Spectra of surface ocean variability and estimation of representation error

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OUTLINE

- Motivation
  - Representation error
  - Using power spectra as a scalable error estimate
- Determining power spectra from altimetry
  - Spatial case
  - Temporal case
  - Preliminary models
  - Tide gauge consistency

Data Assimilation

Merging observations from different systems, each with its own error characteristics.

What is the error of each observation with regards to the model grid values? It must be specified for the assimilation procedures.

This error can be split into two components:

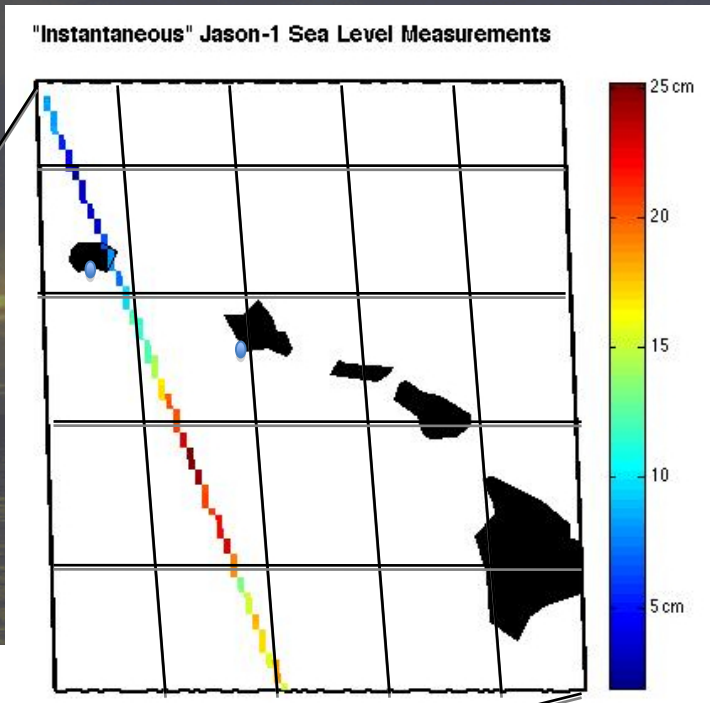
- **Instrument error: uncertainty associated with each measurement.**

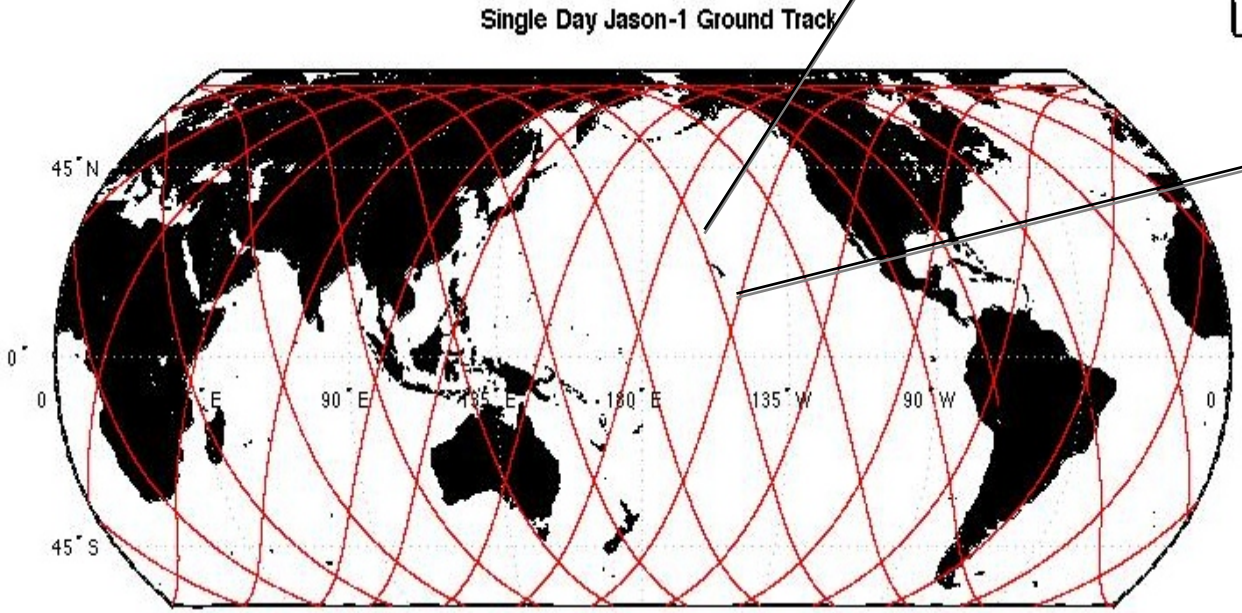
- **Representation error: due to the difference in averaging of the physical field by the model and by different types of the observing systems.**

A typical situation in ocean modeling:

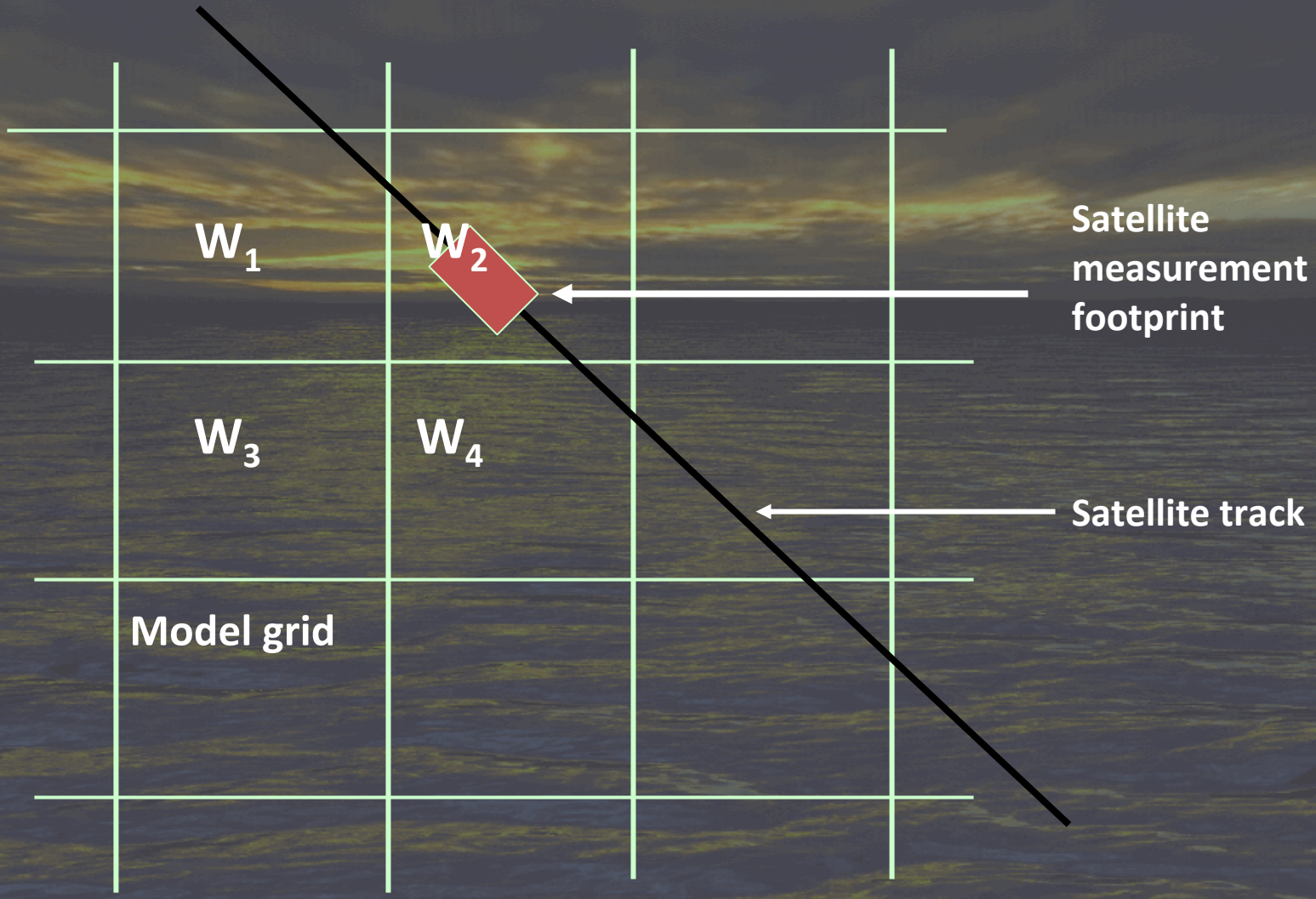
Model: typical grid resolution – 30km x 60km

Data:
Sea surface height altimetry – 6km footprint;
SST – 1-4-25km averages, depending on the product;
In situ observations – local.





Model and data values



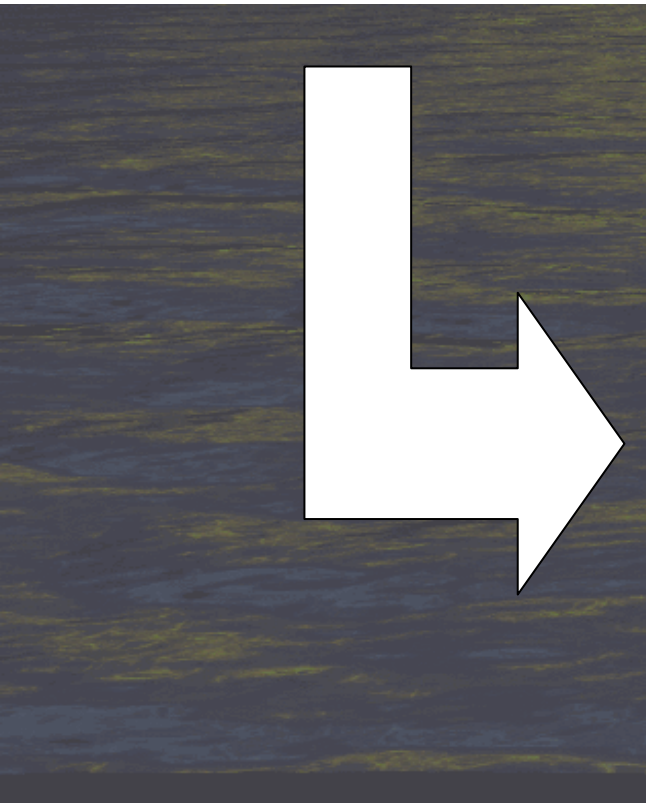
$$S^o = Hs + \varepsilon = W_1s_1+W_2s_2+W_3s_3+W_4s_4+ \varepsilon$$

$$Hs + \varepsilon = s^o$$

$$Hs = \int W(\vec{x})s(\vec{x})d\vec{x}$$

$$s^o = \int W^o(\vec{x})s(\vec{x})d\vec{x}$$

$$\varepsilon = \int (W^o(\vec{x}) - W(\vec{x}))s(\vec{x})d\vec{x}$$



Spectral representation of sampling error

Assume

$$s(\vec{x}) = \int f(\vec{k})e^{2\pi i\vec{k}\vec{x}}d\vec{k},$$

then

$$\varepsilon = \int d\vec{k}f(\vec{k}) \int (W^o(\vec{x}) - W(\vec{x}))e^{2\pi i\vec{k}\vec{x}}d\vec{x} = \int w(\vec{k})f(\vec{k})d\vec{k},$$

where

$$w(\vec{k}) = \int (W^o(\vec{x}) - W(\vec{x}))e^{2\pi i\vec{k}\vec{x}}d\vec{x}.$$

Let $P(\vec{k})$ be a power spectrum of $s(\vec{x})$:

$$\langle f(\vec{k})f(\vec{k'})^* \rangle = P(\vec{k})\delta(\vec{k} - \vec{k'}).$$

Then

$$\langle \varepsilon^2 \rangle = \int P(\vec{k})|w(\vec{k})|^2 d\vec{k}.$$

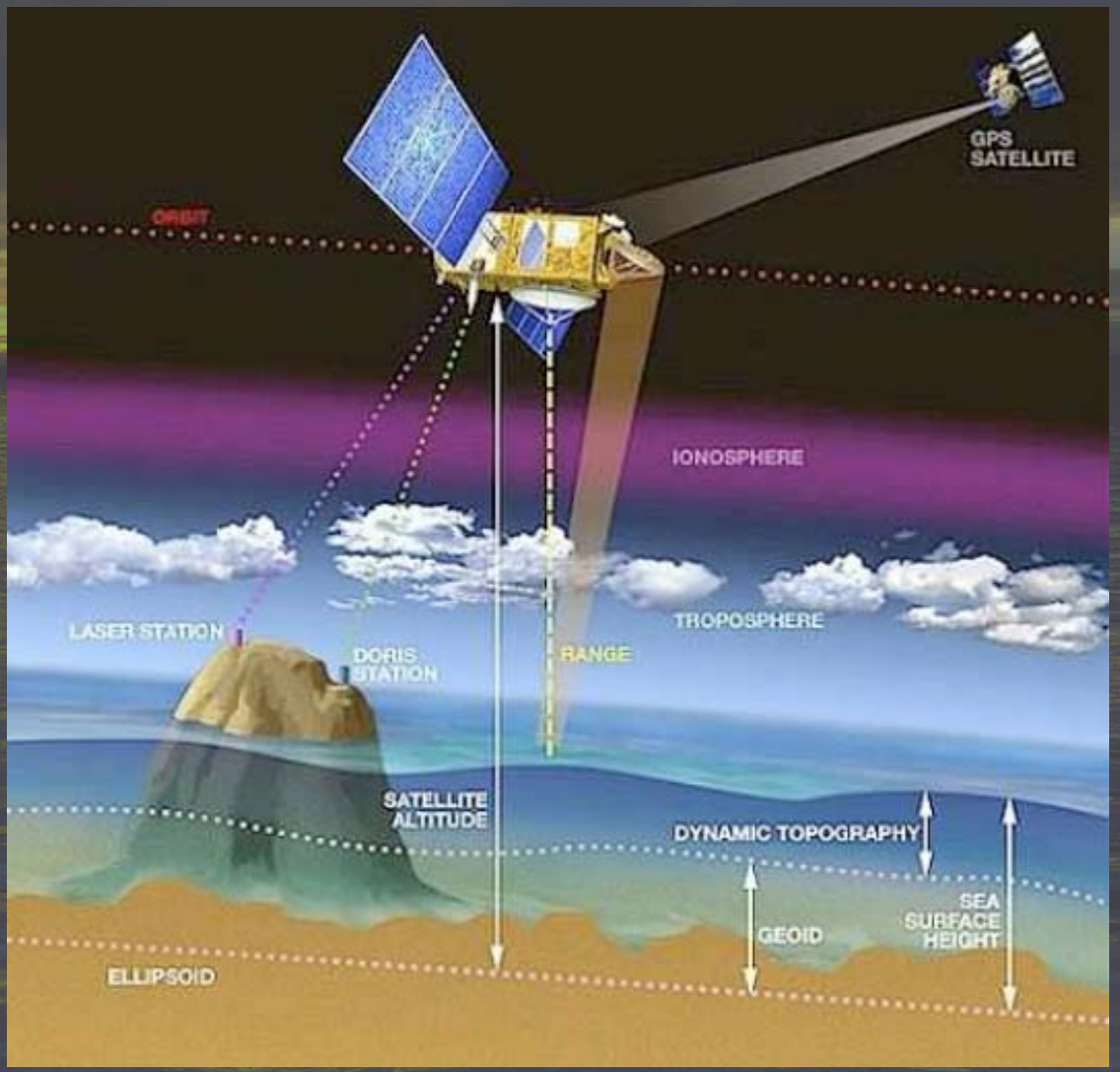
$$\langle \varepsilon^2 \rangle = \int P(\vec{k}) |w(\vec{k})|^2 d\vec{k}.$$

Suppose we want to use a satellite retrieval with a footprint of size l on the ocean surface for the assimilation to a model with the grid size L. By using the footprint value as an estimate for an average over the model grid box, we commit a sampling error. If a wavenumber power spectrum $P(k)$ of the physical field is isotropic and known, the approximate expression for this error is very simple

$$\sigma^2 = \int_{1/L}^{1/l} P(k) dk.$$

The problem becomes one of finding P(k)

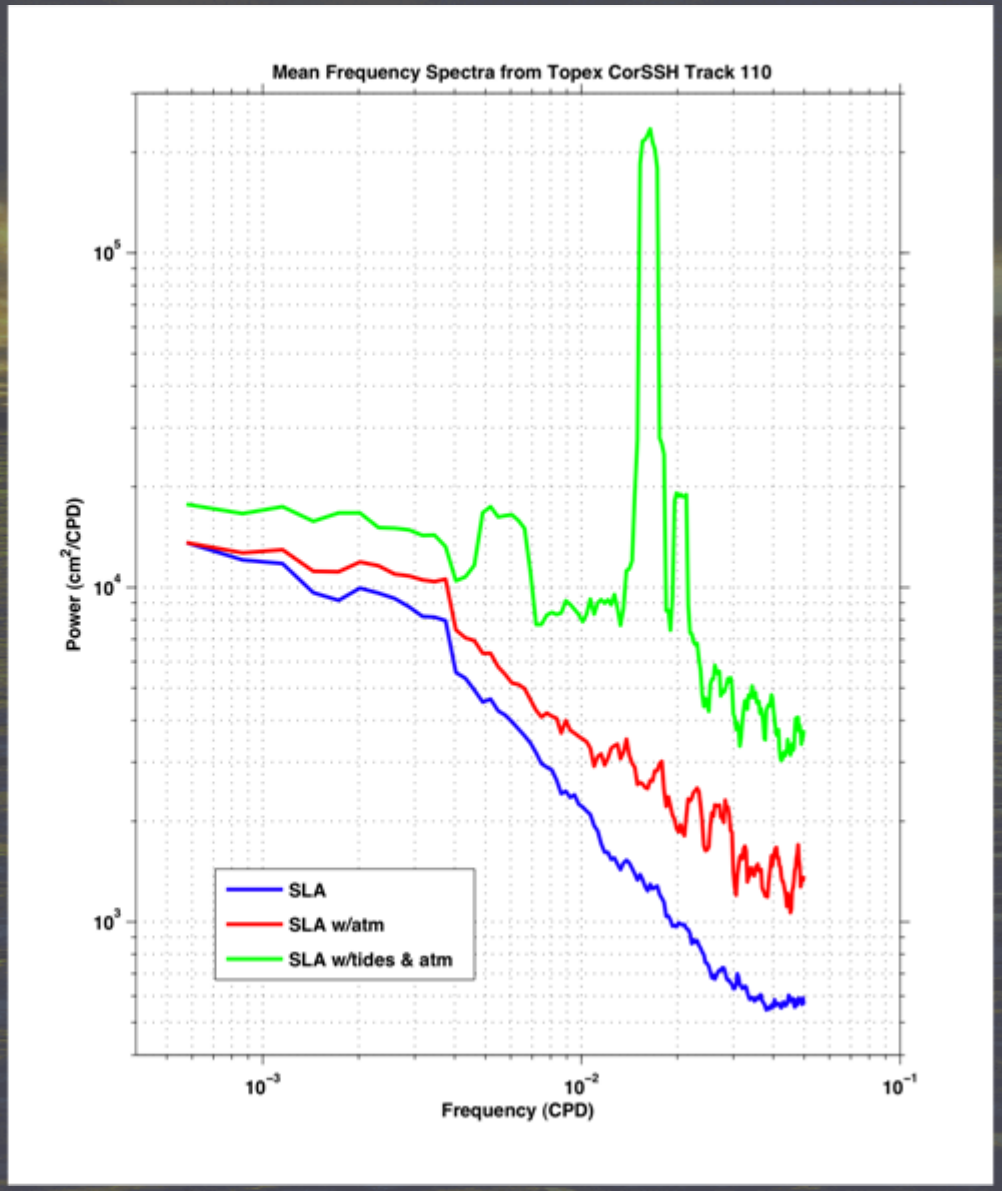
SSH Spectra from Altimetry



Source: AVISO website

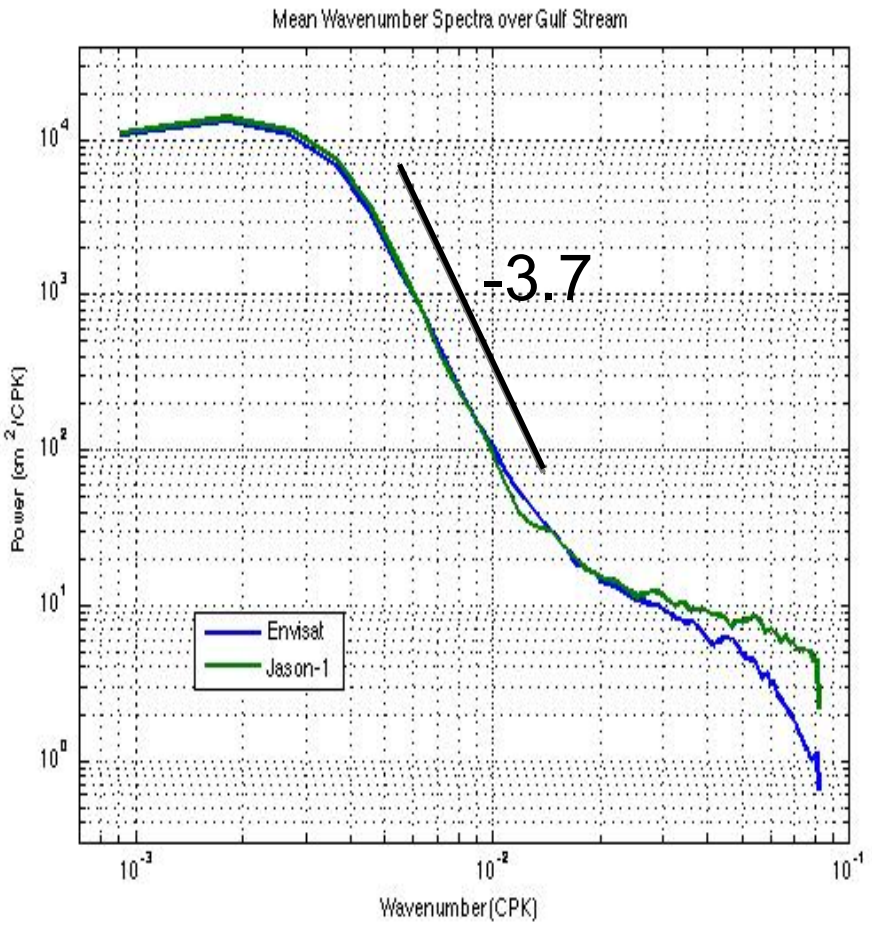
Altimetry Corrections

- Tidal signal removed by FES2004 tidal model
- AVISO Dynamic Atmospheric Correction:
  - Static "Inverted Barometer" response to pressure removed using ECMWF pressure field
  - Dynamic response to wind and pressure simulated by Mog2D FE barotropic model

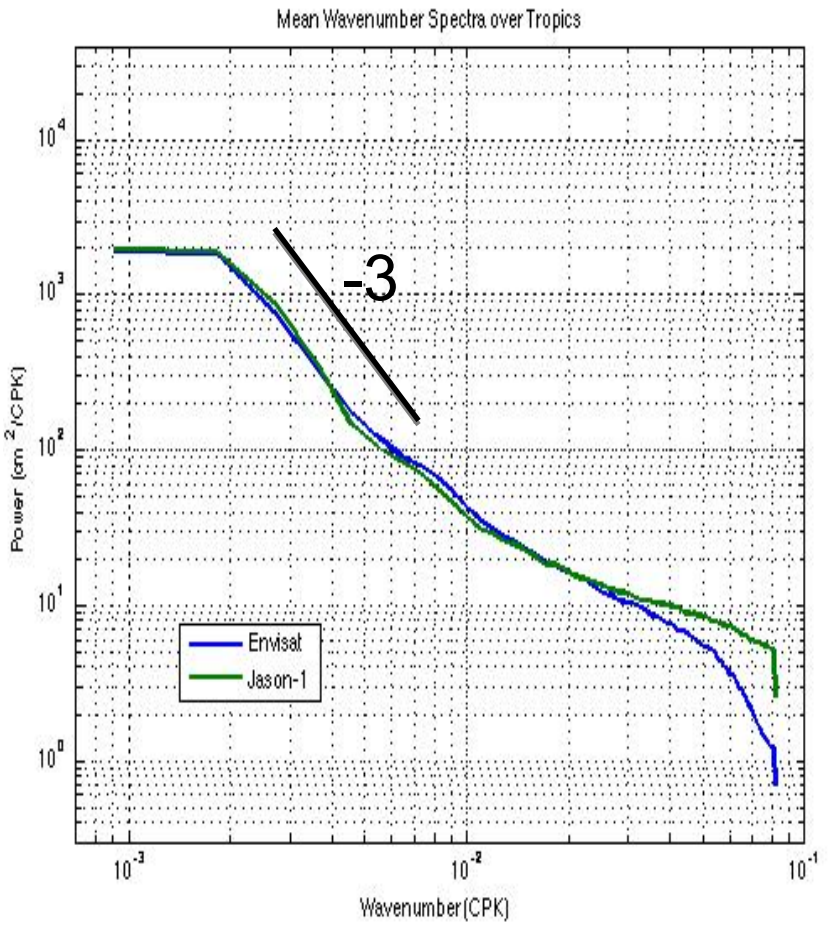


SSH Wavenumber Spectra: Jason and Envisat

Gulf Stream



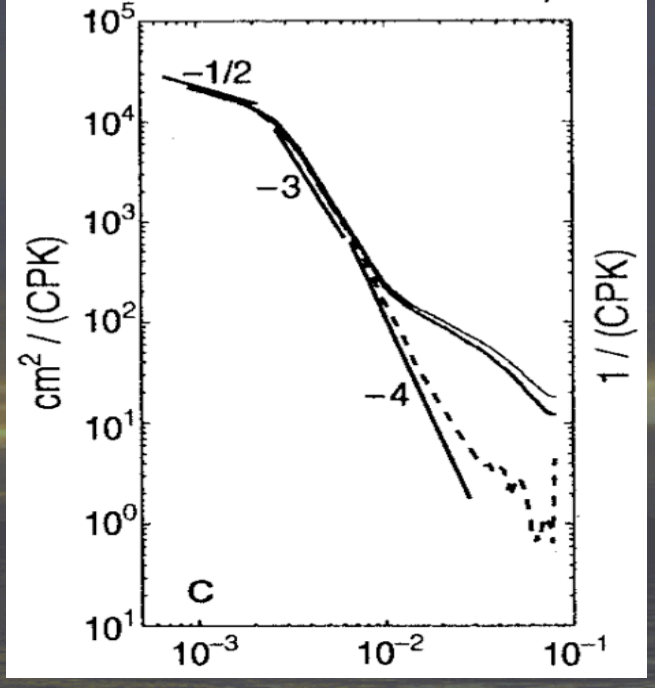
Tropics

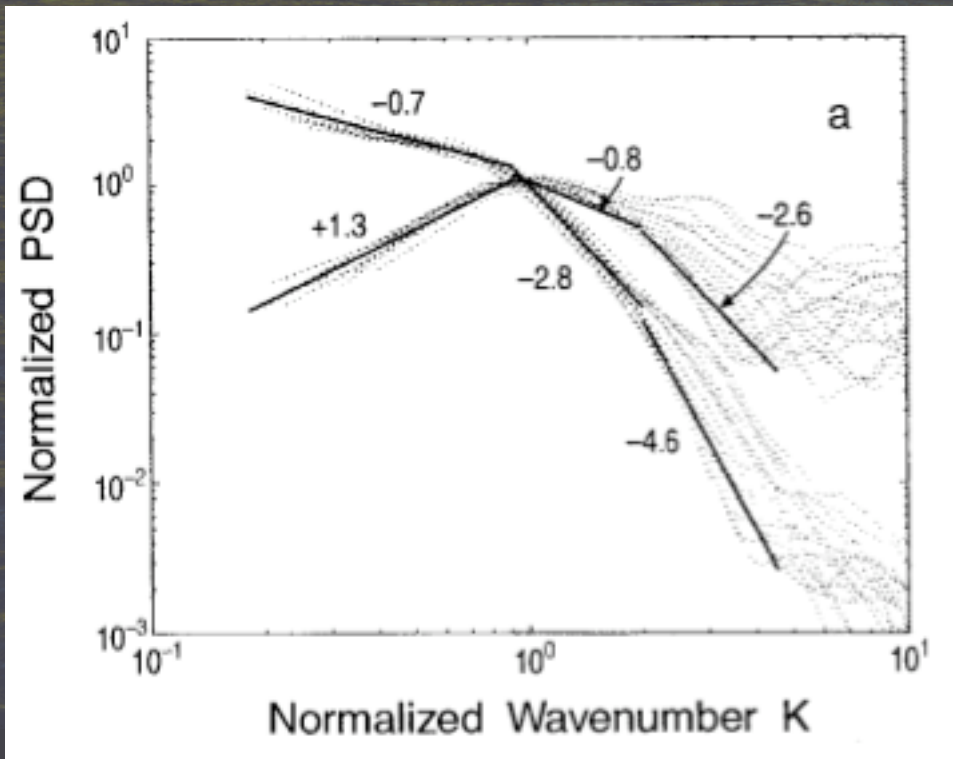


Stammer (1997)

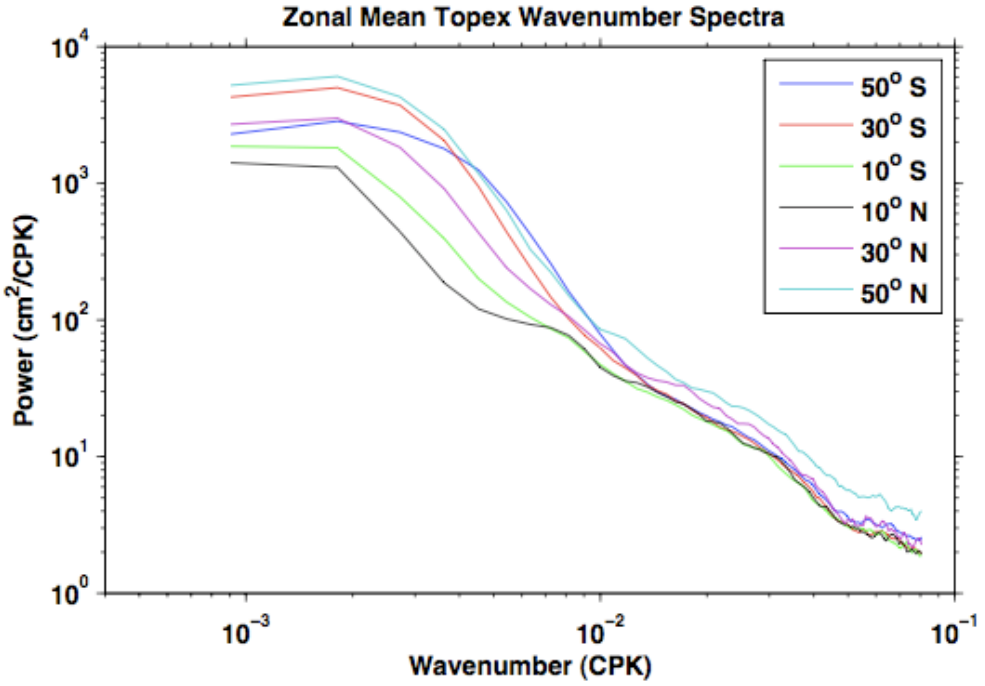
- Optimal filter to remove noise
- Spectral shape independent of latitude when nondimensional wavenumber k’ is used.

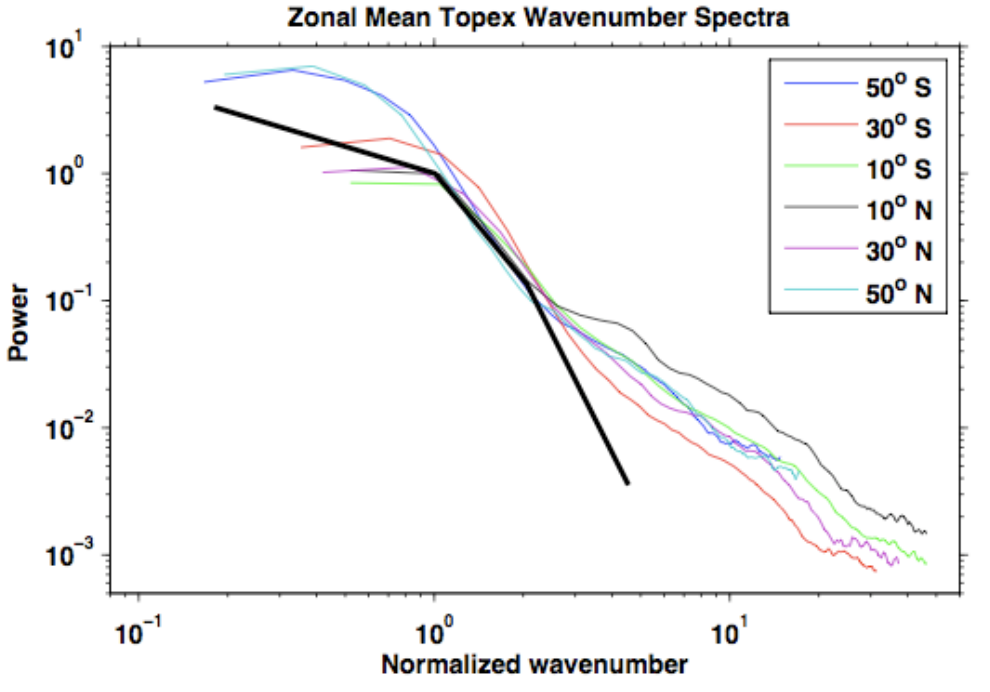
$$k' = k^*[2\pi(0.8+\varphi/25)^*L_{ro}]$$

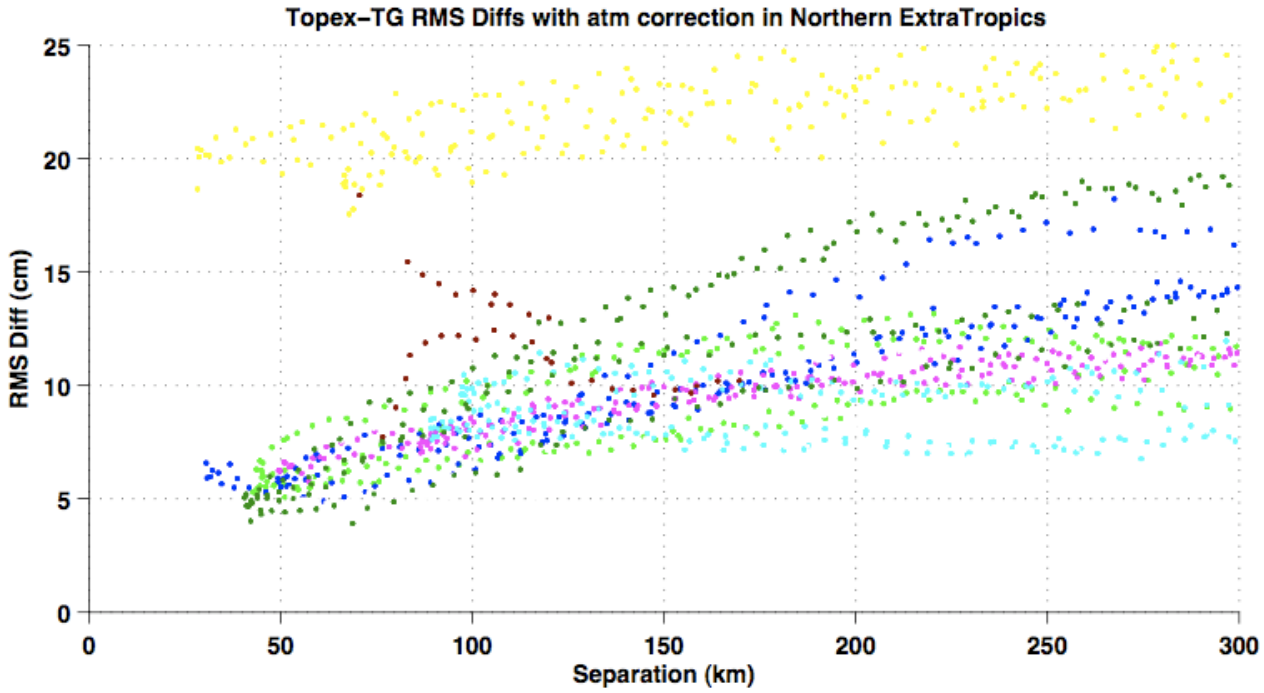


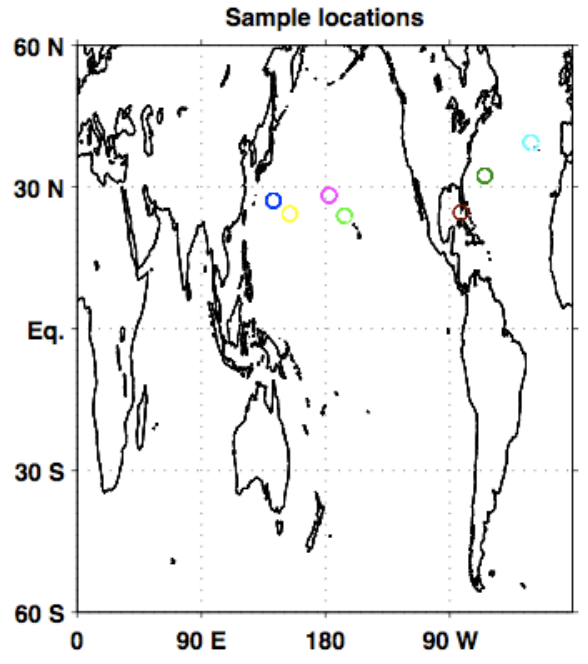


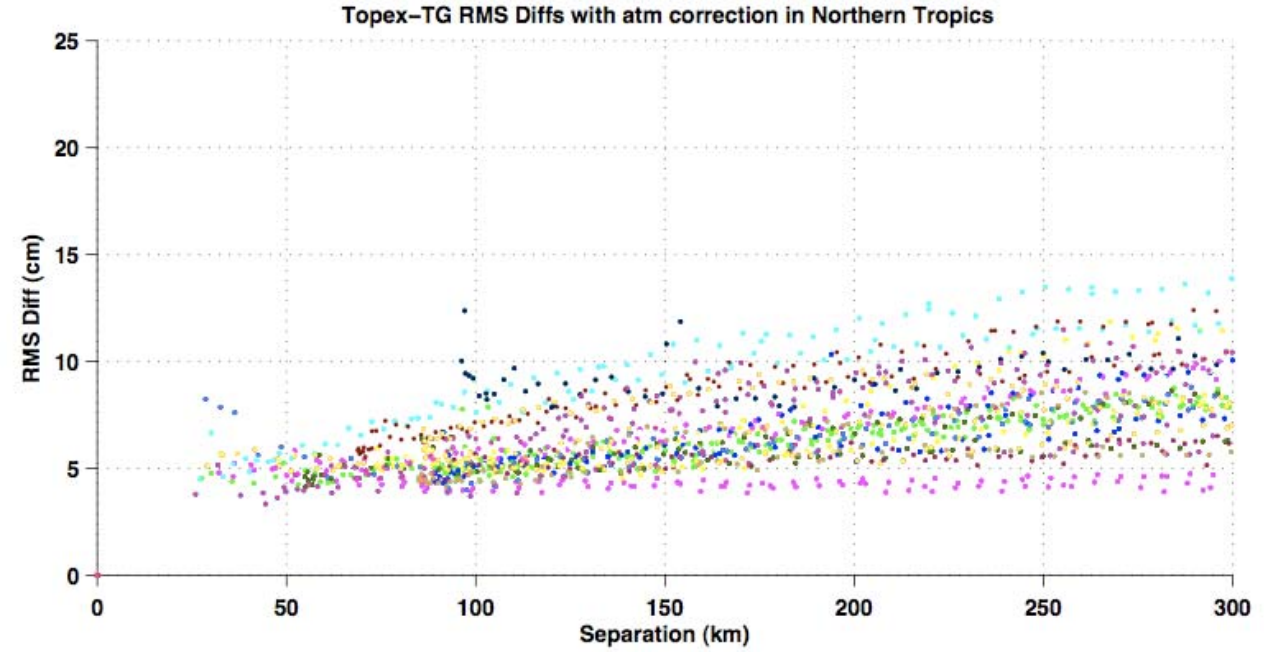
Spectra based on AVISO products with normalized wavenumber






Topex–TG RMS Diffs with atm correction in Northern ExtraTropics
RMS Diff (cm)
Separation (km)
0
5
10
15
20
25
0
50
100
150
200
250
300


Sample locations
60 N
30 N
Eq.
30 S
60 S
0
90 E
180
90 W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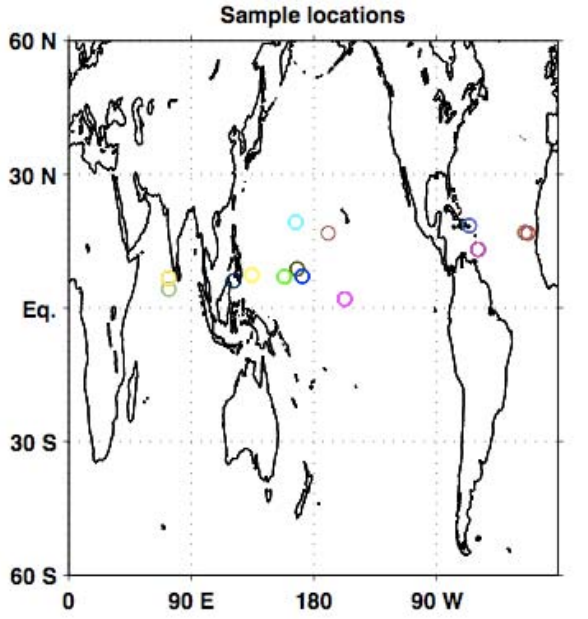
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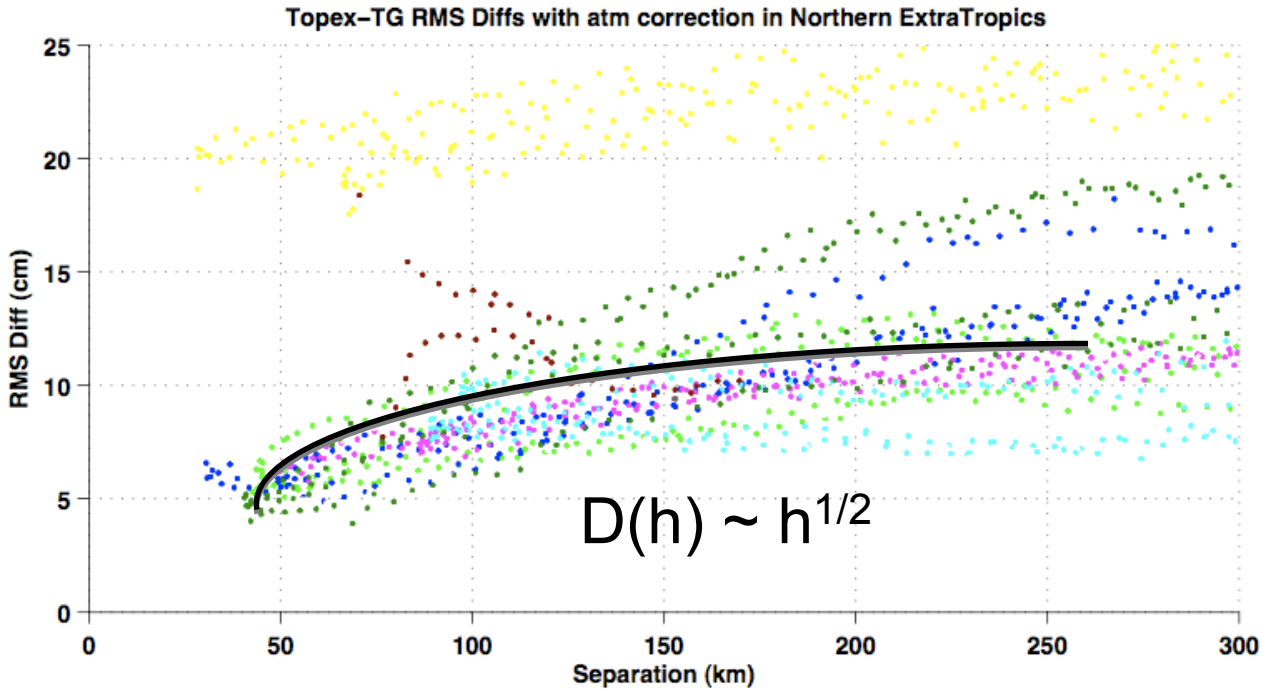
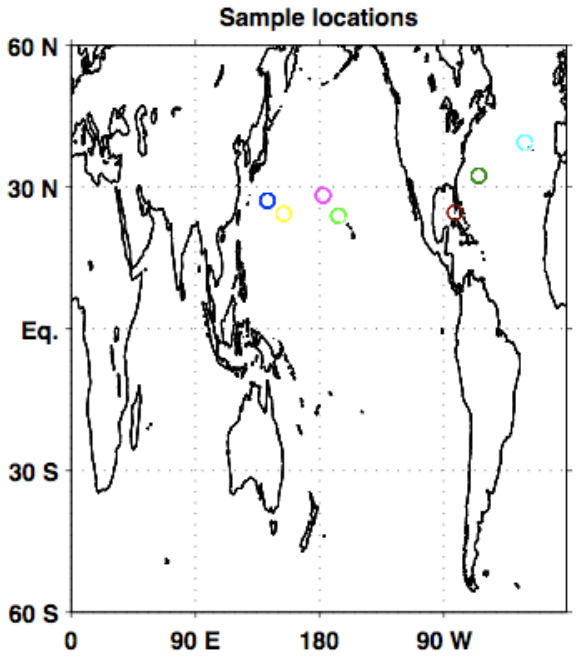




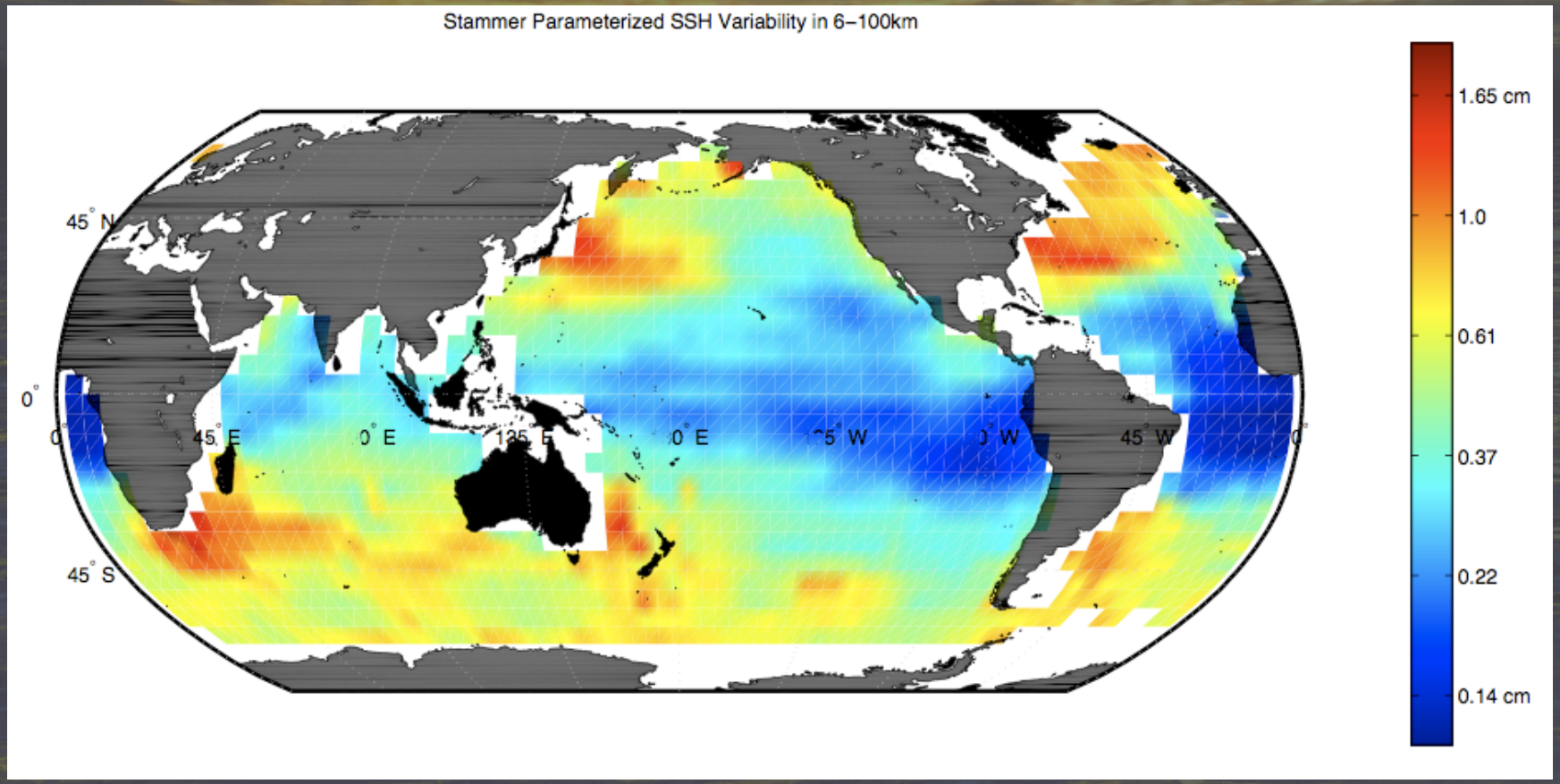
Structure function $D(h) = \text{sqrt } \overline{[s(x+h)-s(x)]^2}$

Where if $P(k) \sim k^{-n}$ and $D(h) \sim h^{p}$

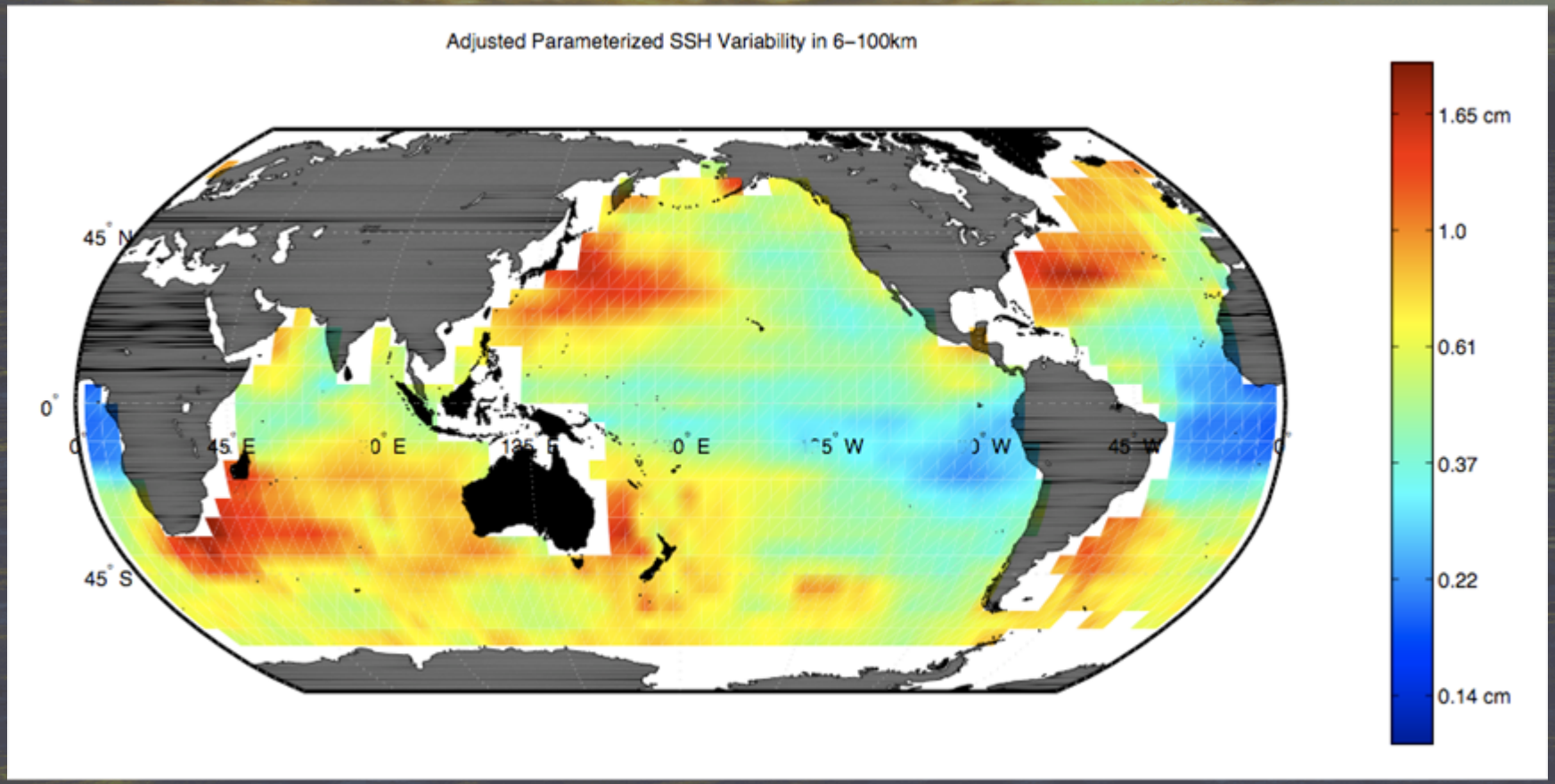
Then $n = p + 1$

Thus a slope of -4.6 at high wavenumbers implies p of 3.6, much larger than observed.

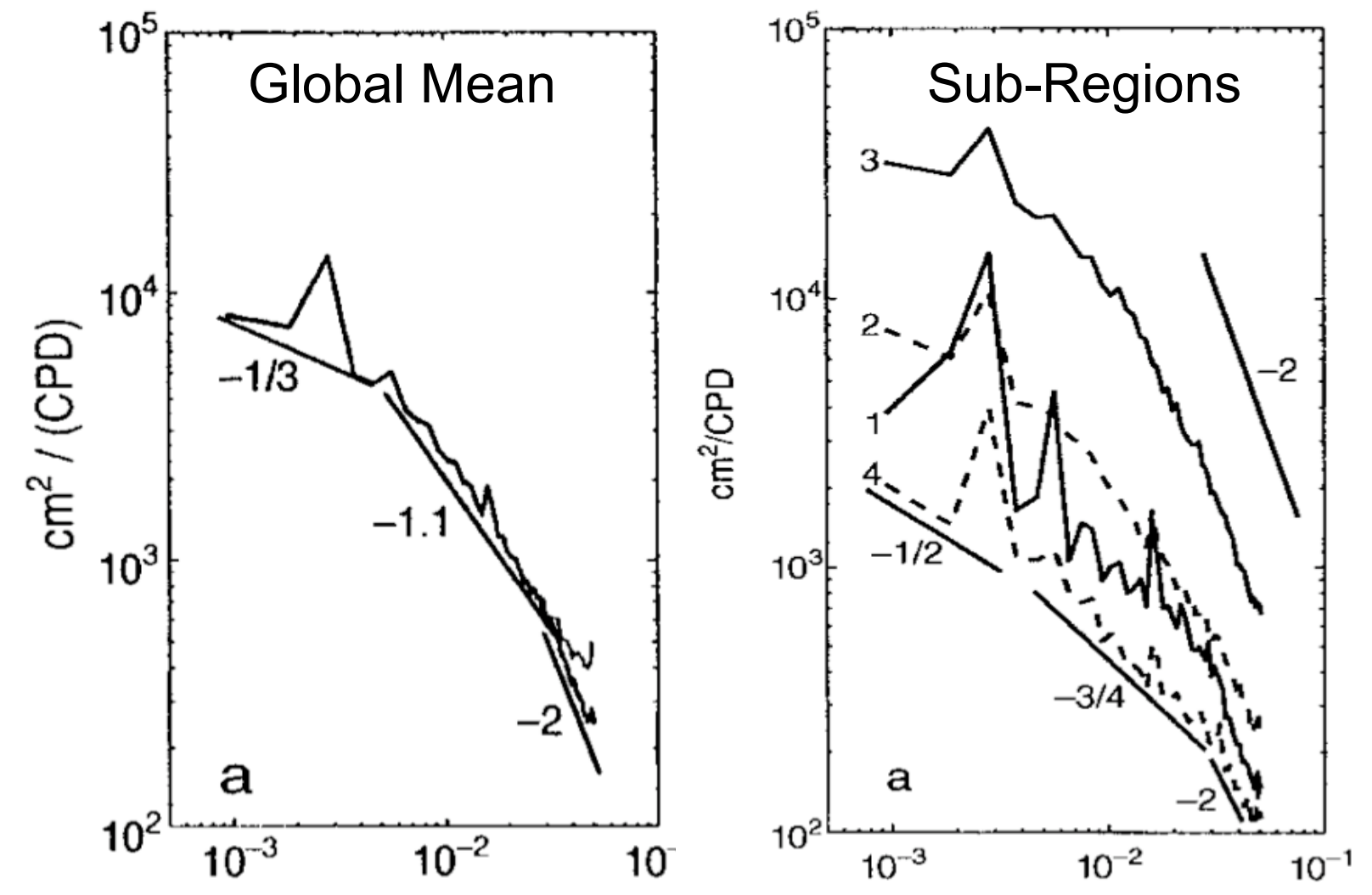
Spatial Sampling Error based on Stammer Parameterization



Spatial sampling error based on adjusted high-wavenumber slope



The Temporal Case

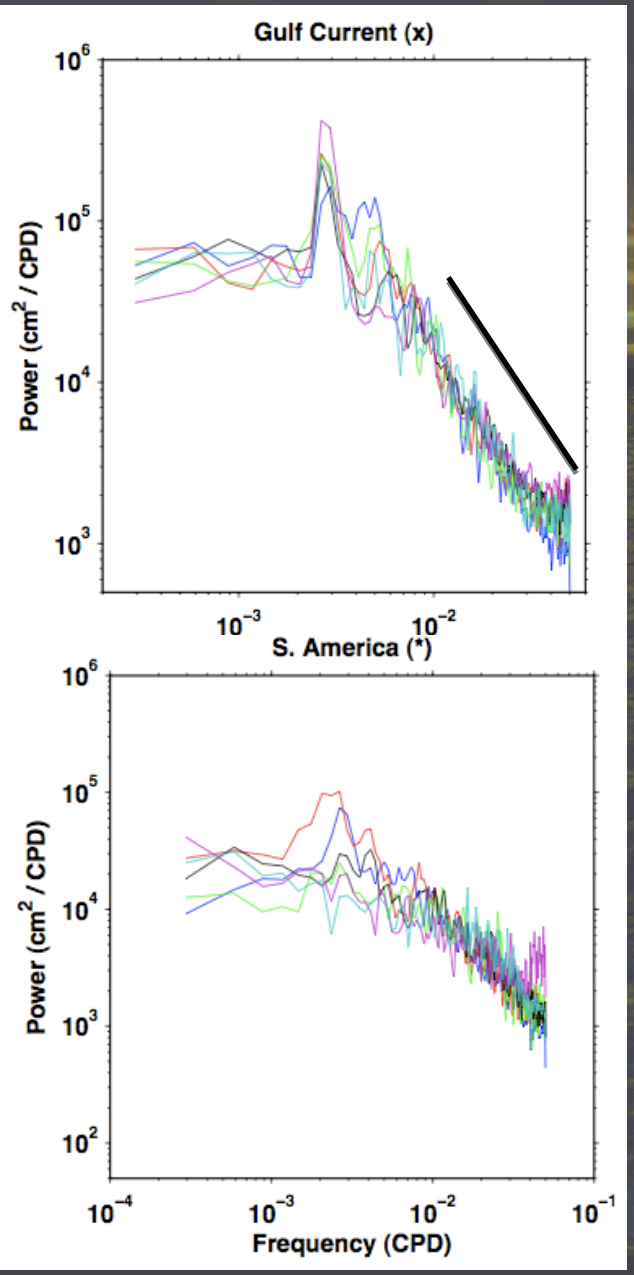


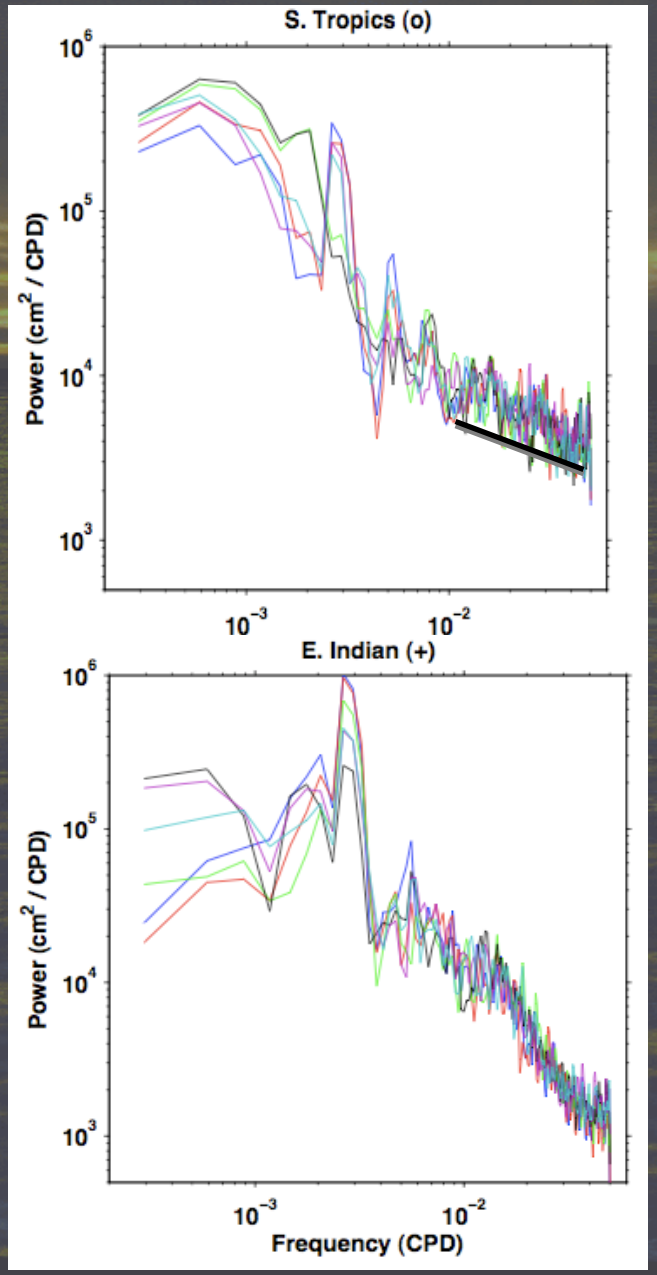
(from Stammer 1997)

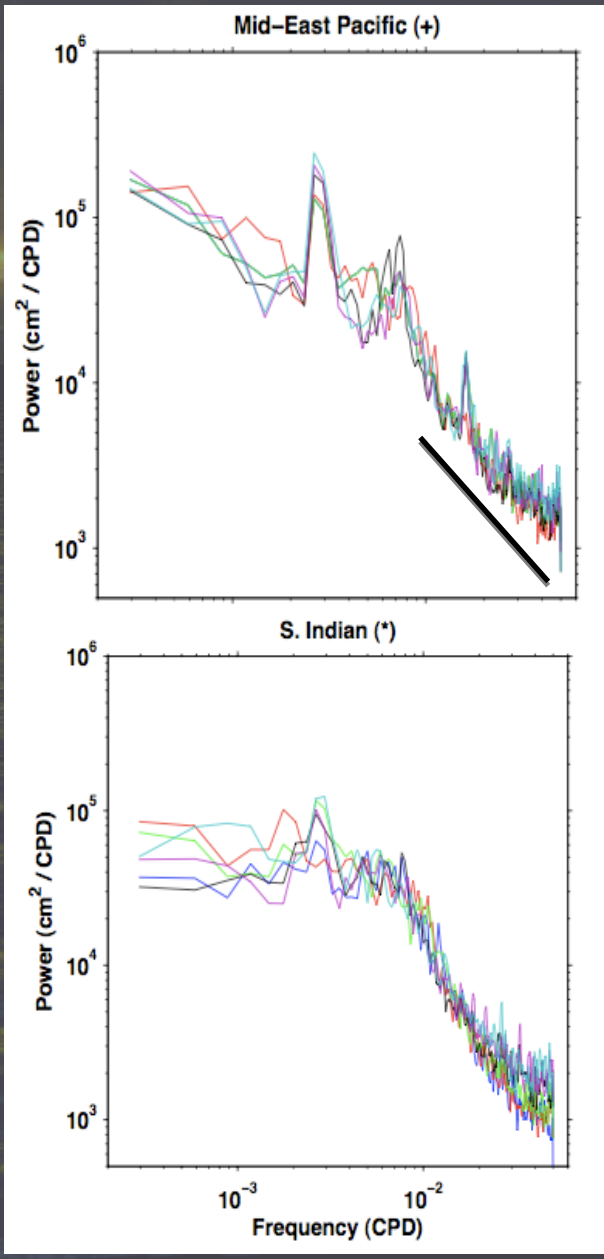
High-Energy

Tropics

Open Ocean

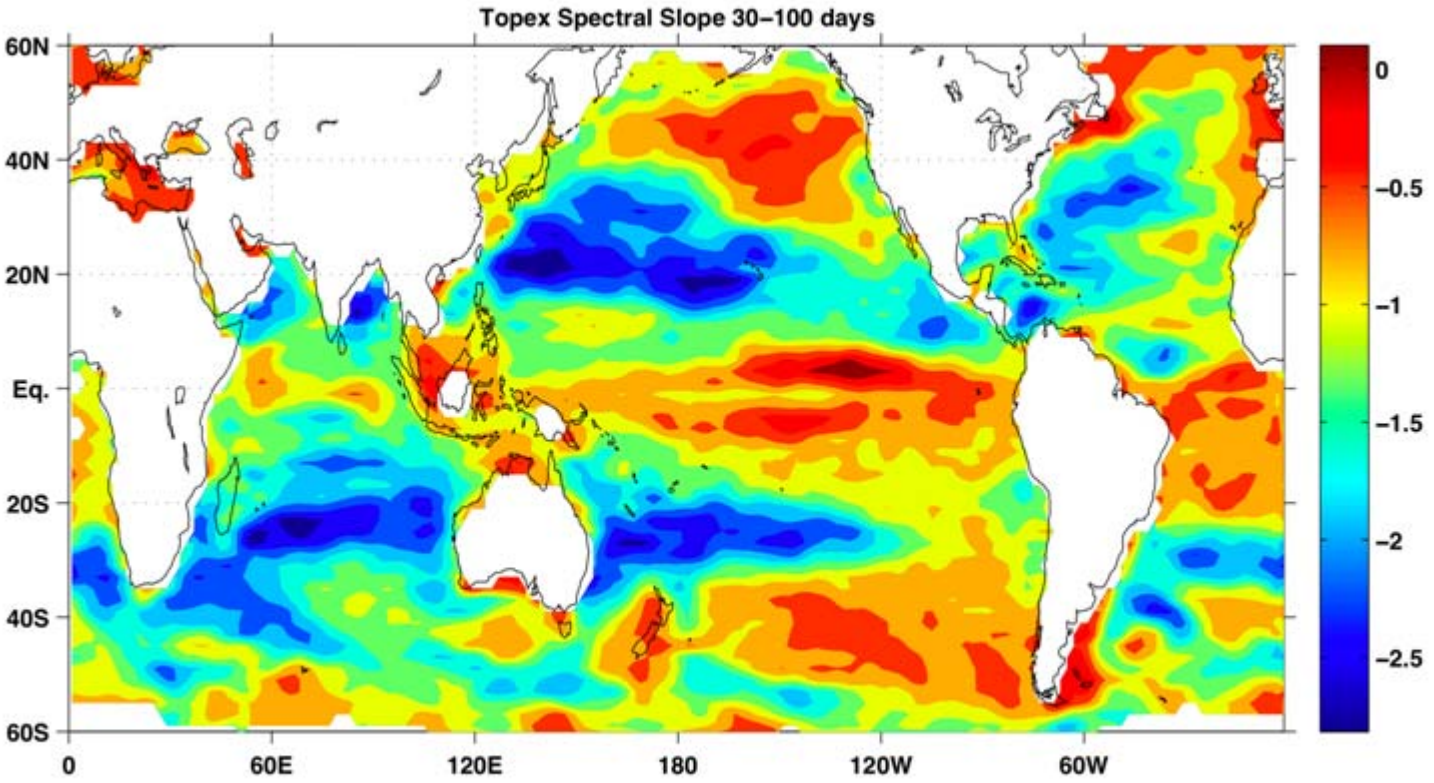






Global map of SSH spectral slope

(30-100 day periods)



Eddy tracks from Chelton, et. al. 2006

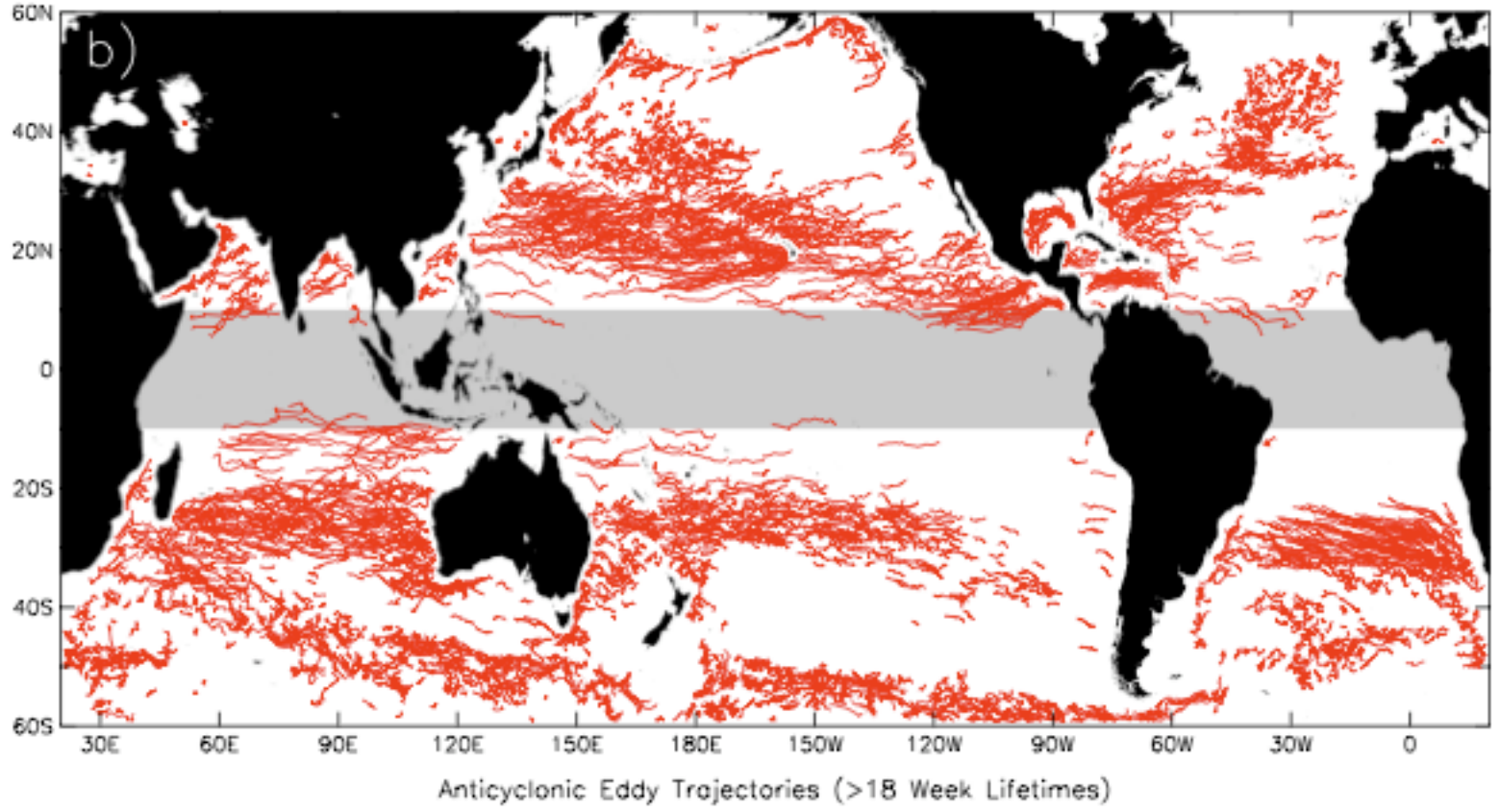
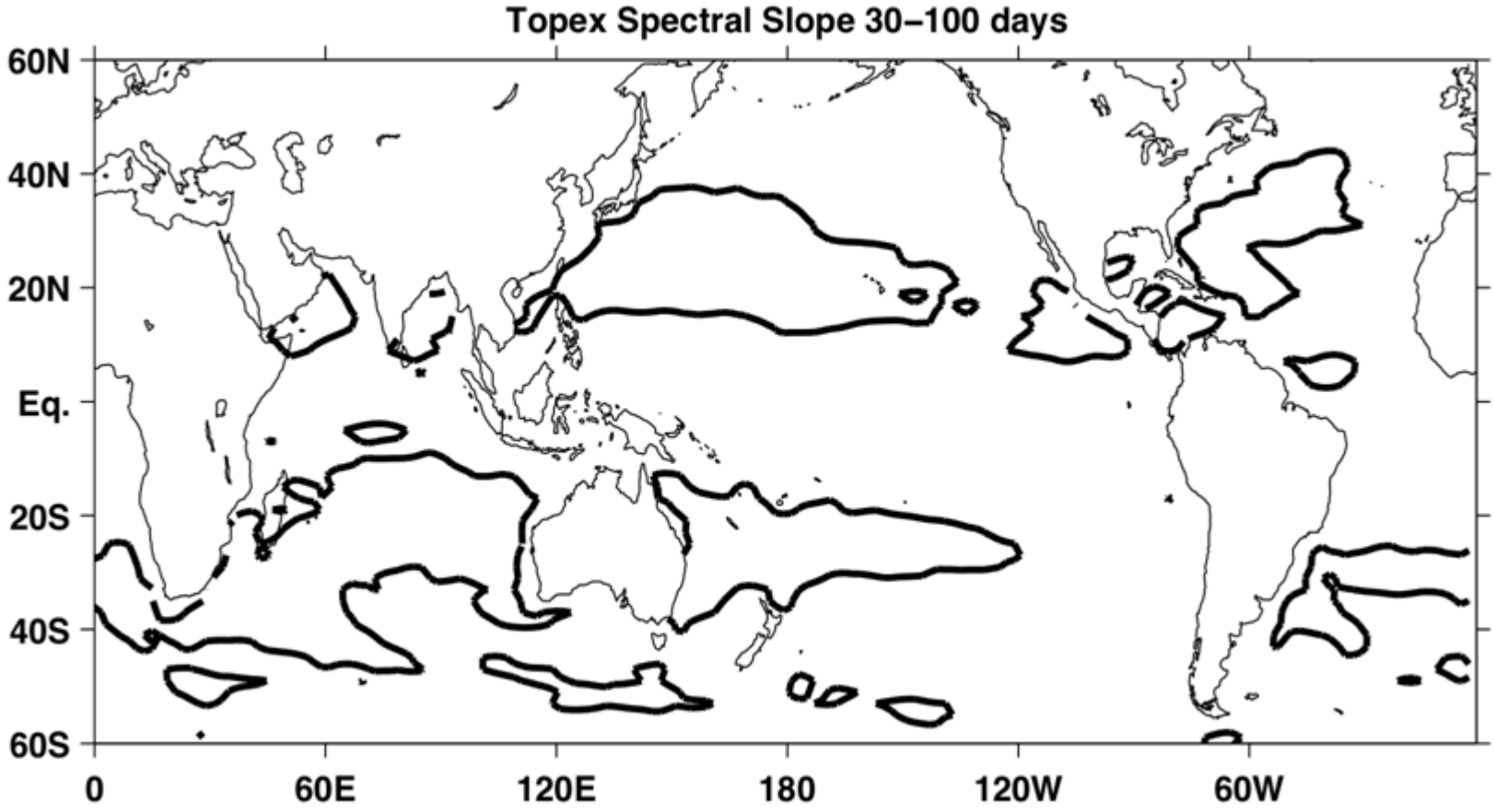


Fig. S5. The trajectories of a) cyclonic and b) anticyclonic eddies with lifetimes longer than 18 weeks and the other tracking criteria summarized in the caption for Fig. 2. Globally there were 5852 such eddies over the 10-year data record, accounting for 6.4% of the total tracked eddies. The numbers of cyclonic and anticyclonic eddies were each 2926. This is an electronic version of Figs. 2c,d in the text.

with -1.75 slope contour



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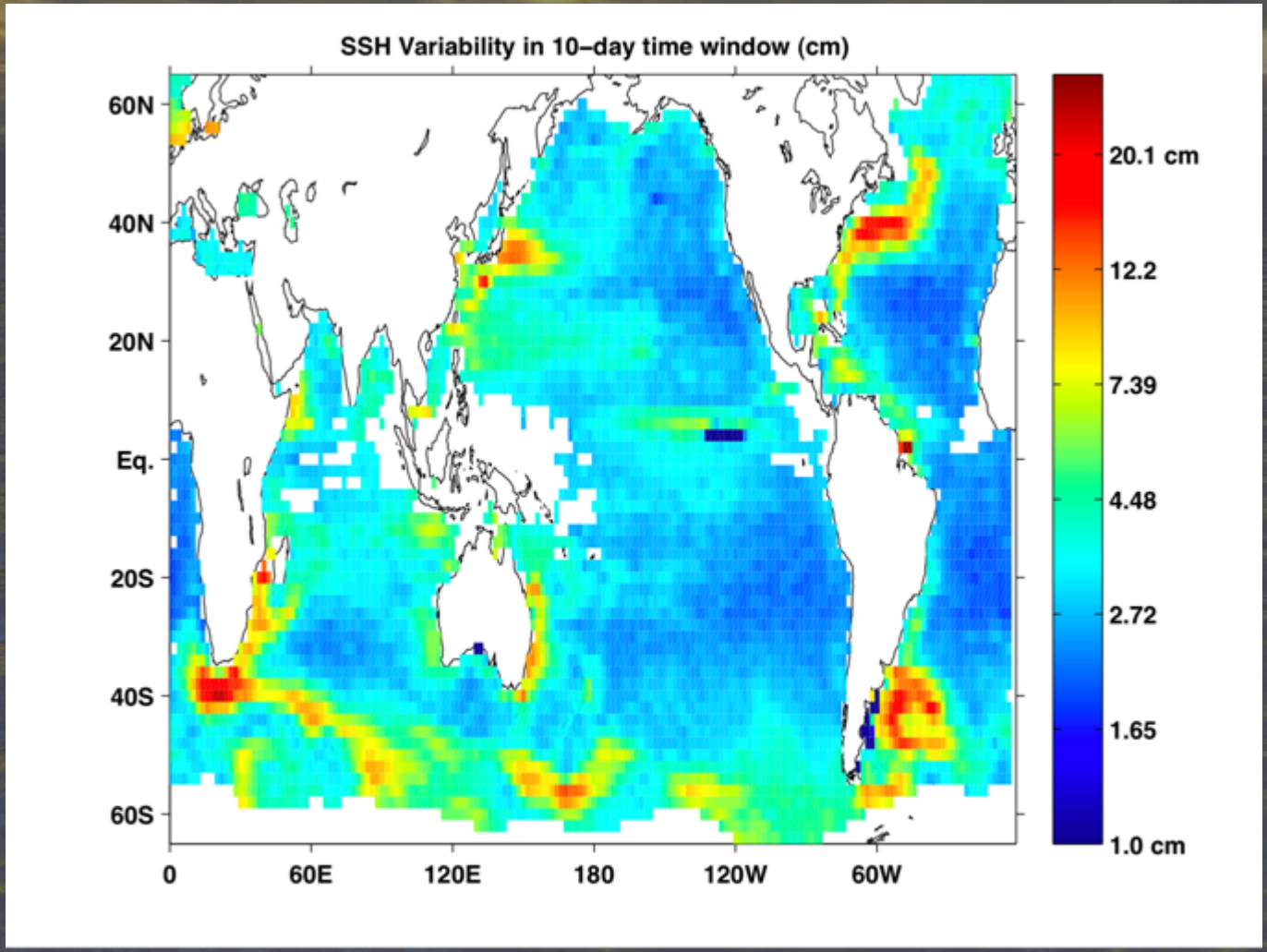
Temporal Error Estimates



Conclusions and Outlook

- **We have made a systematic analysis of global SSH wavenumber and frequency spectra, from altimetry and tide gauges**
- **Spectra were used to estimate variability of SSH within space-time window of data-assimilation**
- **High wavenumber slope of -4.6 from previous work is inconsistent with Topex-tide gauge analysis, and is likely too steep**
- **Interesting correlation between eddy activity and steeper frequency spectral slope**
- **More work is needed to constrain short-term and small-scale variability, and expand work to SST**