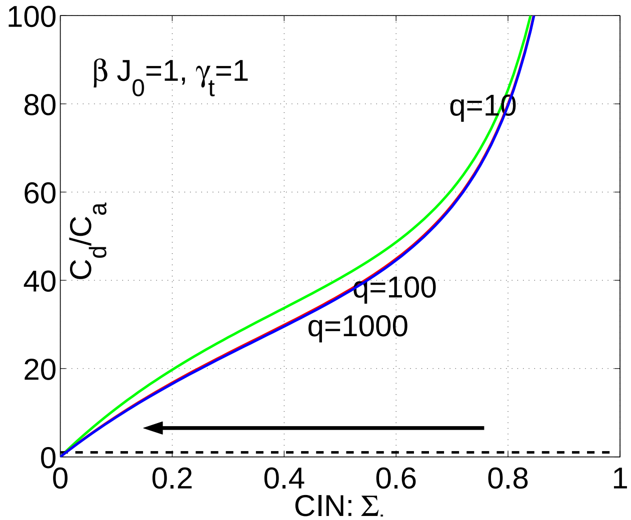
- Single RCEs are stable
- Multiple RCEs: PAC and CIN stable
- 3rd RCE: σ_I-mode is unstable at all wavenumbers

With WISHE: Nonlinear growth of convectively coupled waves (WISHE waves).

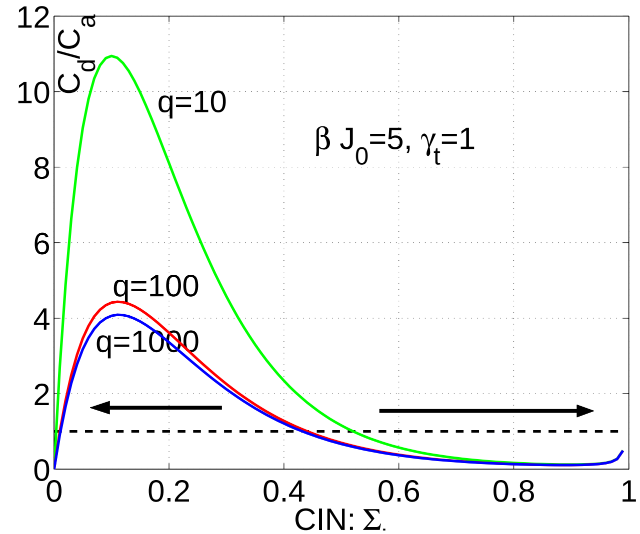
Stochastic dynamics of RCE's:

Mainly three regimes with different levels of intermittency

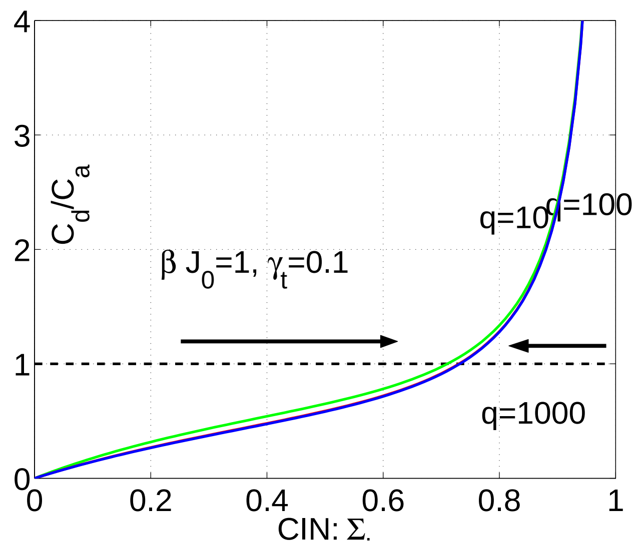
PAC RCE—Rapid but weak oscillations



Multiple RCEs—-High intermittency



(mostly CIN) RCE — Large variance



Single column model

$$\frac{d\theta}{dt} = (1-\sigma_I)\sqrt{R(\theta_{eb}-\gamma\theta)^+} - Q_R^0 - \frac{1}{\tau_R}\theta$$

$$\frac{d\theta_{eb}}{dt} = -\frac{1}{\tau_{eb}}(1-\sigma_I)\sqrt{R(\theta_{eb}-\gamma\theta)^+}(\theta_{eb}-\theta_{em}) + \frac{1}{\tau_e}(\theta_{eb}^*-\theta_{em})$$

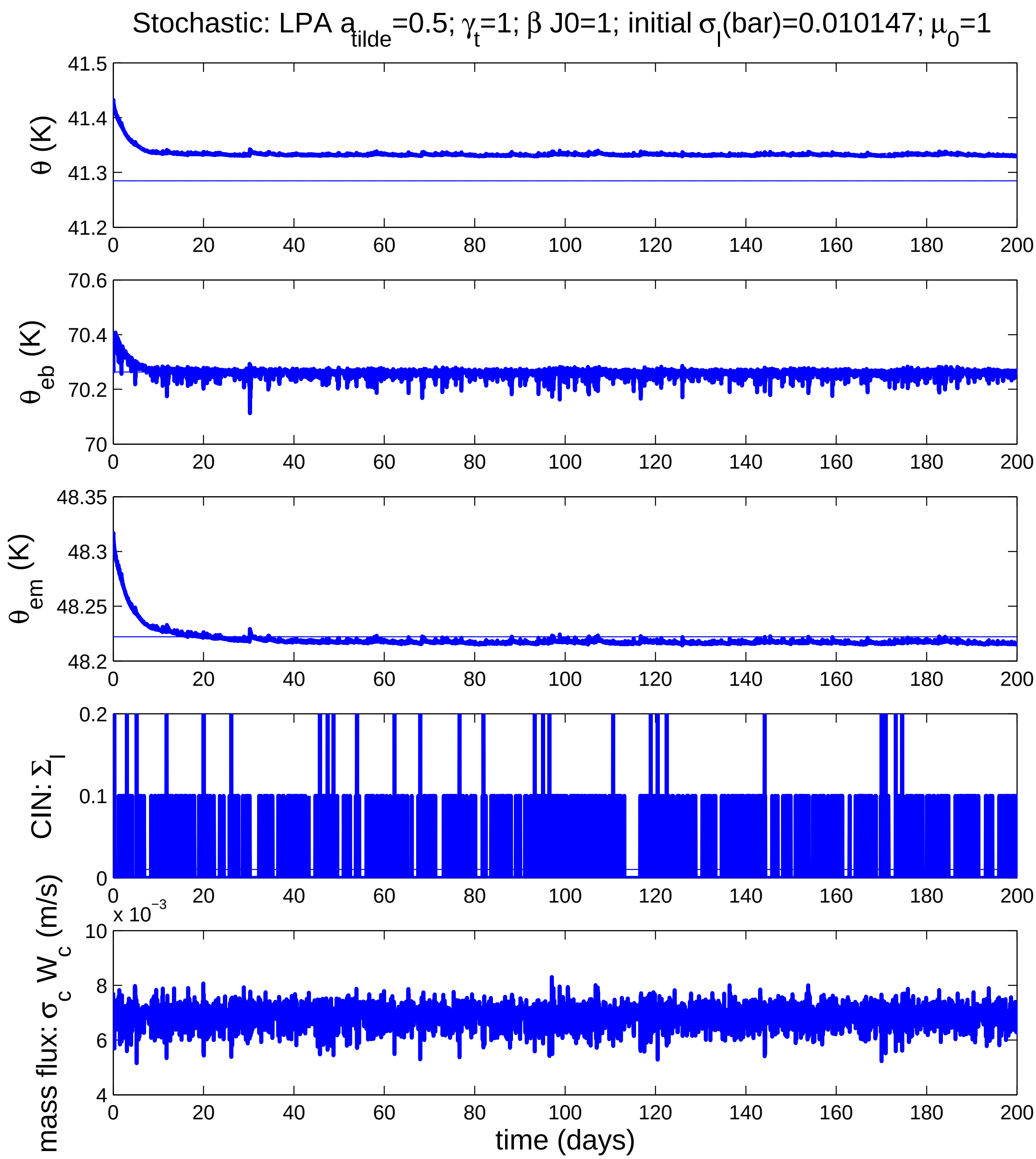
$$\frac{d\theta_{em}}{dt} = \frac{1}{\tau_{em}}(1-\sigma_I)\sqrt{R(\theta_{eb}-\gamma\theta)^+}(\theta_{eb}-\theta_{em}) - Q_R^0 - \frac{1}{\tau_R}\theta$$

Plus

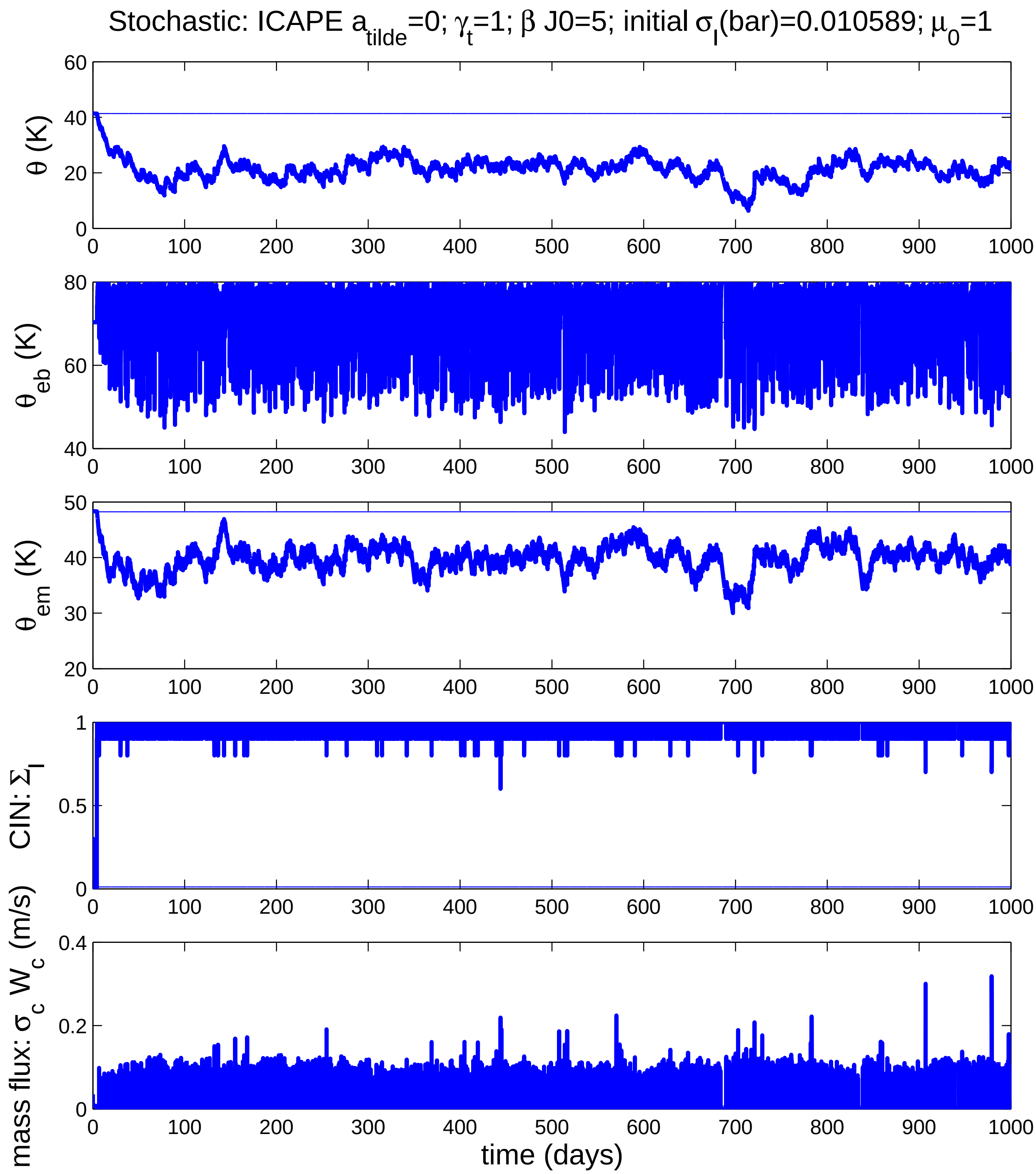
Stochastic birth-death process for σ_I.

Run in three different regimes.

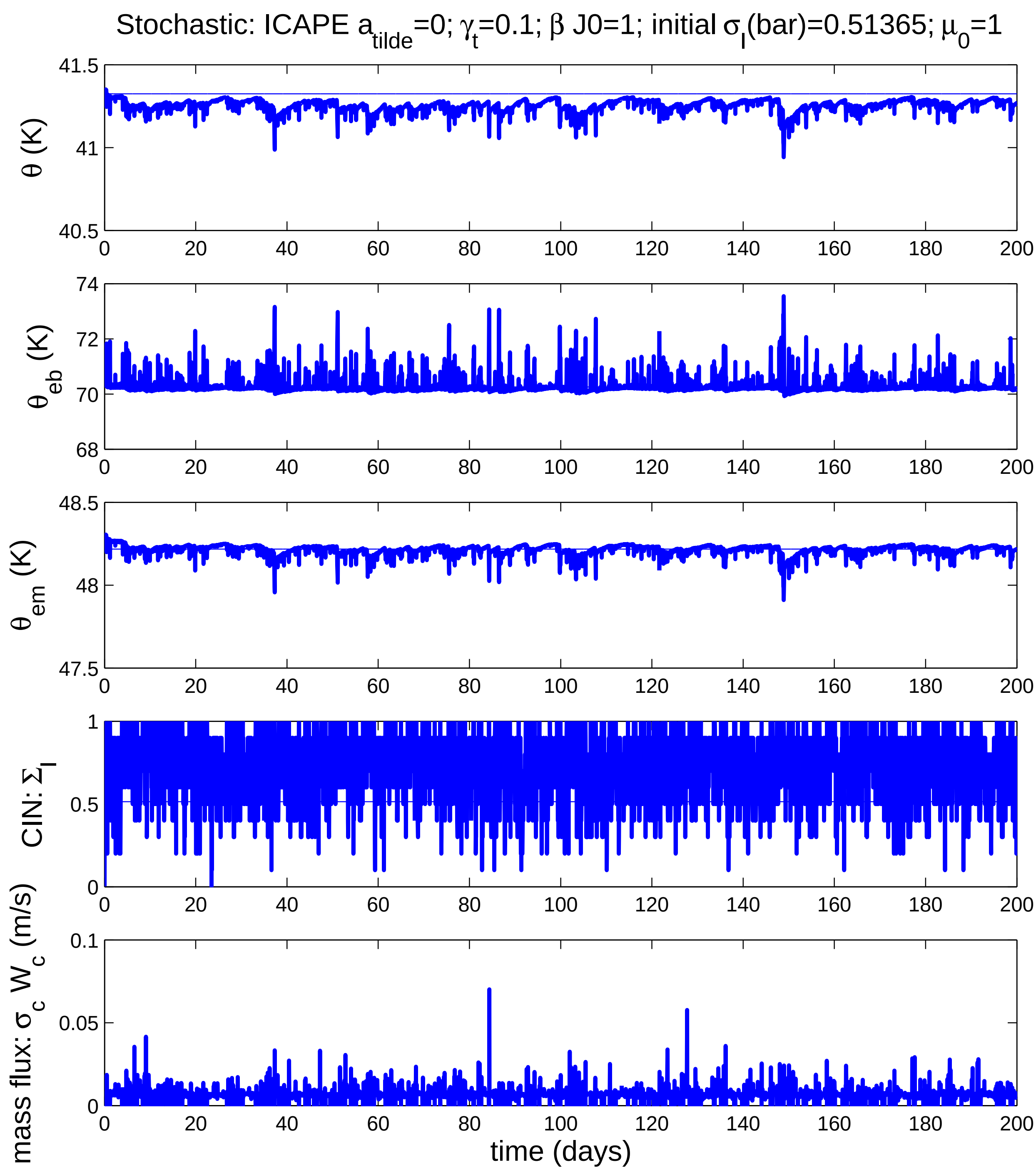
PAC RCE regime: Rapid but weak oscillations



Multiple equilibria regime: Highly intermittent, large amplitude oscillations



Mostly CIN RCE regime: Rapid and large amplitude oscillations, intermittent



Full 2D simulations

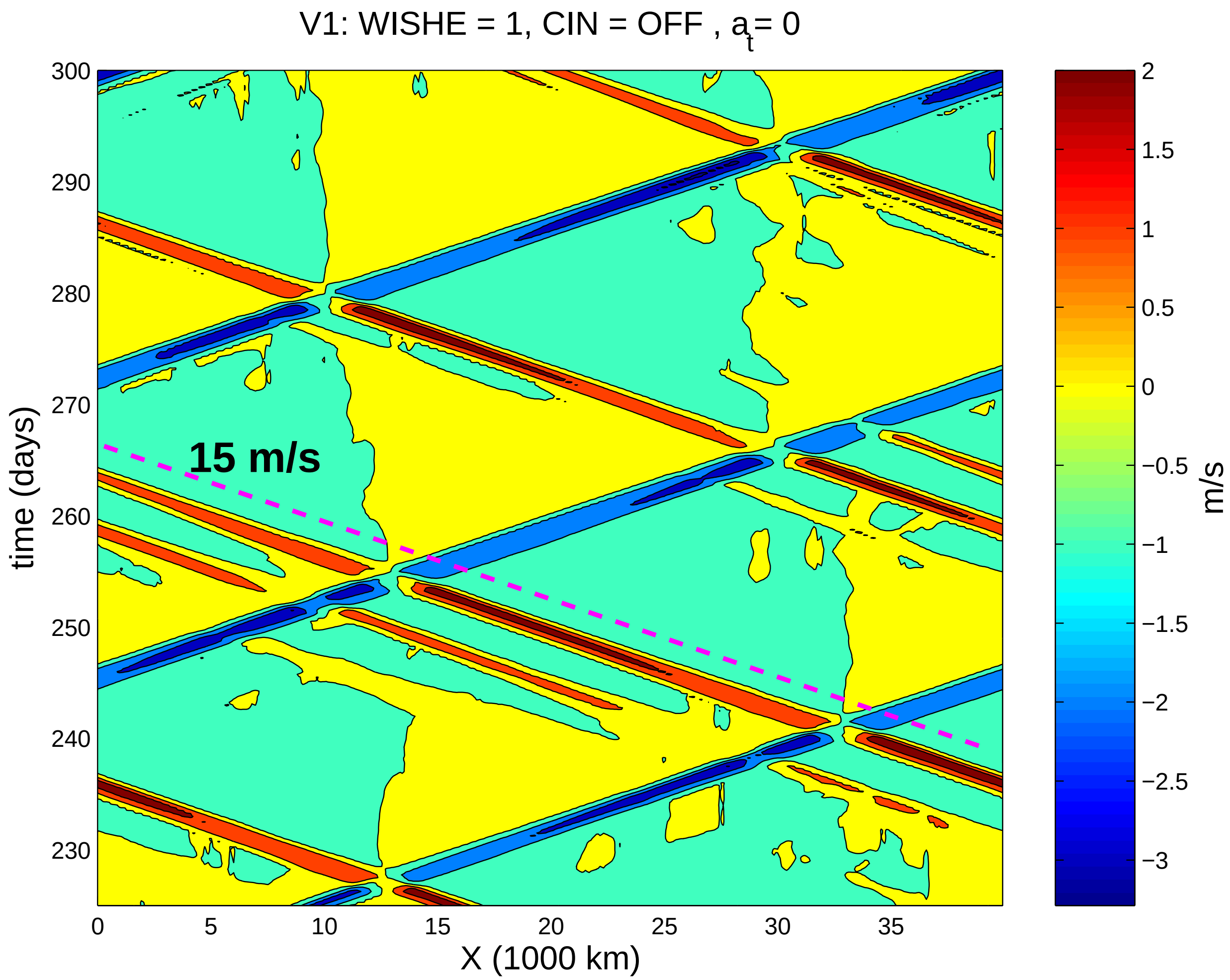
Consider only two regimes.

- Multiple equilibria regime, $\beta J_0 = 5, \tilde{\gamma} = 1$
- Mostly CIN RCE regime, $\beta J_0 = 1, \tilde{\gamma} = 0.1$
- Switch on and off, both WISHE and CIN

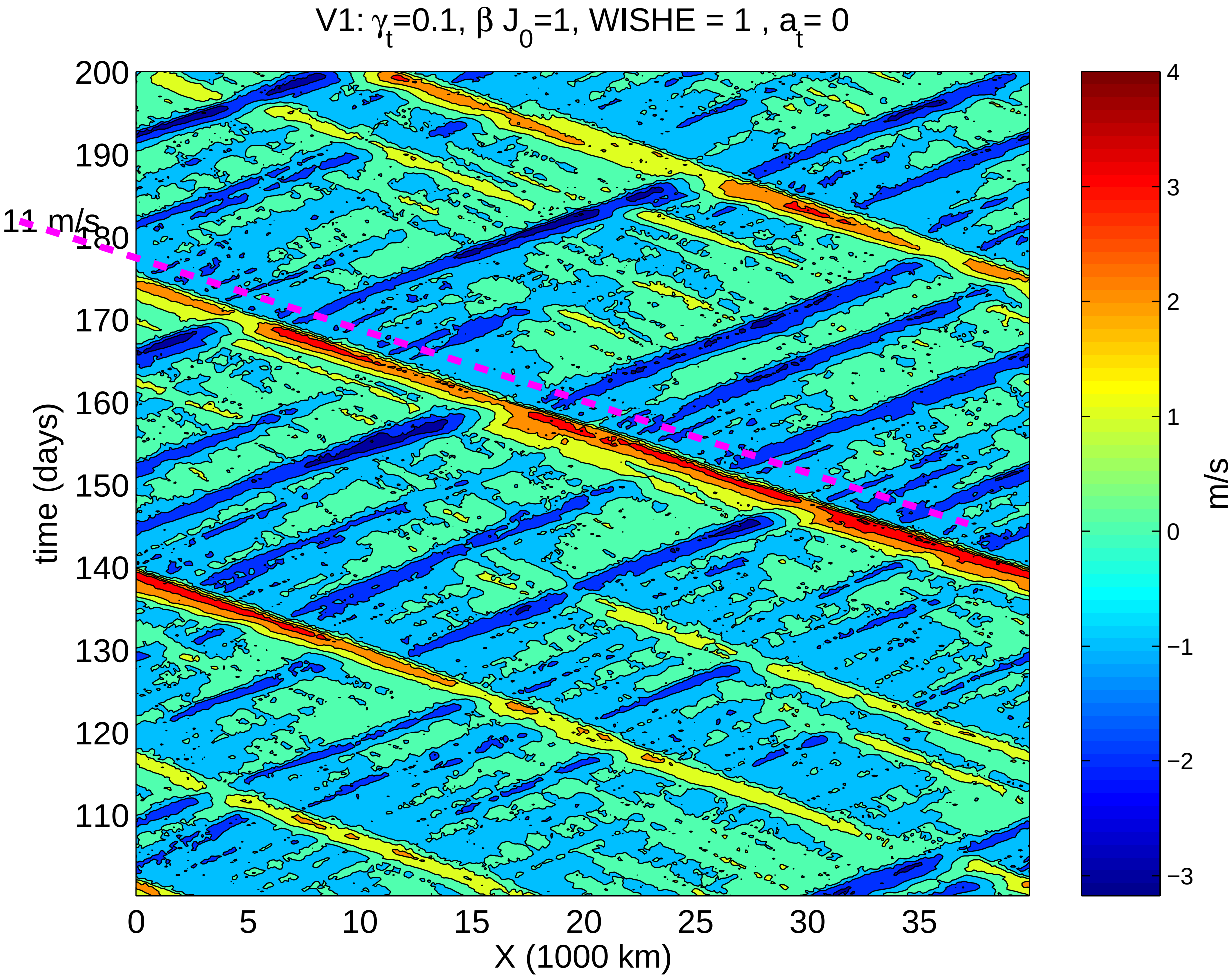
Why use WISHE?

- Amplification and propagation of convectively coupled waves. Otherwise the ICAPE model with single baroclinic mode is linearly and nonlinearly stable.
- How realistic? Not sure.
- Better convective instability mechanisms?
- Stratiform (Mapes 2000, Majda and Shefter, 2001), Multicloud and models (Khouider and Majda, 2006).

WISHE Waves. CIN is OFF (frozen)

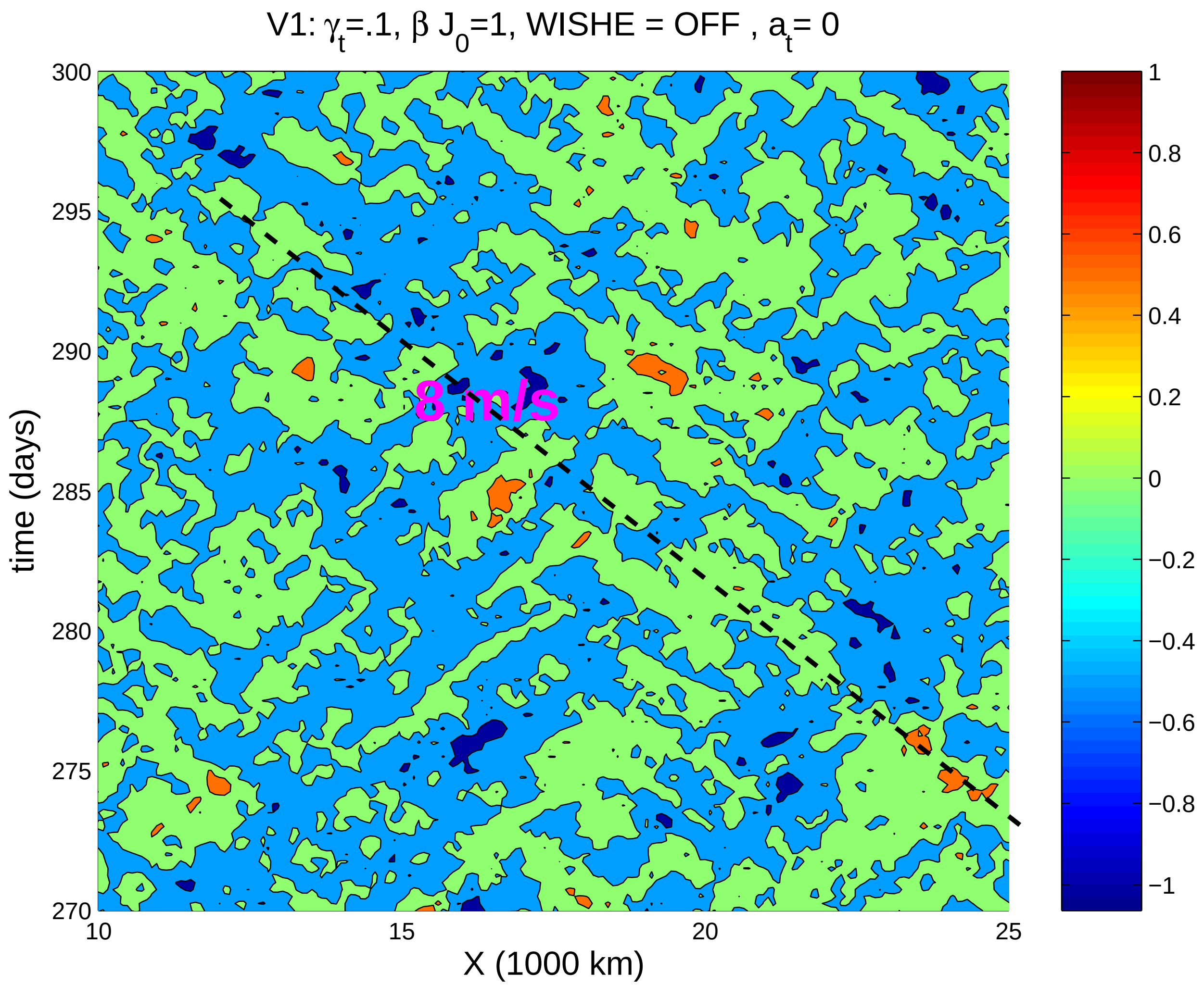


CIN is ON. Mostly CIN RCE regime, $\beta J_0 = 1, \tilde{\gamma} = 0.1$.



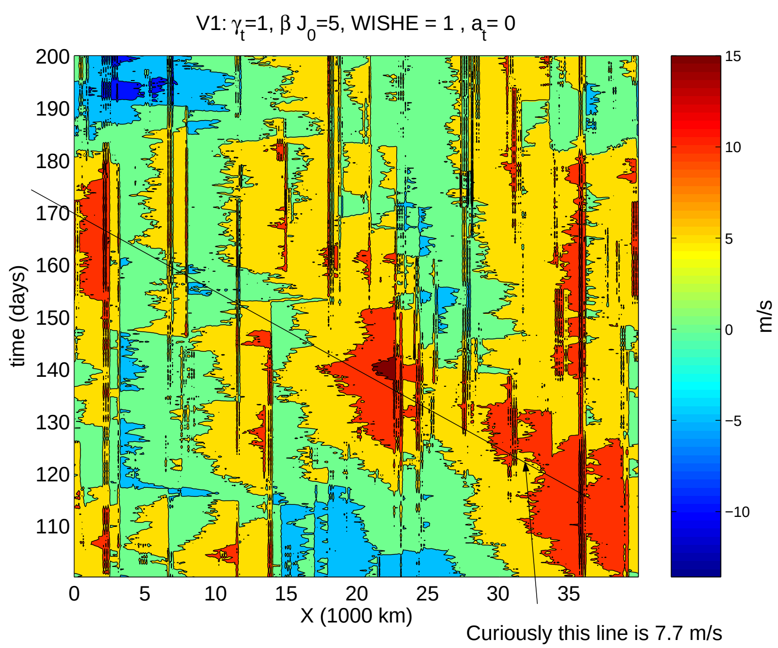
Reduced phase speed.

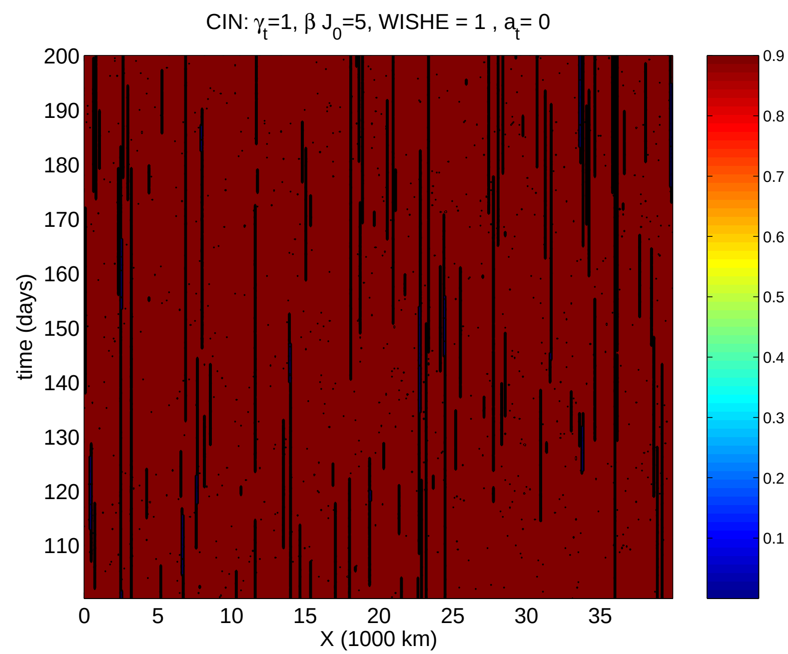
WISHE OFF, Mostly CIN RCE regime, $\beta J_0 = 1, \tilde{\gamma} = 0.1$.



Intermittent bursts of convection. Phase reduced even more 8 m/s.

Multiple RCE, $\beta J_0 = 5, \tilde{\gamma} = 1$

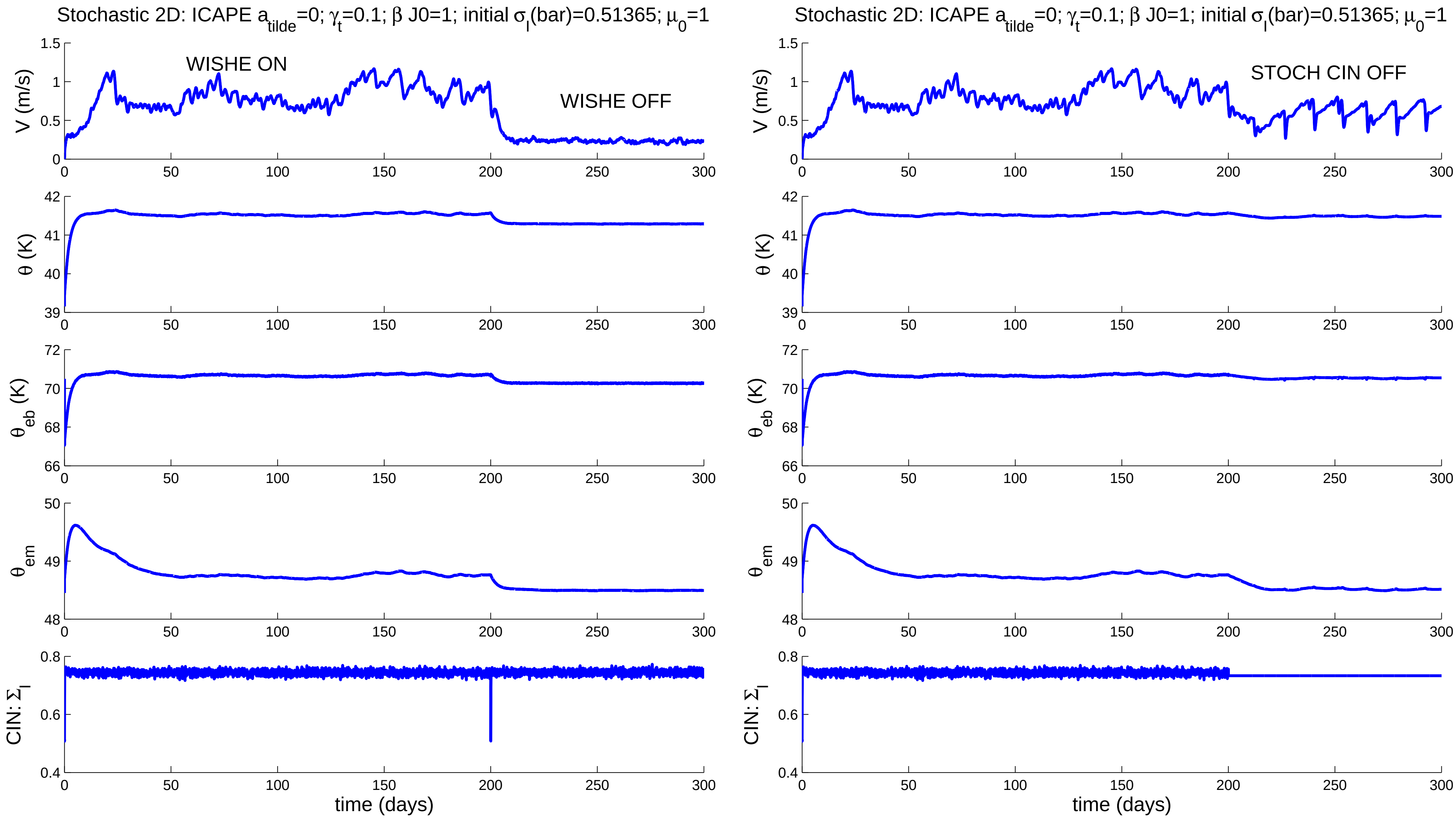




Highly intermittent strong bursts.

Is this the MJO?

RMS Time series



Concluding Remarks

- Coarse graining of systematic microscopic lattice model
- Birth-death process for CIN within a coarse (GCM) grid cell
- Toy GCM: Effects both tropical waves and climate
- Mean field RCEs: PAC, CIN, multiple equilibria, different stability features and Stochastic dynamics, 3 regimes
- Multiple RCEs, statistically CIN state, intrmittent long excurtions to PAC RCE
- CIN (dominated, single) RCE regimes: Highly intermittent with strong oscillations
- Bursts of strong convective events in an environment which is otherwise dominated by CIN
- Effect on WISHE waves: intermittency, reduced phase speed, increased wave amplitude.

Stochastic multicloud model

- based on multivariable markov chain
- three cloud types, interacting with each other and with large scale/resolved variables.
- will be presented at next-week's workshop.