Electron transport and heating in semiconductor devices and circuits

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- Introduction
- Device modeling
- Circuit modeling
- Numerical examples
Introduction

Compact modeling approach:
• Describe devices by substituting circuits (compact models)
• Solve compact model using fitting parameters

Multiphysics modeling approach:
• Combine semiconductor and circuit models
  (Tischendorf 2003, Bartel/Günther 2003)
• Here: include also lattice temperature effects
Introduction

Coupled device-circuit model:

Device temperature

Electron transport

Heating in circuit elements

Circuit elements

① Electron transport: energy-transport model
② Device temperature: heat equation
③ Circuit elements: circuit model
④ Heating in circuit elements: thermal network model
1 Device modeling

Energy-transport equations

Derivation from Boltzmann eq. with dominant elastic scattering (Stratton 1962, Ben Abdallah/Degond 1996)

Balance equations:

$$\partial_t n - \text{div } J_n = 0$$
$$\frac{3}{2} \partial_t (nT) - \text{div } S_n = -J_n \cdot \nabla V + \frac{3}{2} \frac{n(T-T_0)}{\tau}$$

Current equations:

$$J_n = \nabla (\mu_n(T_L) T_L n) - \mu_n(T_L) T_L \frac{n}{T} \nabla V$$
$$S_n = \frac{3}{2} (\nabla (\mu_n(T_L) T_L nT) - \mu_n(T_L) T_L n \nabla V)$$

- $T$ electron temperature, $T_L$ lattice temperature
- Mobility $\mu_n(T_L) = \mu_0(T_0/T_L)^{\alpha}$: $\alpha$ depends on elastic scattering model (Degond/A.J./Pietra 2000)
Energy-transport equations: electrons and holes

\[ \partial_t n - \text{div} J_n = R(n, p), \quad J_n = \nabla n - \frac{n}{T} \nabla V \]

\[ \frac{3}{2} \partial_t (nT) - \text{div} S = -J_n \cdot \nabla V + \frac{3}{2} \frac{n(T-T_0)}{\tau} \]

\[ S = \frac{3}{2} (\nabla (nT) - n \nabla V) \]

\[ \lambda^2 \Delta V = n - p - C(x) \]

\[ \partial_t p - \text{div} J_p = R(n, p), \quad J_p = \nabla p + p \nabla V \]

Numerical difficulties:

- Convection dominance due to high electric fields
- Positivity preservation of \( n, p \) and \( T \), current conservation
- Iterative scheme for nonlinear discrete system
Device modeling

Discretization of stationary equations

\[-\text{div} \, J + cn = f, \quad J = \nabla n - \frac{n}{T} \nabla V \quad \text{in } \Omega = \bigcup_i K_i\]

Formulation in extensive variables $n, \frac{3}{2} n T$:

- Slotboom var. $z = e^{-V/T} n$, $T|_{K_i} = \text{const.} \Rightarrow J = e^{V/T} \nabla z$
- Mixed finite elements: $z_h \in L^2$ piecewise constant, $J_h \in H_{\text{loc}}(\text{div})$ Marini-Pietra ($P_2$ with 3 DOF) $\rightarrow$ M-matrix
- Decoupling Gummel method (Newton in Poisson equation)

Formulation in dual entropy variables $u_1 = \frac{\mu - V}{T}$, $u_2 = -\frac{1}{T}$:

- Chem. potential $\mu$: $n = T^{3/2} e^{\mu/T} \Rightarrow$ eliminates convection
- Mixed finite elements: $z_h \in L^2$ piecewise constant, $J_h \in H_{\text{loc}}(\text{div})$ Raviart-Thomas $\rightarrow$ M-matrix
- Full Newton or Gummel with vector extrapolation
Device modeling

3D Single-Gate MESFET (Gadou/A.J. 2008)

Closed state

Open state
Device modeling

3D Gate-All-Around MESFET (Gadau/A.J. 2008)

Geometry

Temperature

Thermal energy

Current density
2 Lattice heat modeling

- Define internal energy $u$ and energy flux $J_u$:
  \[
  u = \text{electric} + \text{lattice} + \text{conduction-band} + \text{thermal energies}
  \]
  \[
  J_u = \text{displacement} + \text{Fourier} + \text{particle} + \text{dissipated-power} + \text{band-energy fluxes}
  \]
- Conservation equation $\partial_t u - \text{div} J_u = 0 \Rightarrow$ heat equation
  \[
  \rho_L c_L \partial_t T_L - \text{div} (\kappa_L \nabla T_L) = H
  \]
  \[
  H = -\frac{3}{2} \frac{n(T - T_L)}{\tau} + R(n, p)(E_c + \frac{3}{2}T) - S_L(T_L - T_0)
  \]
  material density $\rho_L$, lattice heat capacity $c_L$, radiation $S_L(T_L - T_0)$, conduction-band energy $E_c$
- Coupling to energy-transport through energy relaxation
1D bipolar diode: forward bias 1.5 V (Brunk/A.J. 2008)

- Temperature minimum around junction
- Almost constant lattice temperature due to high heat conductivity
- Lattice heating decreases current density
• Kirchhoff’s current law: $A i = 0$
  $A$: matrix of node-to-branch relations, $i$: branch currents

• Kirchhoff’s voltage law: $v = A^\top e$
  $v$: branch voltages, $e$: node potentials

• Current-voltage characteristics:

\[ v_R = R i_R, \quad i_C = C \frac{dv_C}{dt}, \quad v_L = L \frac{di_L}{dt} \]

Modified nodal analysis: source current $i_s$, voltage $v_s$

\[ A_C C A_C^\top \frac{de}{dt} + A_R R^{-1} A_R^\top e + A_L i_L + A_V i_V + A_I i_s = 0 \]

\[ L \frac{di_L}{dt} - A_L^\top e = 0, \quad A_V^\top e = v_s \]

• Differential-algebraic equations of index 1 or 2

• Numerical methods: Rosenbrock-Wanner (Günther), BDF
Thermal network modeling

Heating of thermal nodes:
• Thermal nodes = resistors, contact nodes...
• ODE for node temperature $\hat{T}$:
  \[ \hat{M} \frac{d}{dt} \hat{T} = \text{source terms} + \text{radiation} + \text{dissipation power} \]
• Dissipation power = current $\times$ electric potential

Heating of distributed elements:
• Distributed elements = electric lines, 1D devices...
• PDE for distributed temperature $T^d$:
  \[ M \partial_t T^d = \partial_x (\kappa_L \partial_x T^d) + \text{radiation} + \text{dissipation power} \]

Coupling to circuit and device models:
• Electro-thermal coupling: dissipation powers
• Thermal-device coupling: through boundary conditions
Model overview

Complete electro-thermal circuit-device model

Nonisothermal semiconductor device model
Energy-transport equations, heat equation for lattice temperature

coupled through semiconductor current and circuit voltages

coupled through semiconductor heat flux and boundary terms

Electric circuit model
Network equations from modified nodal analysis

coupled through electro-thermal sources and resistances

Thermal network model
Heat equations for lumped and distributed elements

→ Nonlinear partial differential-algebraic equations (PDAE)

• Discretization: BDF-2 and Marini-Pietra finite elements

• Iteration: Gummel-type inner loop for nonisothermal semiconductor model, fixed-point-type outer loop
**Numerical example**

**Clipper:** (Brunk/A.J. 2008)

- Used for entrance protection
- Output voltage should be less than 4 V
- Input frequency of 10 GHz

![Diagram of Clipper circuit](image)

- **Lattice temperature**
- **Output signal**

![3D lattice temperature graph](image)

![Output signal graph](image)
Novelties:
• Modeling of nonisothermal semiconductor-circuit system
• Numerical solution of complete nonlinear system

Final result:
Lattice heating may lead to undesired large output signal in clipper circuit