Small-Scale and Short-Term Variability in the Ocean: Use of its Statistics for Error Modeling

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in collaboration with:

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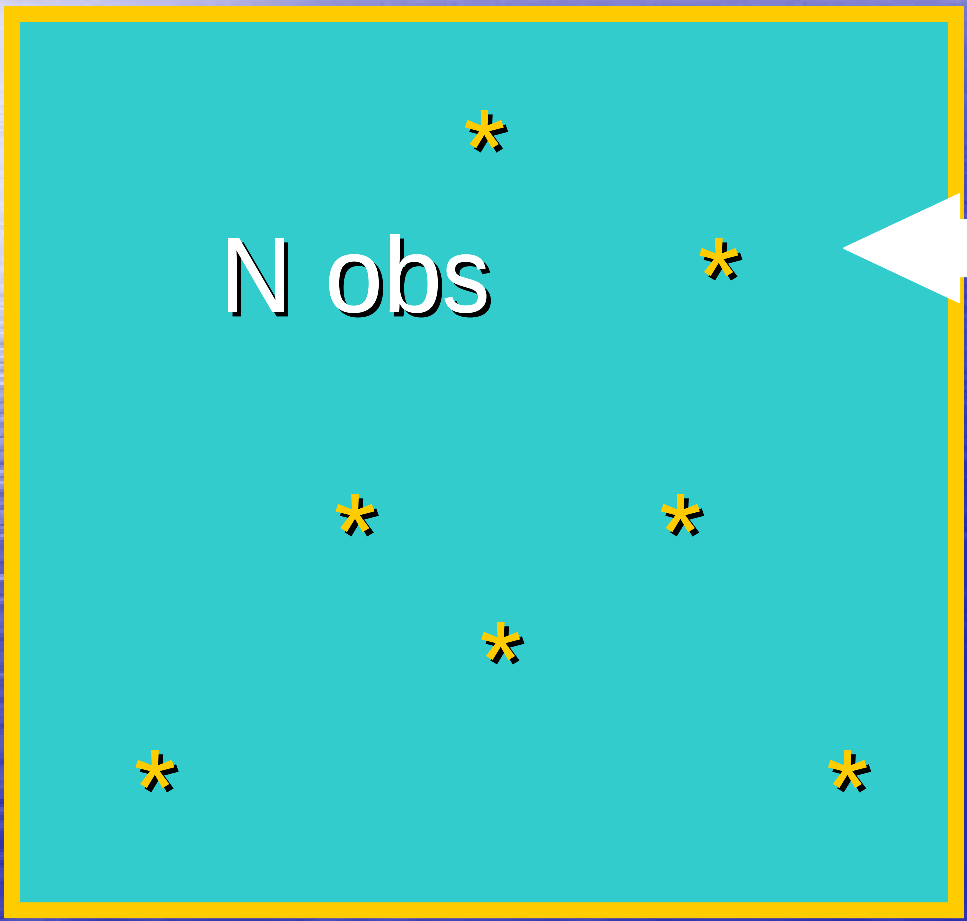
What is the error in the binned obs mean (as estimates of the “true” bin area average)?

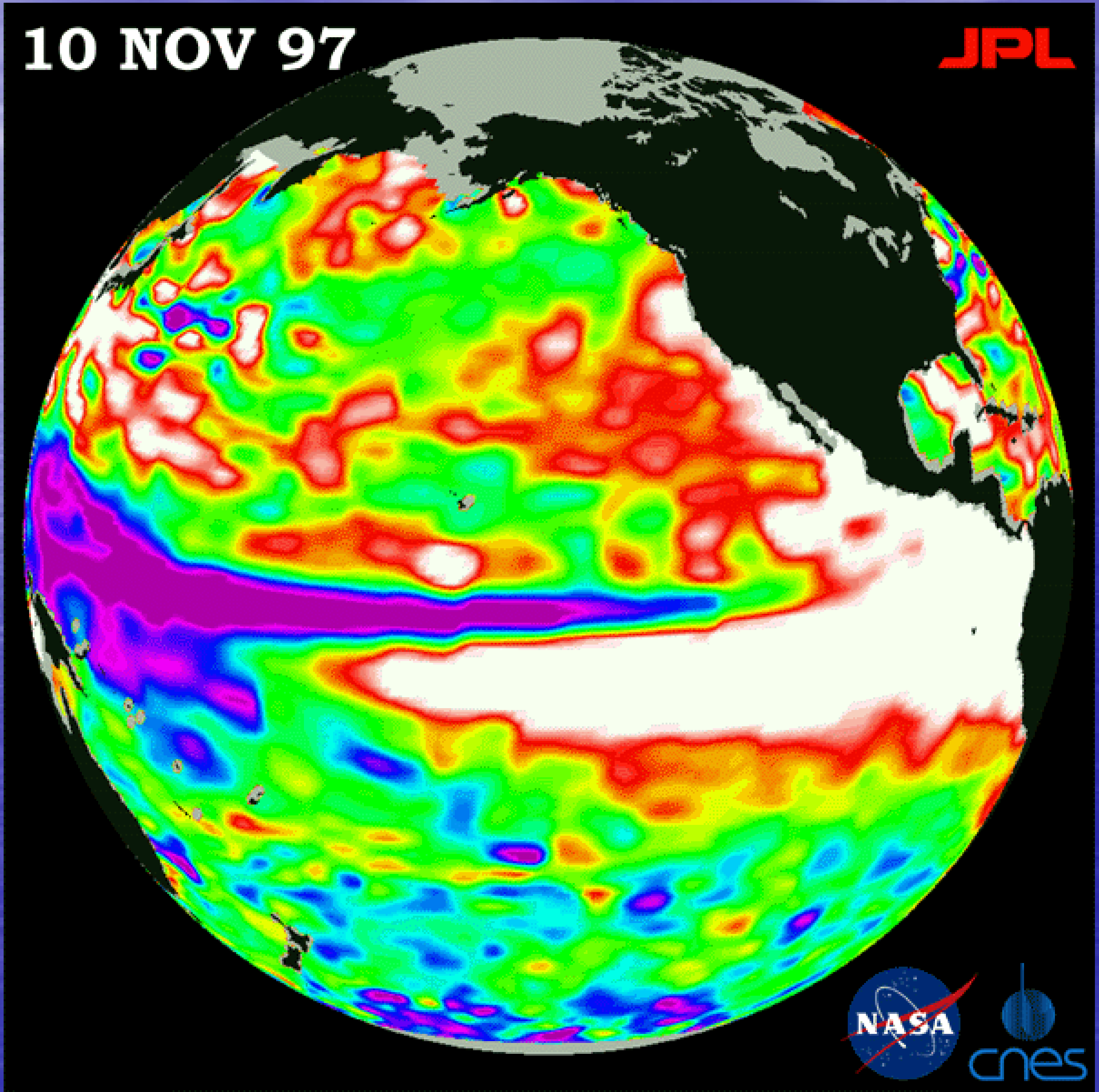


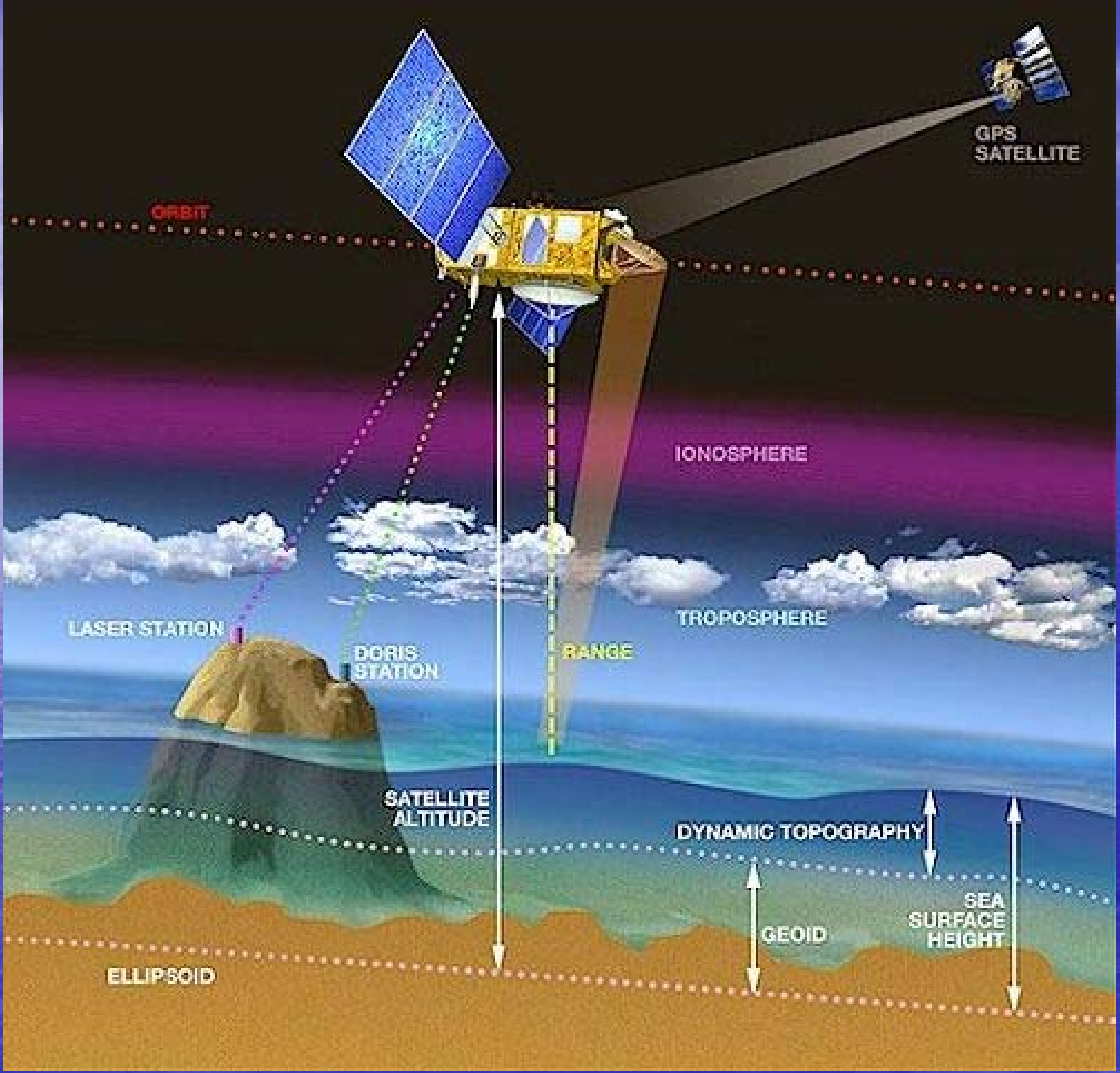
F(x,y) [or F(x,y,t)]

Error variance for the mean of N observ is

$$\sigma^2/N$$

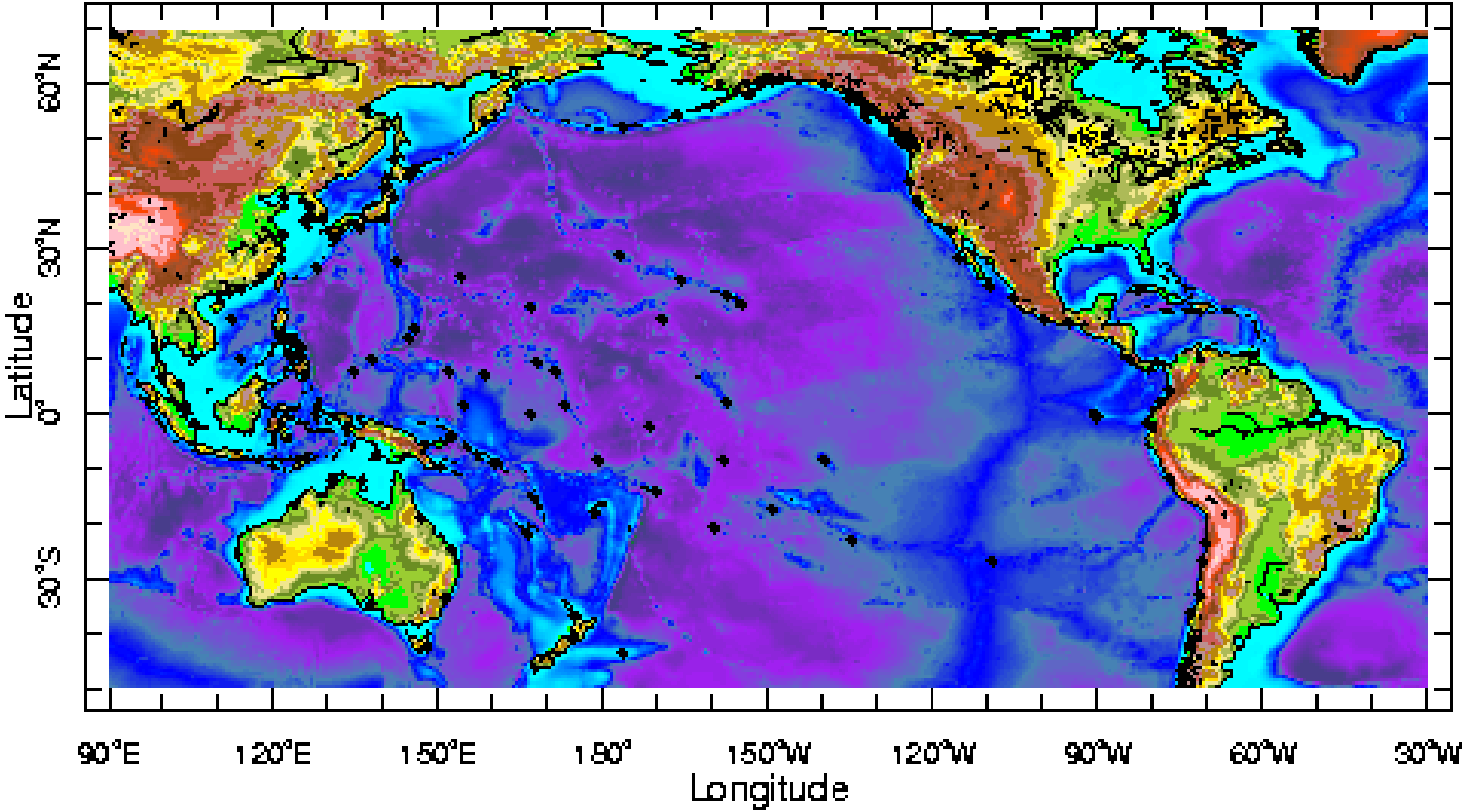
Sea Surface Height Anomaly

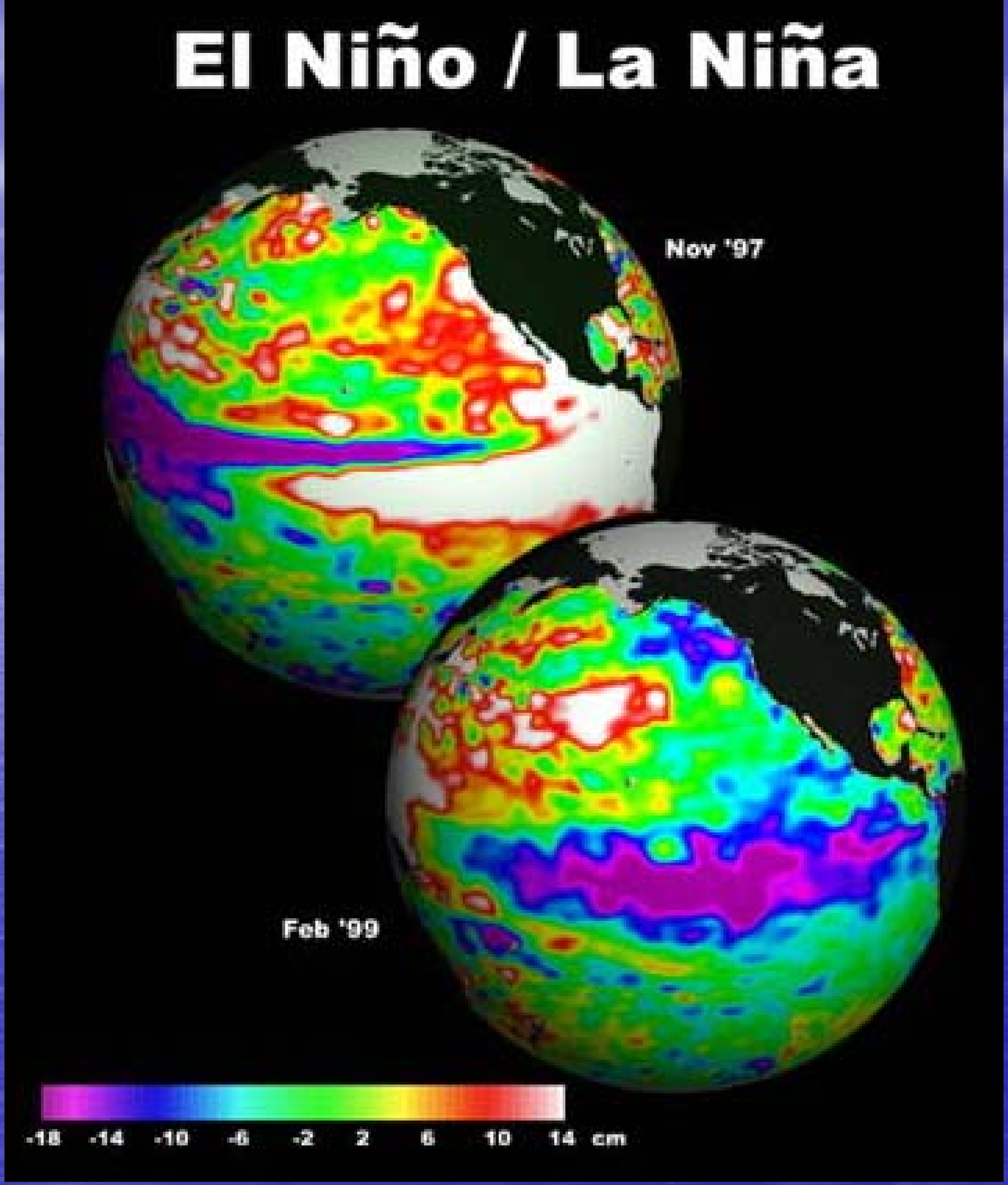




Altimetry principle from http://www.jason.oceanobs.com/html/alti/principe_uk.html

Pacific Ocean tide gauge stations network (from University of Hawaii sea level center)





From http://topex-www.jpl.nasa.gov/science/images/el-nino-la-nina.jpg

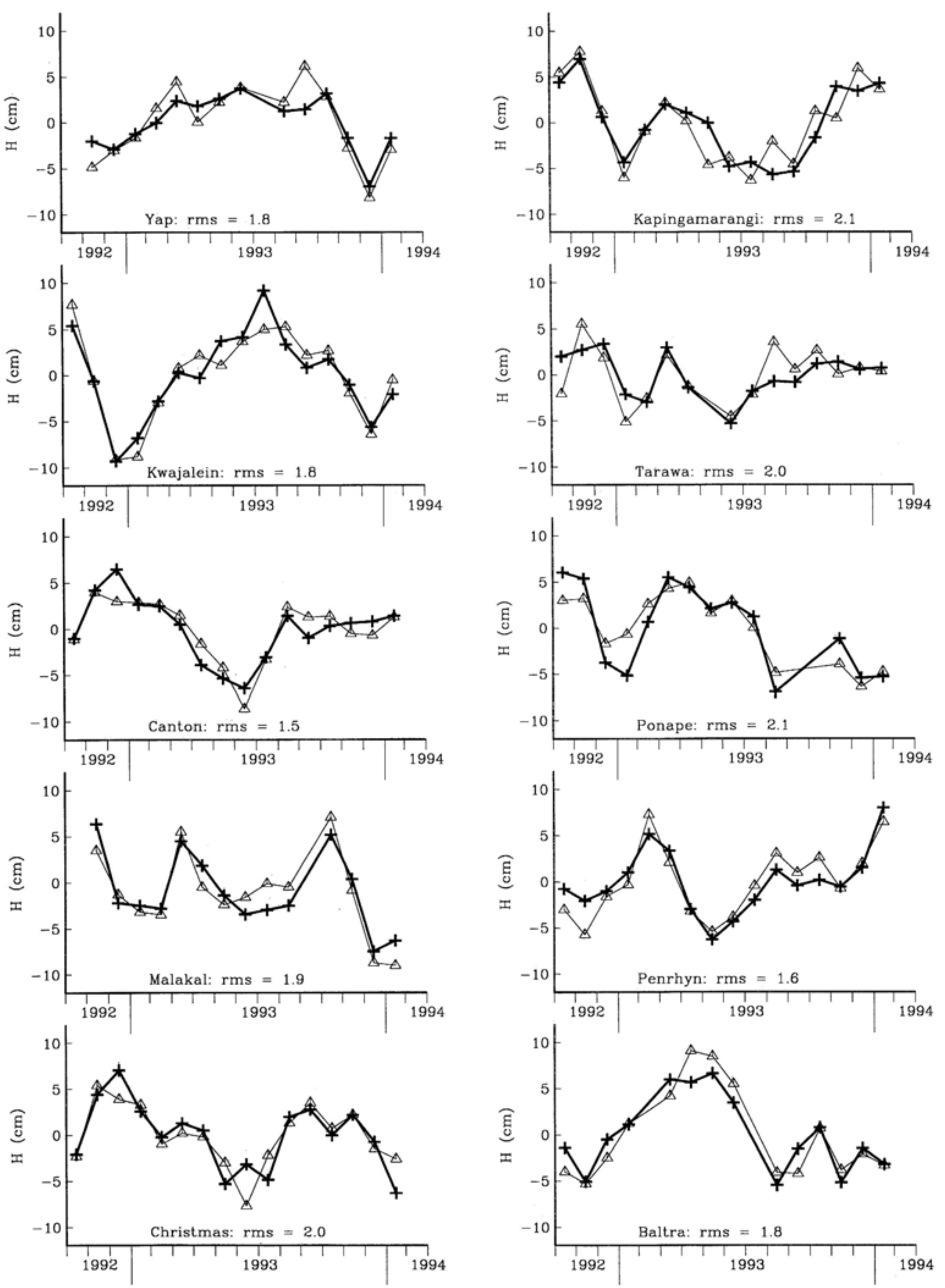


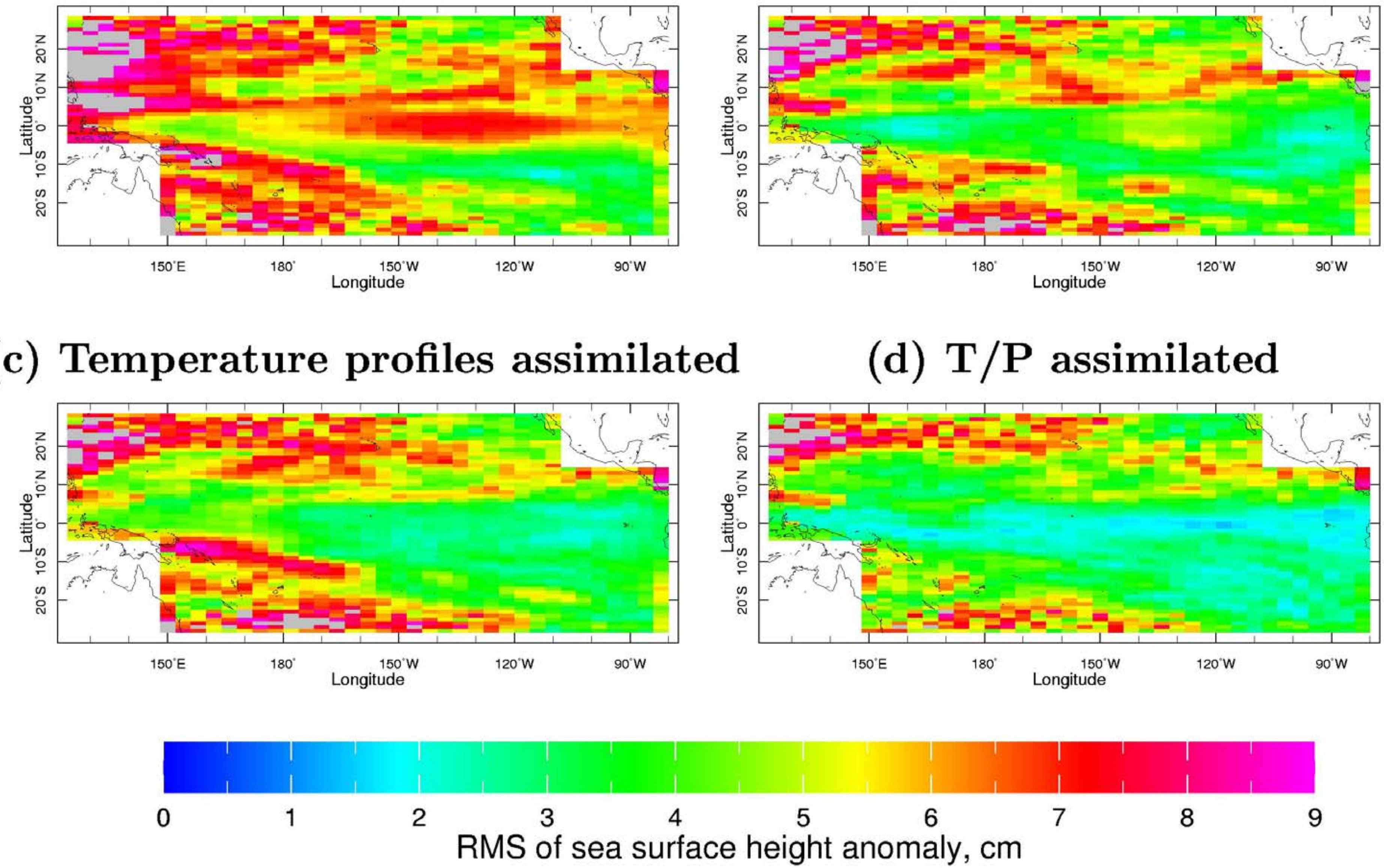
Figure 5. Time series from 10 island gauges (thin line) within 10° of the equator at which particularly good agreement was found with T/P data (heavy line). The existence of so many examples such as this argues that T/P is achieving accuracies at the 2-cm level for monthly mean heights in 4 × 1 cells, without orbit adjustment.

Comparison of 4x1 degree T/P altimetry averages with tide gauge data (from Cheney et al 1994)

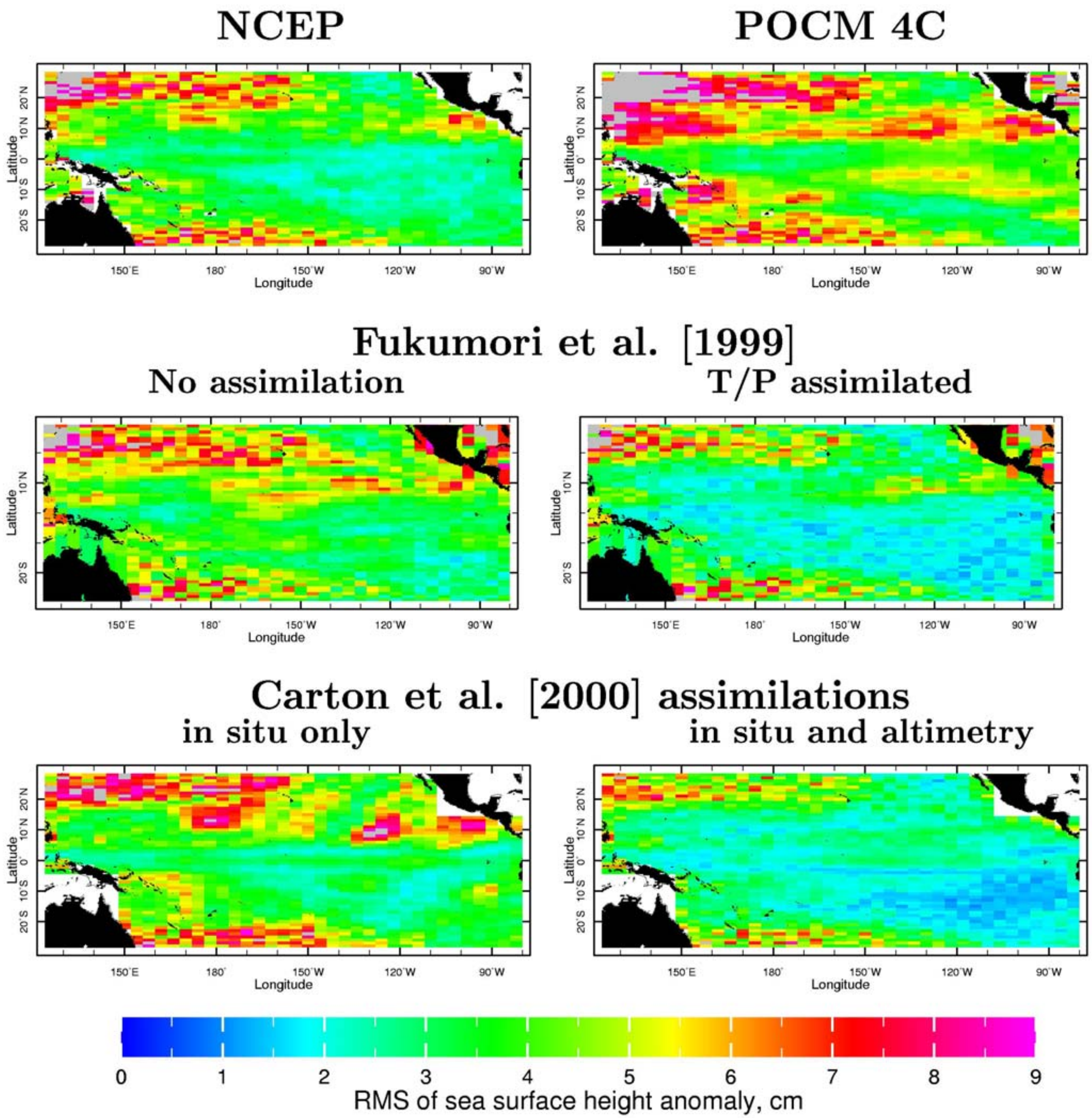
Altimetry missions from http://www.jason.oceanobs.com/html/missions/welcome_uk.html

Sea level height anomaly: RMS[T/P – Linear Model]

(a) No assimilation | (b) Tide gauges assimilated | (c) Temperature profiles assimilated | (d) T/P assimilated



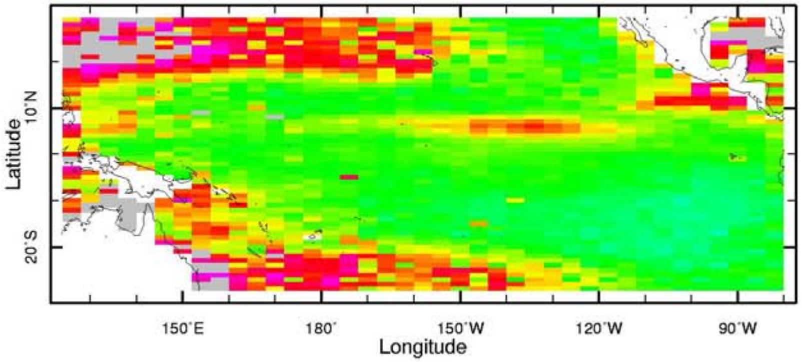
Sea level height anomaly: RMS[T/P – GCMs



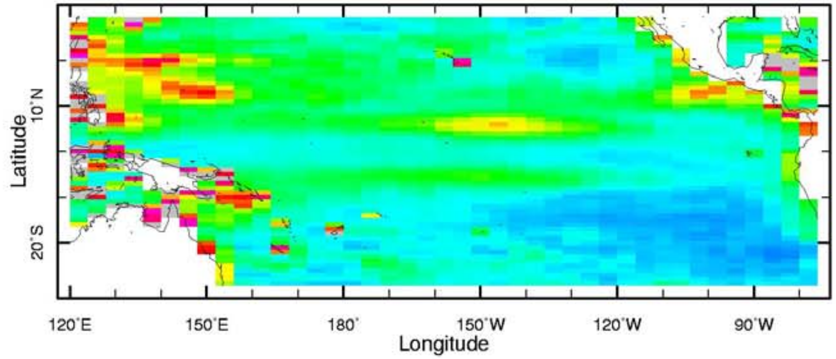
Small-scale variability and gridded altimetry error

Variability inside $4^o \times 1^o$ monthly bins: $\sigma_{4^o \times 1^o \times 1\,\mathrm{month}}$

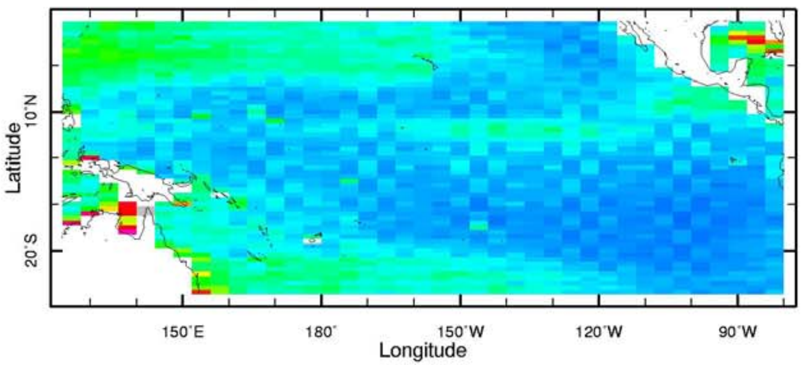
(a) Topex



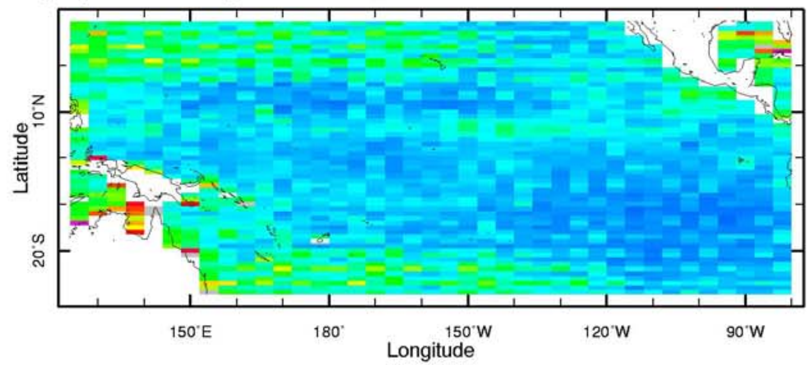
(b) POCM 4C

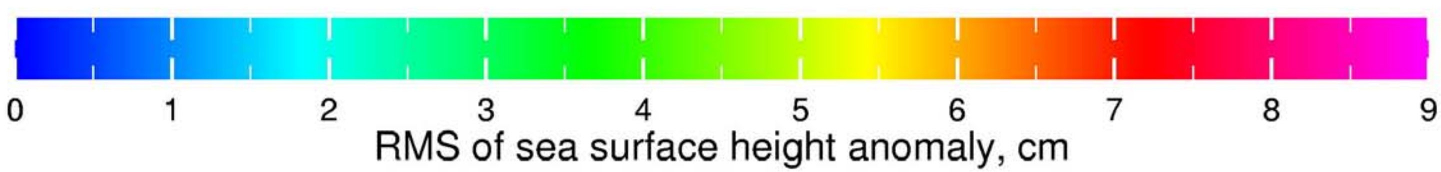


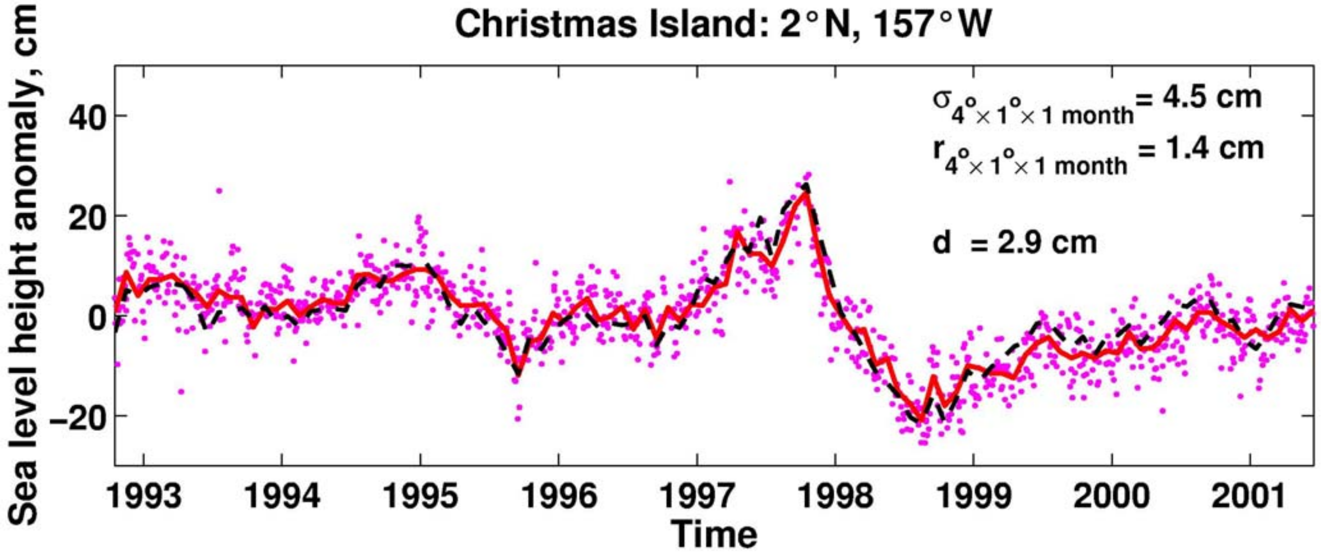
(c) Est err of gridded values: $r_{4^o \times 1^o \times 1\,\mathrm{month}}$

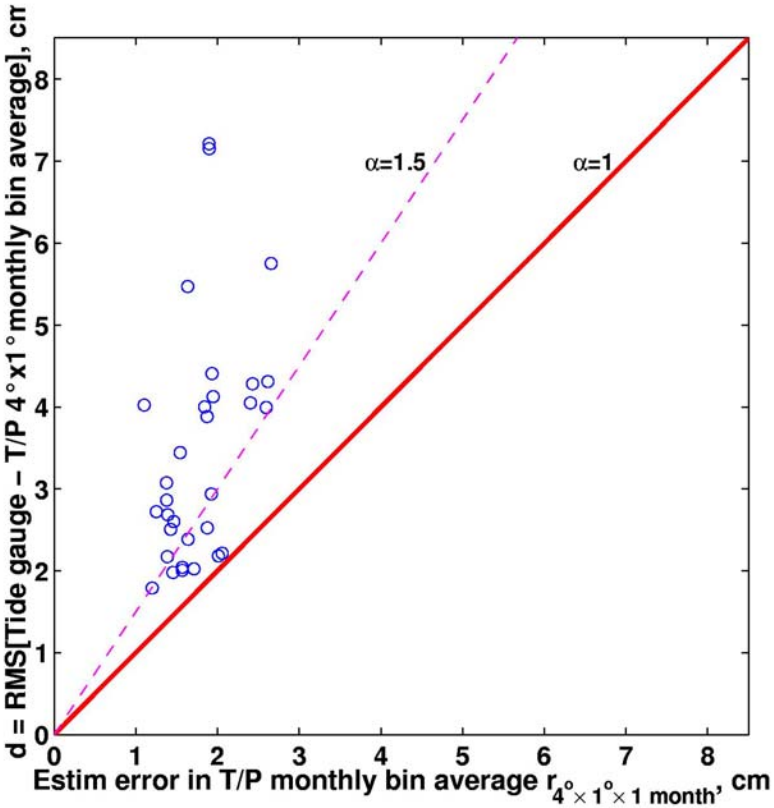


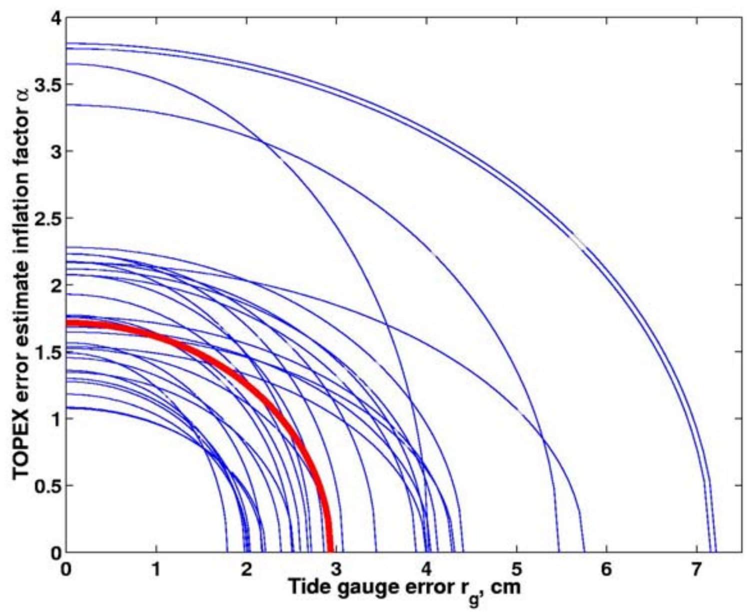
(d) RMS[Cheney et al. - Ducet et al.]





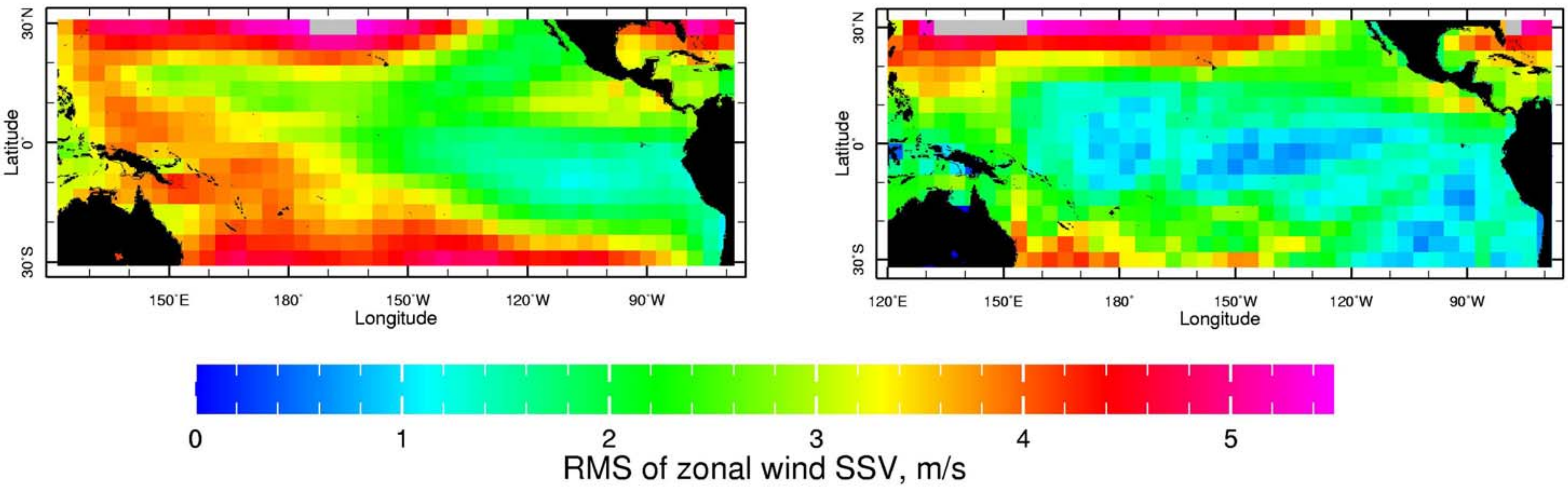

Christmas Island: 2°N, 157°W
$\sigma_{4° \times 1° \times 1\ month}$ = 4.5 cm
$r_{4° \times 1° \times 1\ month}$ = 1.4 cm
d = 2.9 cm
Sea level height anomaly, cm
Time
1993
1994
1995
1996
1997
1998
1999
2000
2001


d = RMS[Tide gauge – T/P 4°x1° monthly bin average], cm
Estim error in T/P monthly bin average $r_{4° \times 1° \times 1\ month}$, cm
α=1.5
α=1


TOPEX error estimate inflation factor α
Tide gauge error r_g, cm

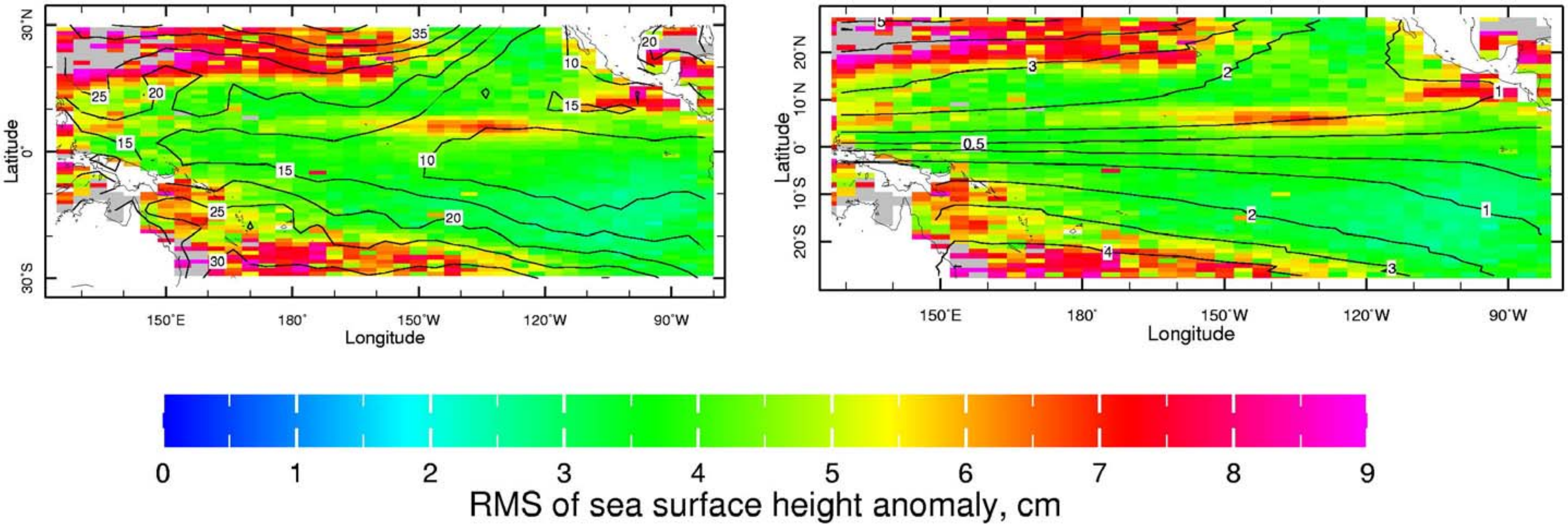
Small-scale variability in zonal wind $\sigma_{4^o \times 4^o \times 1\ \mathrm{month}}$

(a) NSCAT　　　　(b) COADS



Small-scale variability in wind and sea level height

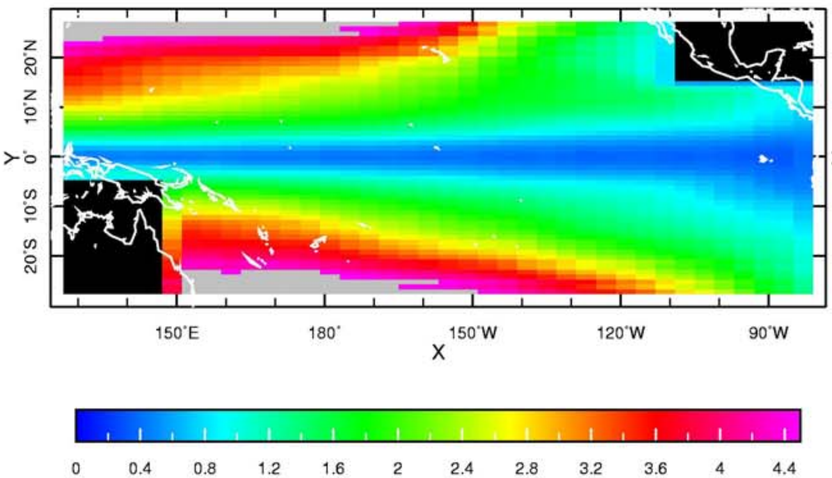
(a) SSV of zonal pseudostress (b) Simulated sea level height SSV



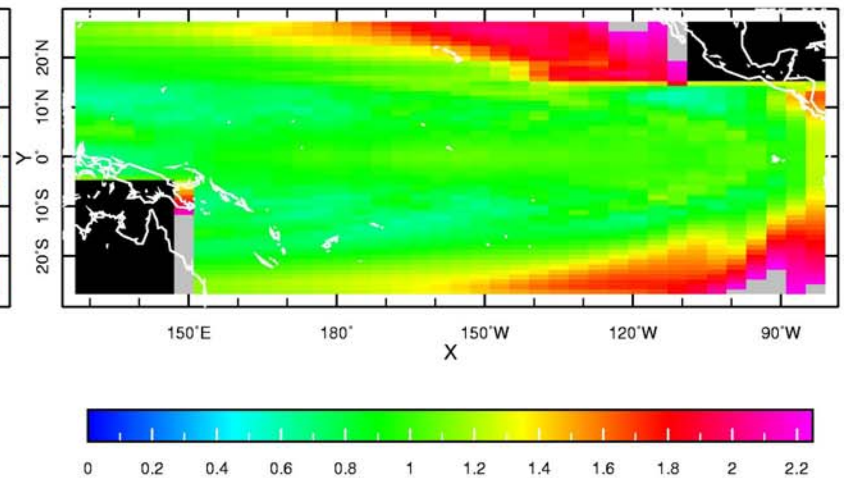
Monte Carlo experiments with a linear model

Variability inside $4^o \times 1^o$ monthly bins: $\sqrt{\langle \sigma^2_{4^o \times 1^o \times 1\ \mathrm{month}}(s) \rangle_{\mathrm{months}}}$

(a) $1^o \times 1^o$ scaled noise

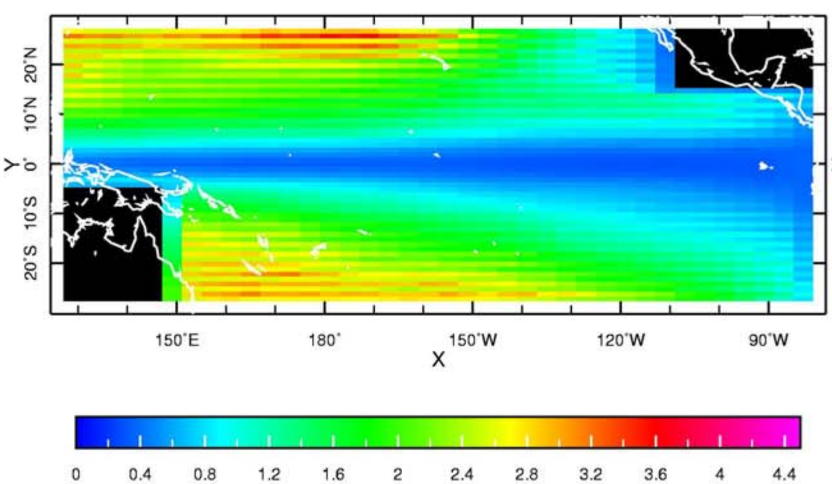


(b) $20^o \times 10^o$ scaled noise

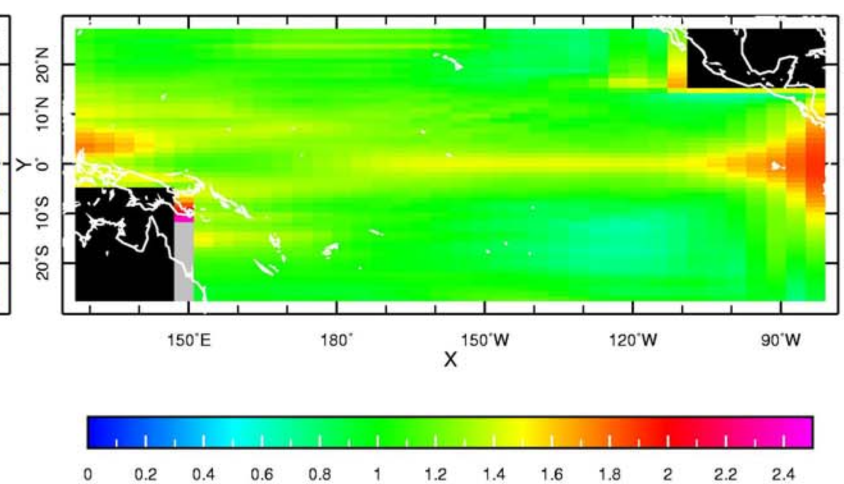


Variability of $4^o \times 1^o$ monthly means: $\sigma_{\mathrm{months}}([s]_{4^o \times 1^o \times 1\ \mathrm{month}})$

(c) $1^o \times 1^o$ scaled noise



(d) $20^o \times 10^o$ scaled noise

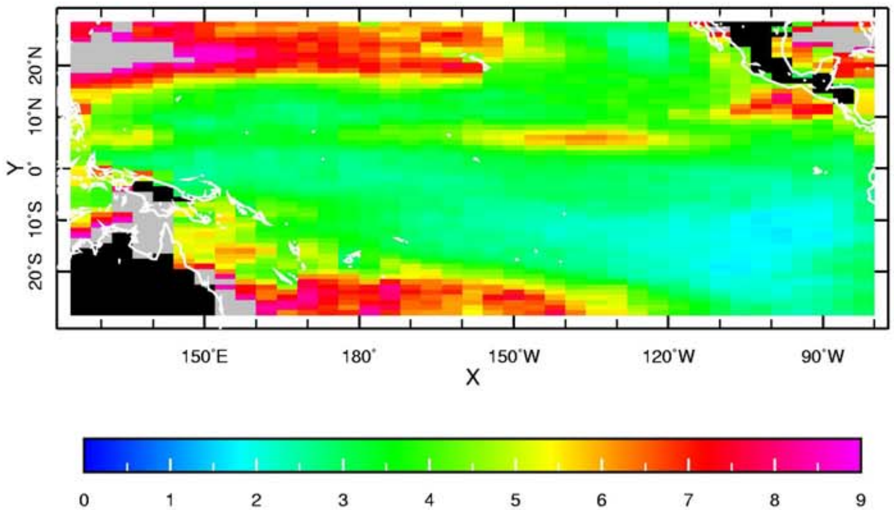


Interesting result:

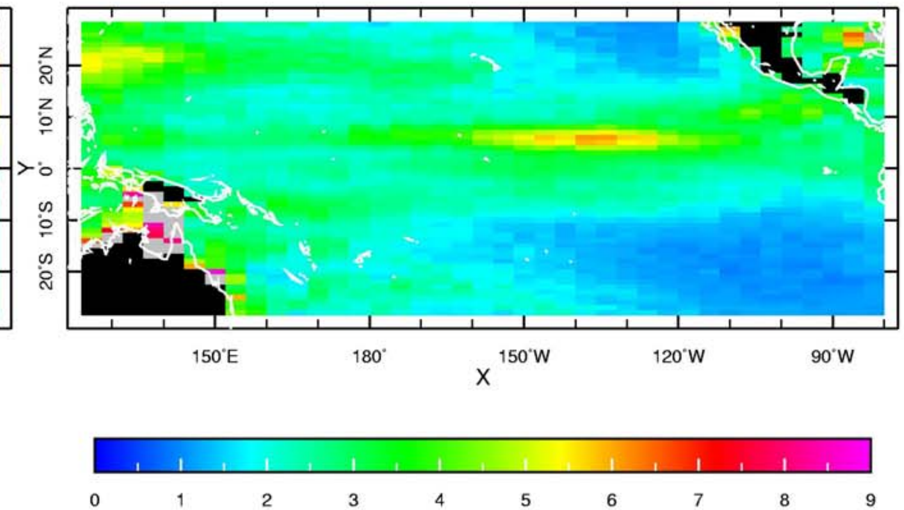
Despite great differences in complexity, all tropical Pacific sea level height simulation and assimilation products that we analyzed to date have similar error patterns. Their common features can be traced to the spatial energy distribution in the small-scale and short-term variability of ocean sea level.

Time-space separation of small-scale sea level height variability

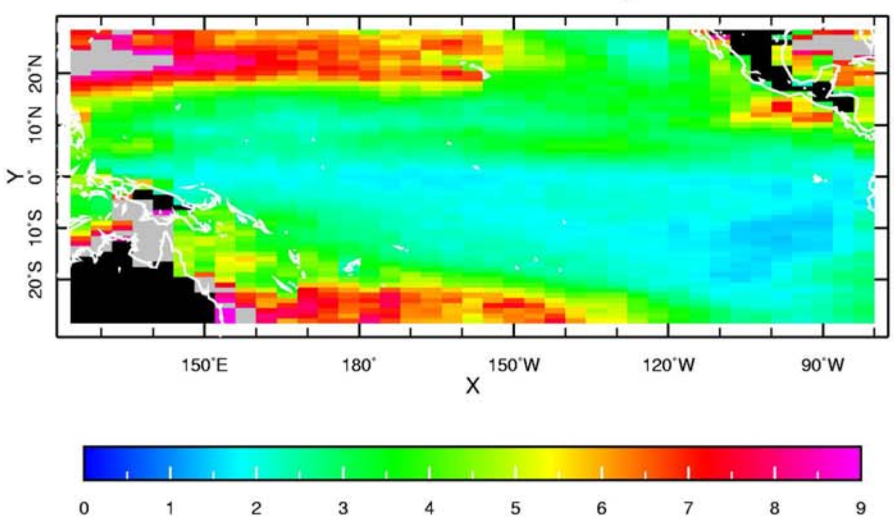
(a) Total SSV $\sigma_{4^o\times1^o\times1\,\mathrm{month}}(s)$



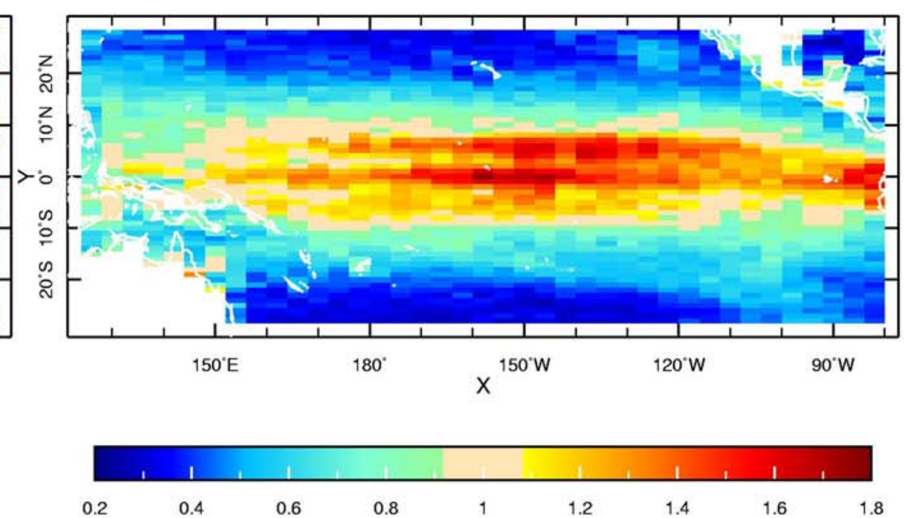
(b) Short-term temporal variability $\sigma_{1\,\mathrm{month}}([s]_{4^o\times1^o})$

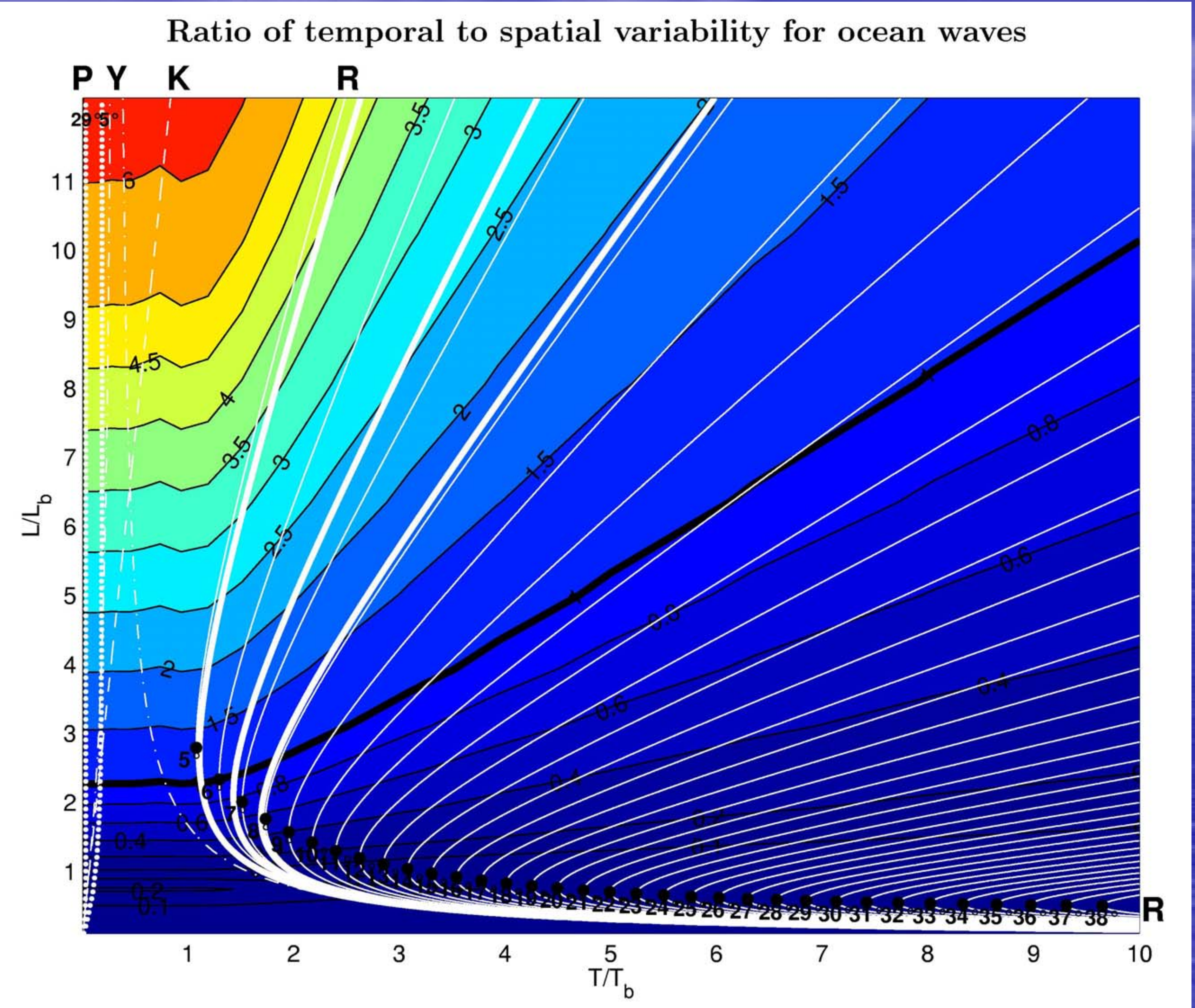


(c) Spatial variability inside bins $\sqrt{[\sigma^2_{4^o\times1^o}(s)]_{1\,\mathrm{month}}}$



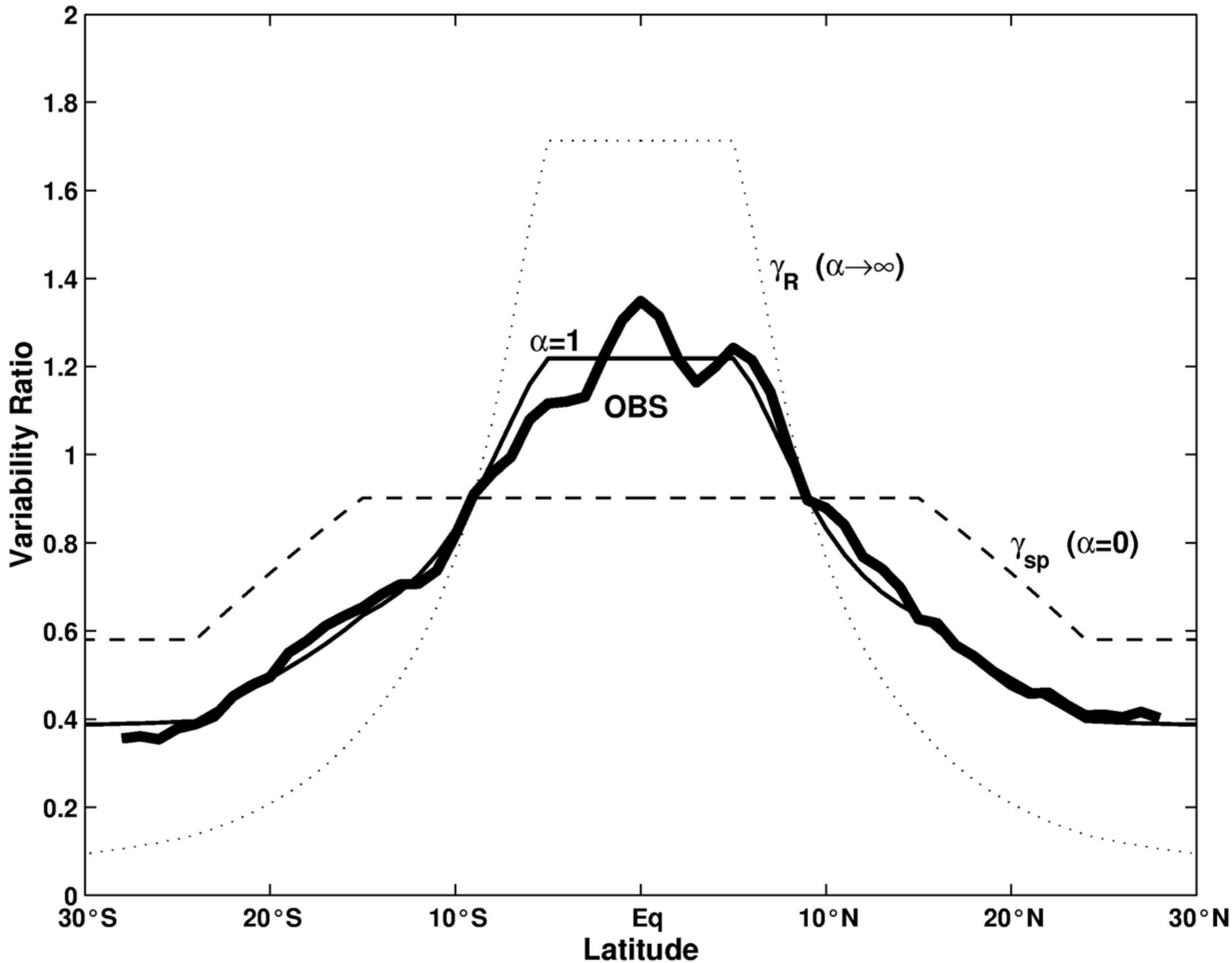
(d) Ratio γ **of temporal to spatial variability**

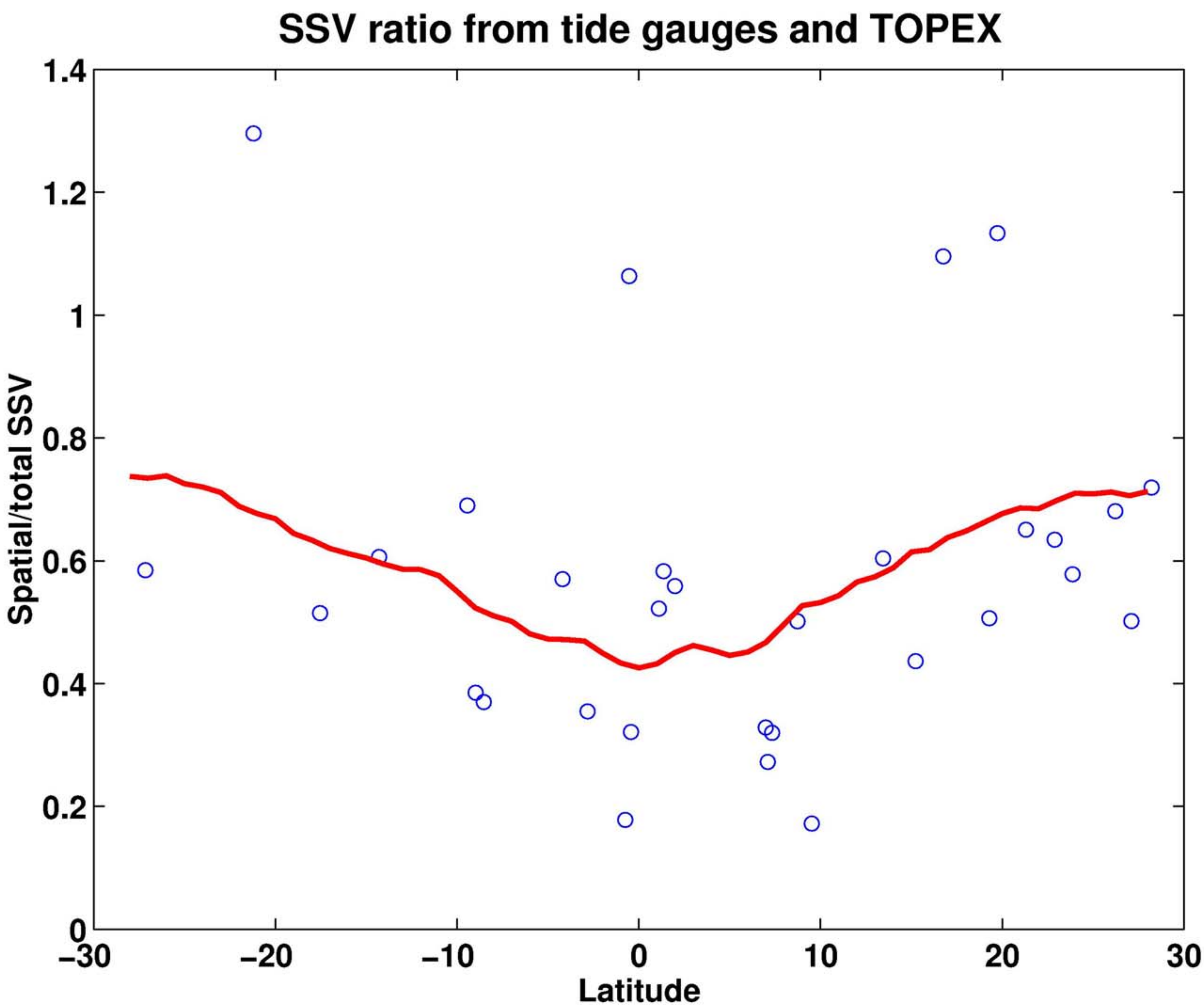




Ratio of temporal to spatial variability for ocean waves
P Y K R
L/L_b
T/T_b
R

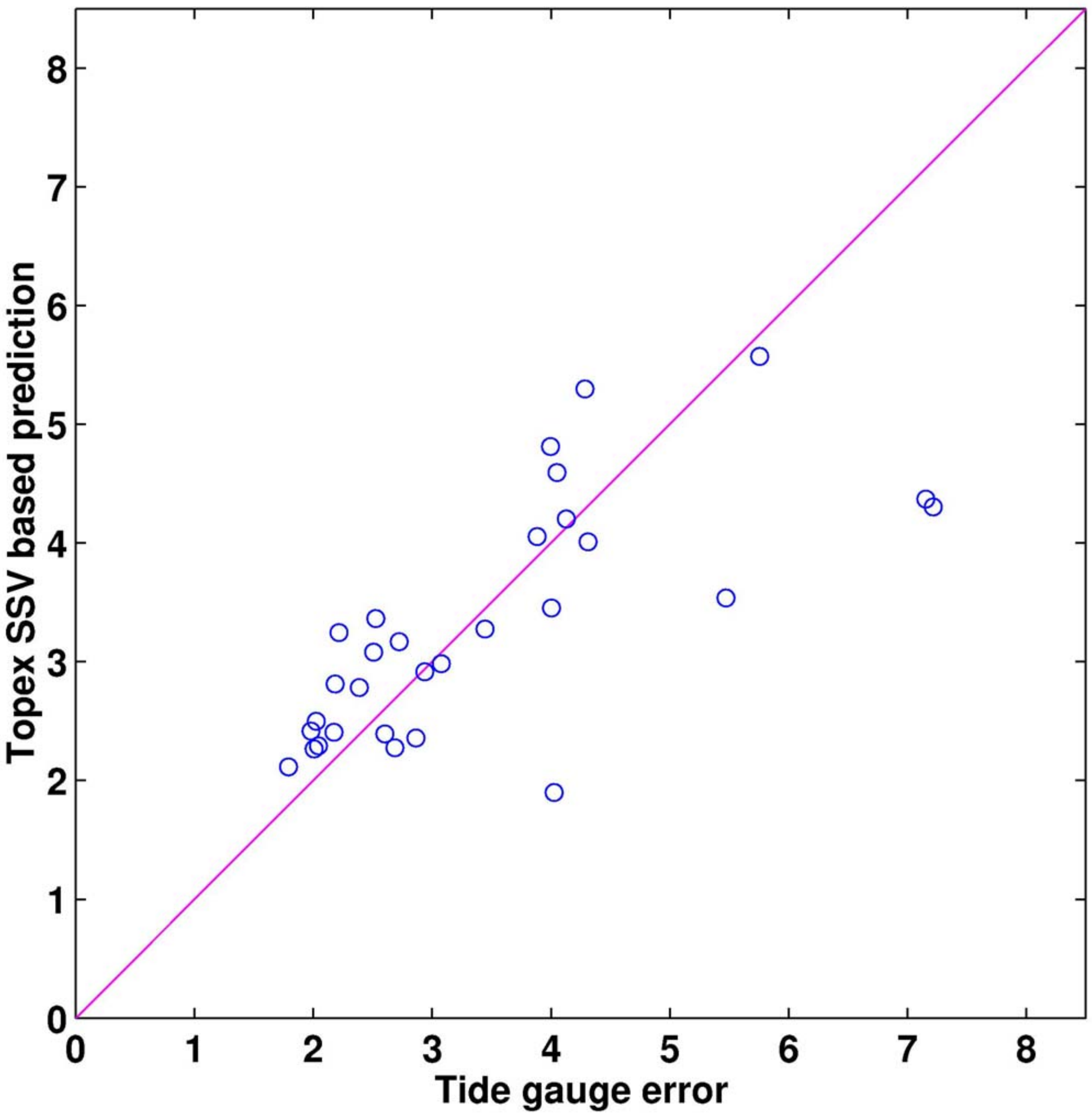
Meridional profile of the ratio of temporal to spatial variability in the ocean

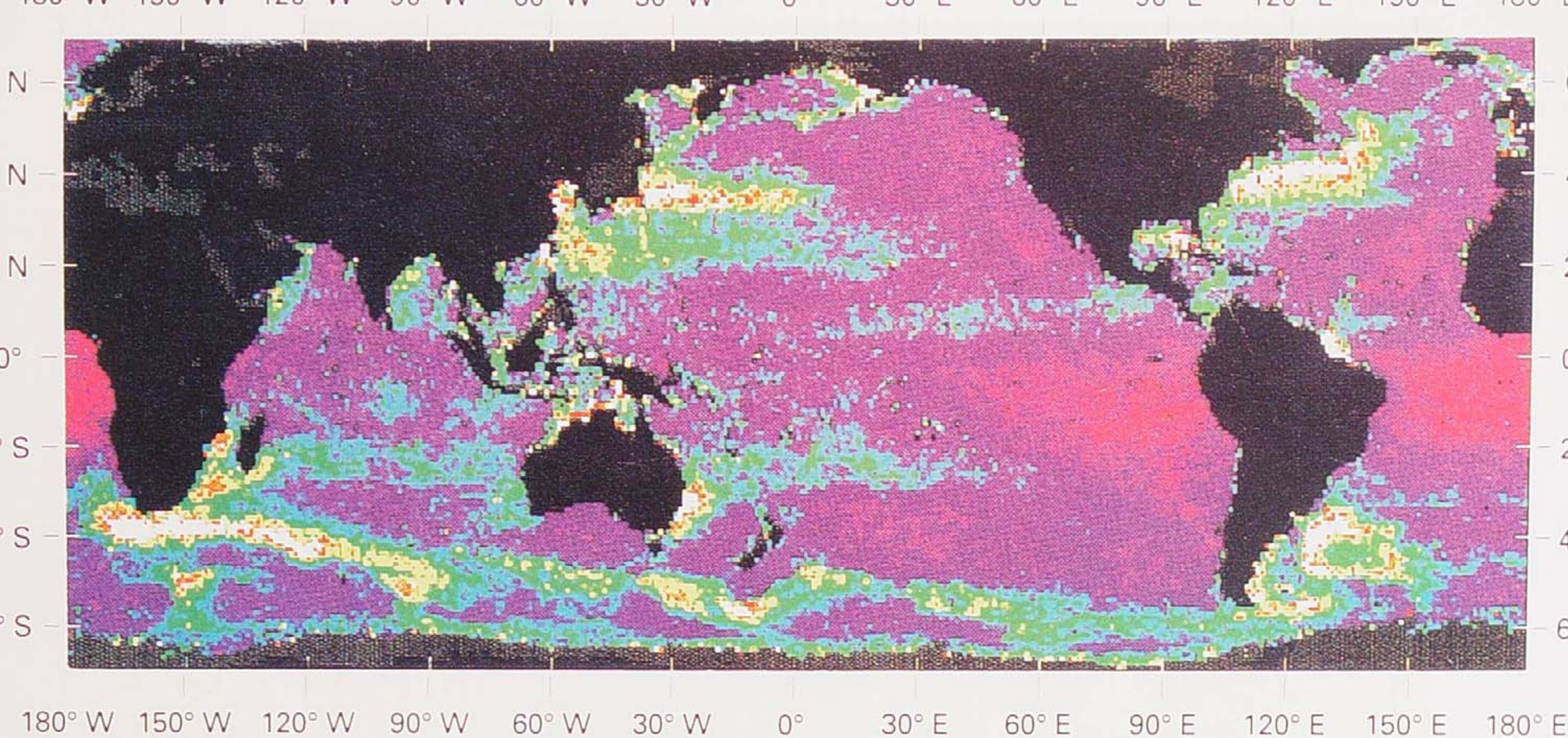




SSV ratio from tide gauges and TOPEX
Spatial/total SSV
Latitude
1.4
1.2
1
0.8
0.6
0.4
0.2
0
−30
−20
−10
0
10
20
30

Verification of tide gauge model error



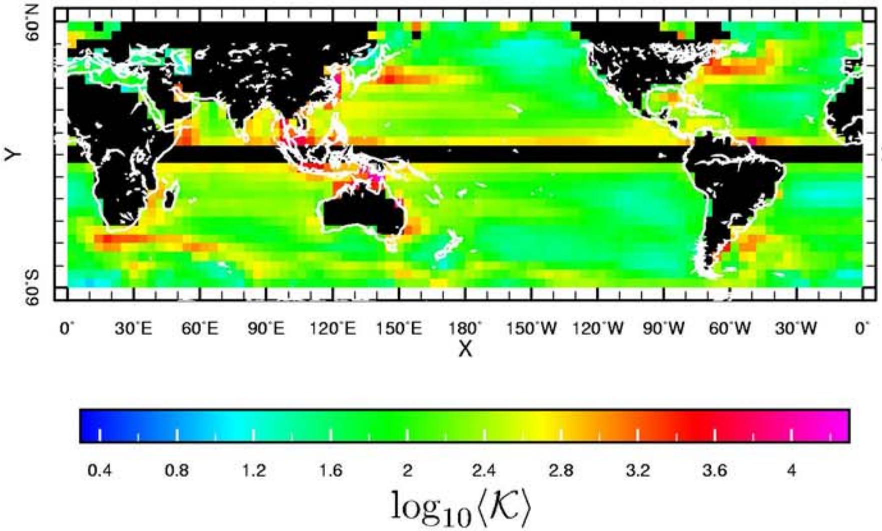


M. Tomczak & J.S. Godfrey book

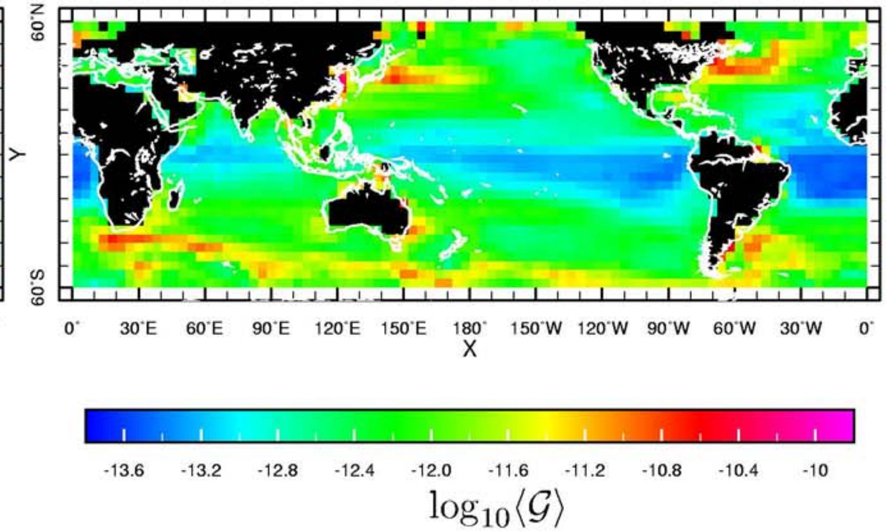
“Regional Oceanography: an Introduction”

Fig. 4.8. Annual mean of eddy energy in the ocean as observed by satellite altimeter during December 1986 - November 1987. The eddies are detected by measuring the shape of the sea surface, which bulges downward in cyclonic and upwards in anticyclonic eddies as explained in Figs 2.7 and 3.3. The quantity shown is the standard deviation of observed sea level (cm) from the mean sea level over the observation period. From Fu *et al.* (1988).

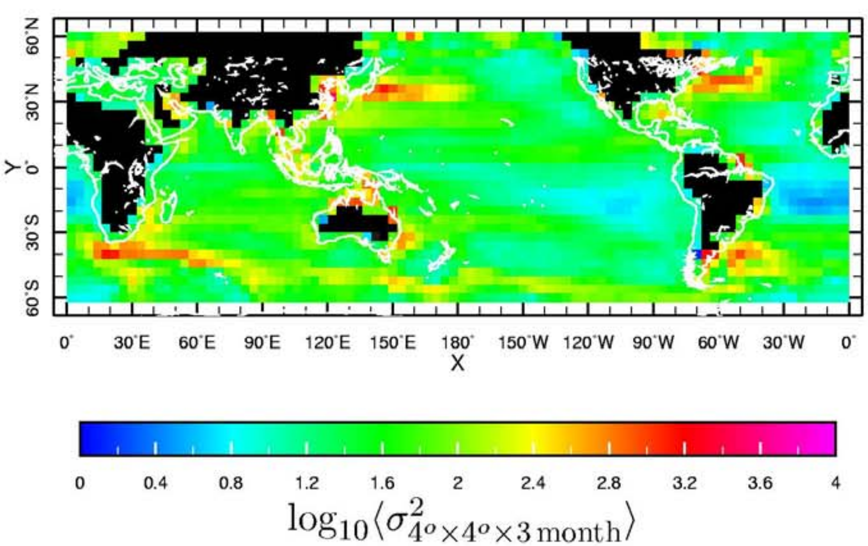
(a) Geostr kinetic energy $\langle \mathcal{K} \rangle$, $(\mathrm{cm/s})^2$



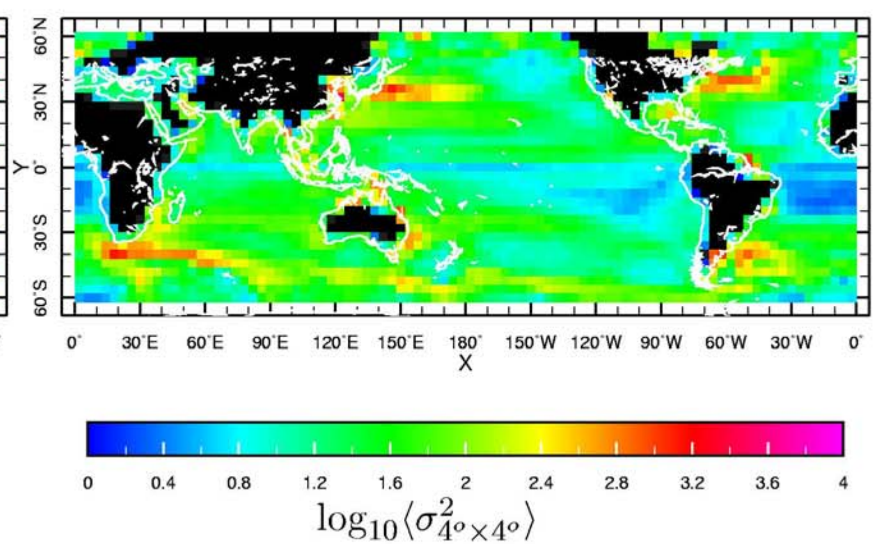
(b) Squared gradient $\langle \mathcal{G} \rangle$



(c) $\langle \sigma^2_{4^o \times 4^o \times 3\,\mathrm{month}} \rangle$, cm^2



(d) $\langle \sigma^2_{4^o \times 4^o} \rangle$, cm^2



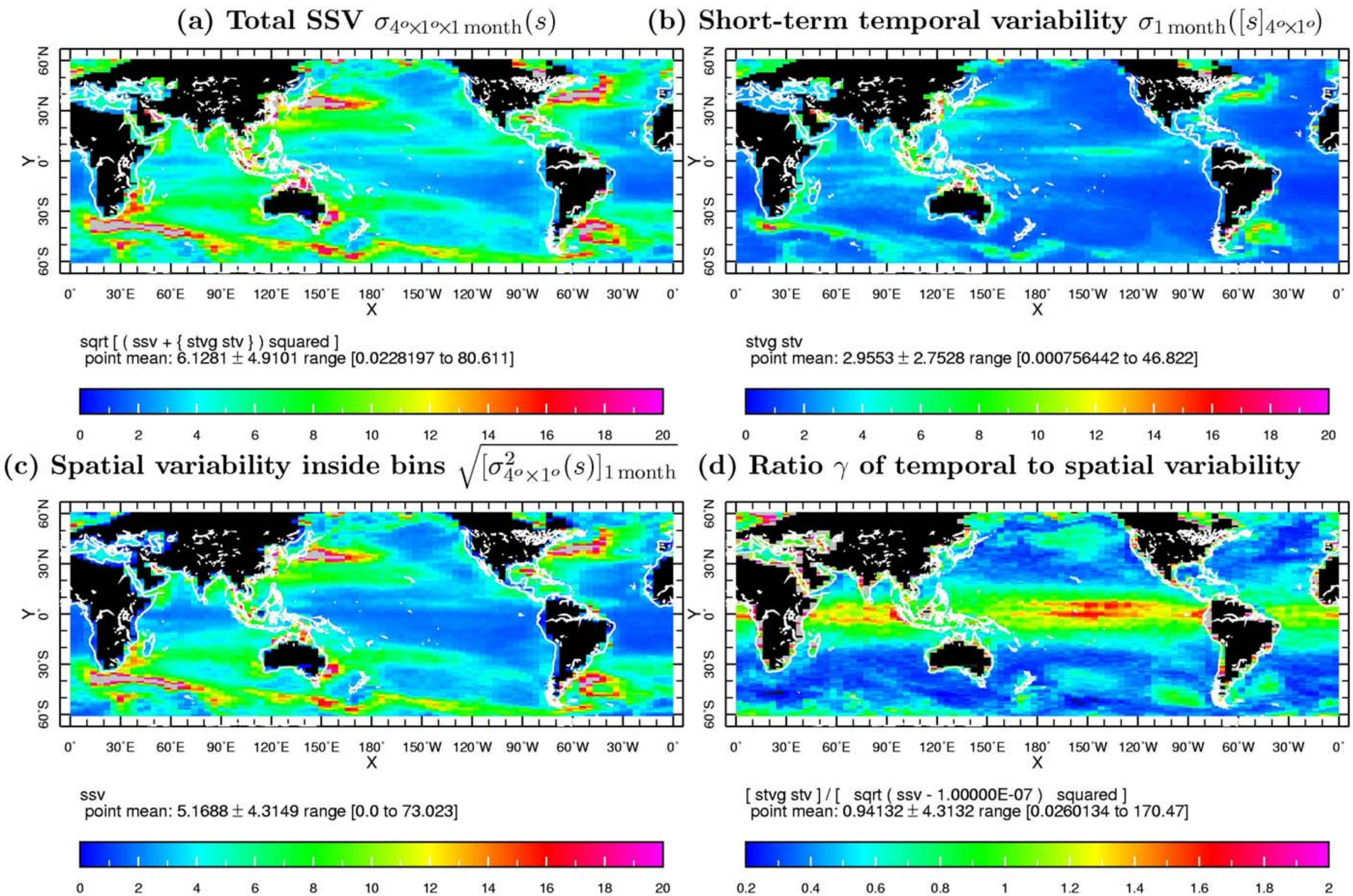
Connection between surface geostrophic kinetic energy and small-scale variability in sea surface height:

$$<\sigma^2>=C(f/g)^2 <K>,$$

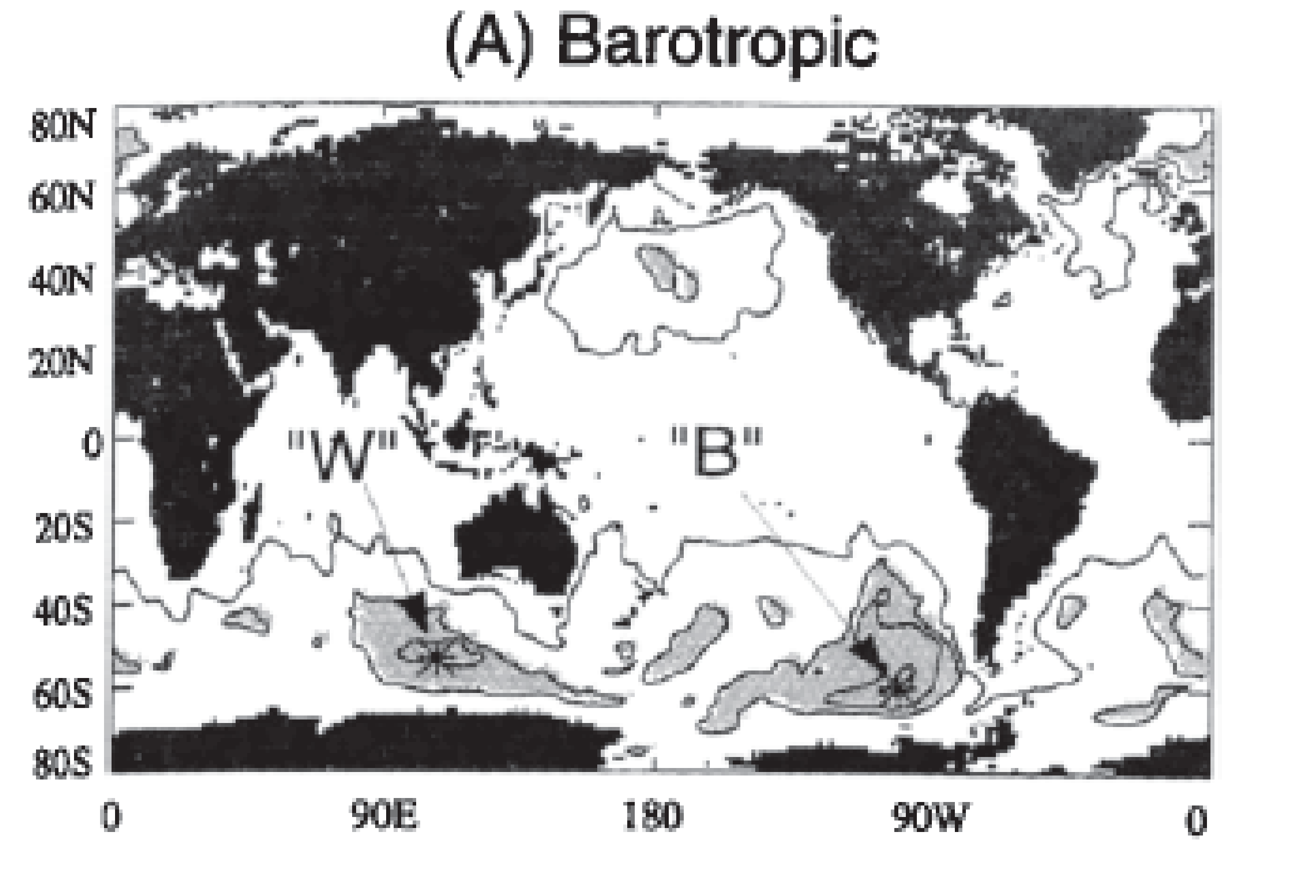
where $C=\alpha (L_x^2+L_y^2)/6$, and α depends on the wavenumber power spectrum of the ocean sea surface height. Parameter α describes how small differences in sea surface height scale to the $L_x X L_y$ box

TOPEX [*Ducet et al.* 2000]

Time-space separation of small-scale sea level height variability



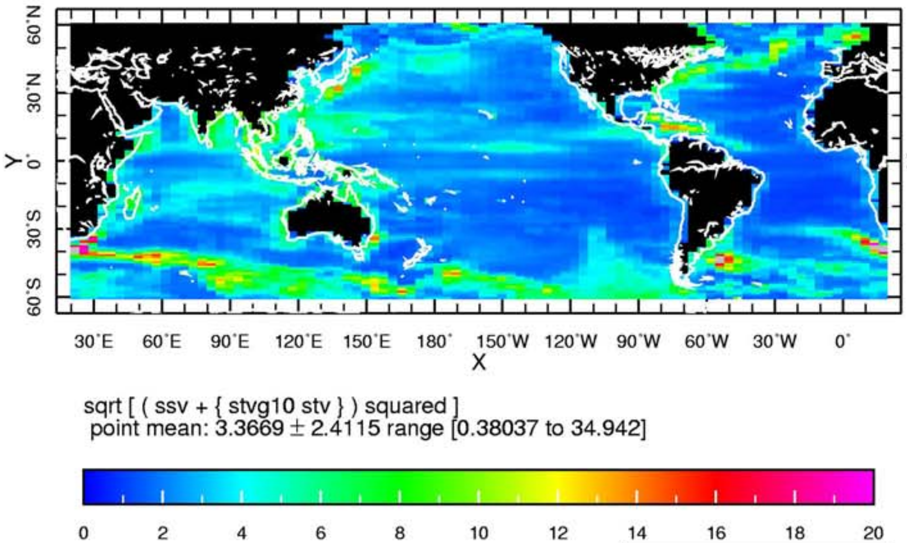
Fukumori et al. [1998] in the simulations using MOM have found major regional differences in frequency content of sea surface height variability connected to barotropic vs baroclinic variability prevalence.



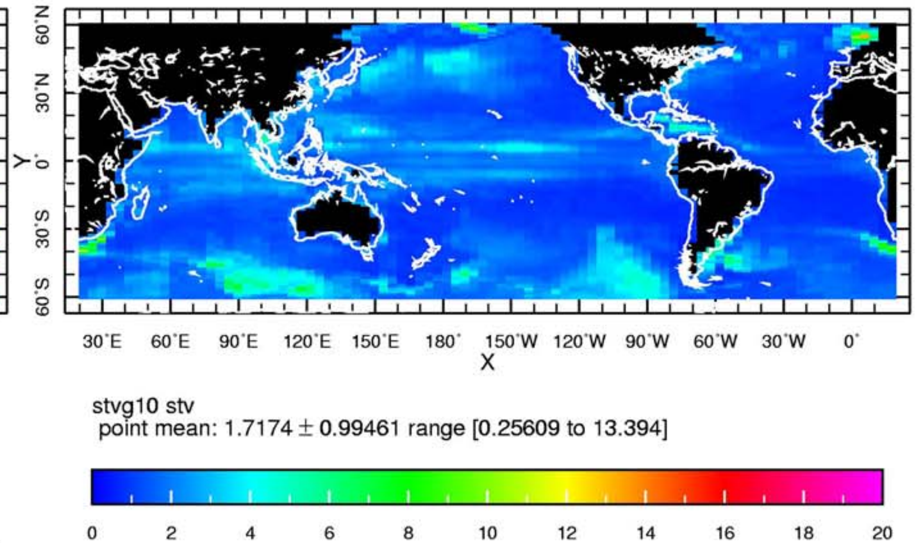
POCM 4C model [*Tokmakian and Challenor* 1999]

Time-space separation of small-scale sea level height variability

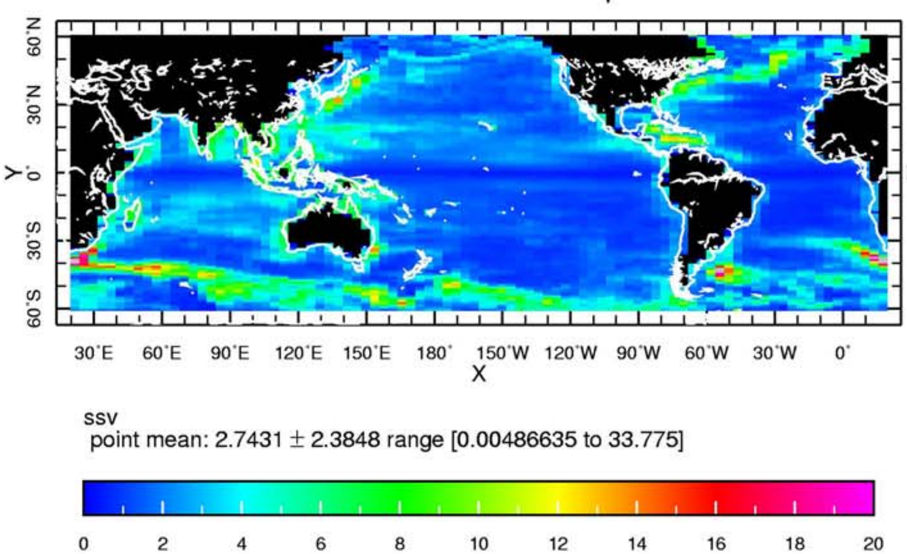
(a) Total SSV $\sigma_{4^o\times1^o\times1\,\mathrm{month}}(s)$



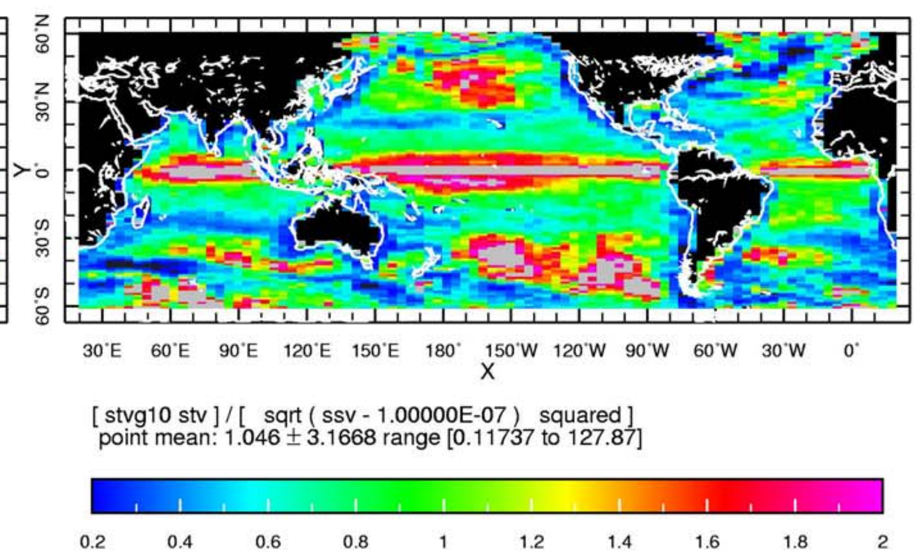
(b) Short-term temporal variability $\sigma_{1\,\mathrm{month}}([s]_{4^o\times1^o})$

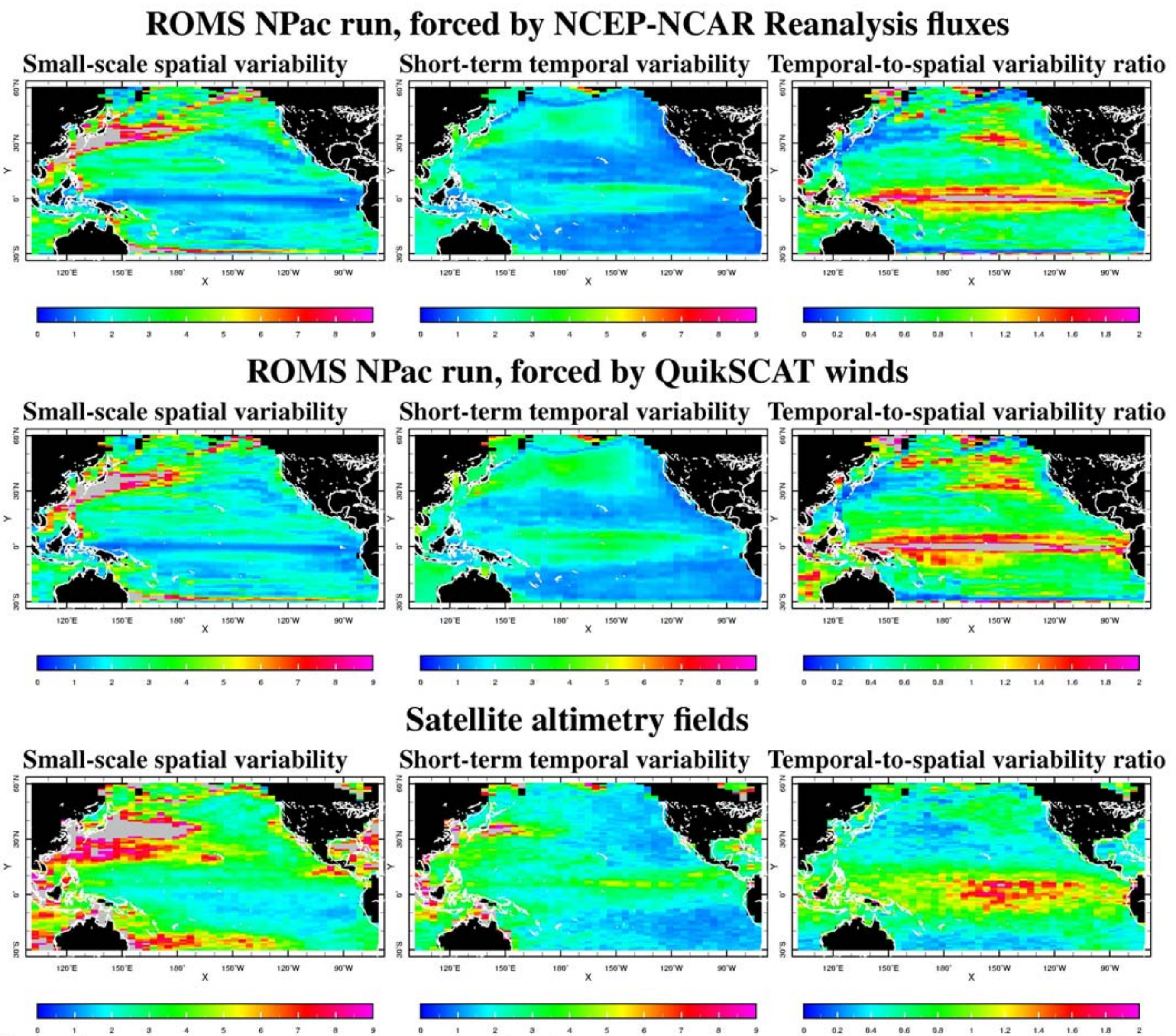


(c) Spatial variability inside bins $\sqrt{[\sigma^2_{4^o\times1^o}(s)]_{1\,\mathrm{month}}}$

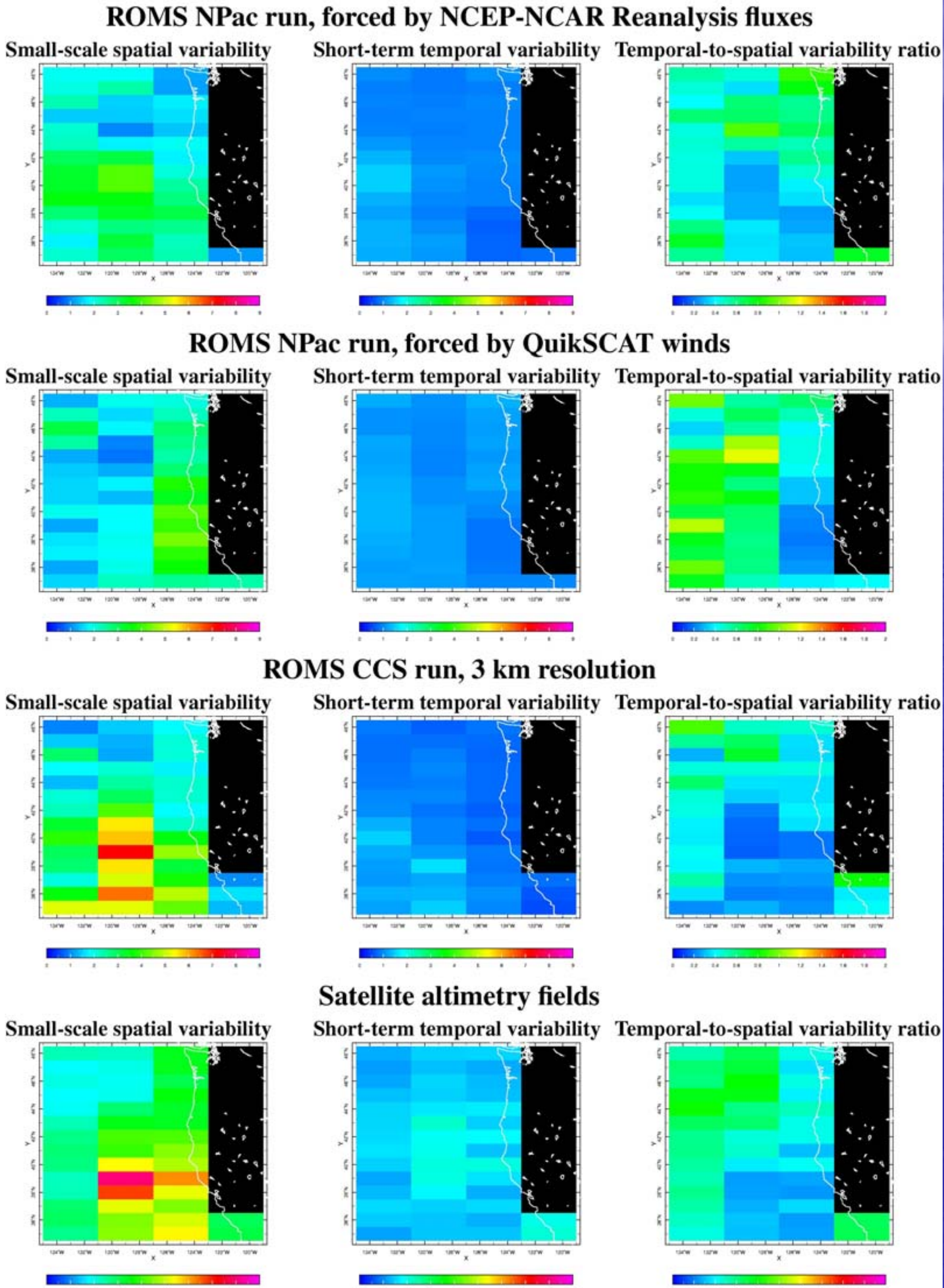


(d) Ratio γ **of temporal to spatial variability**





Curchitser et al., JGR, 2005



Resolving mesoscale features is essential for simulating realistic levels of spatial and temporal variability at these scales: current submesoscale parametrizations cannot substitute for model resolution in terms of these statistics.

Figure: Spatial and temporal variability and their ratio within 4ºX1ºX(1 month) bins (following *Kaplan et al. 2004; Curchitser et al.2005*)

Results:

- Despite great differences in complexity, all tropical Pacific sea level height simulation and assimilation products that we analyzed to date have similar error patterns. Their common features can be traced to the spatial energy distribution in the small-scale and short-term variability of ocean sea level.

- We analyzed the ratios of temporal and spatial contributions to the small-scale sea surface height variability. These are useful for characterizing dominant local scales and regimes of ocean motion as well as for modeling observational error. The model ratios are very different from those based on observations.