Electron transport and heating in semiconductor devices and circuits

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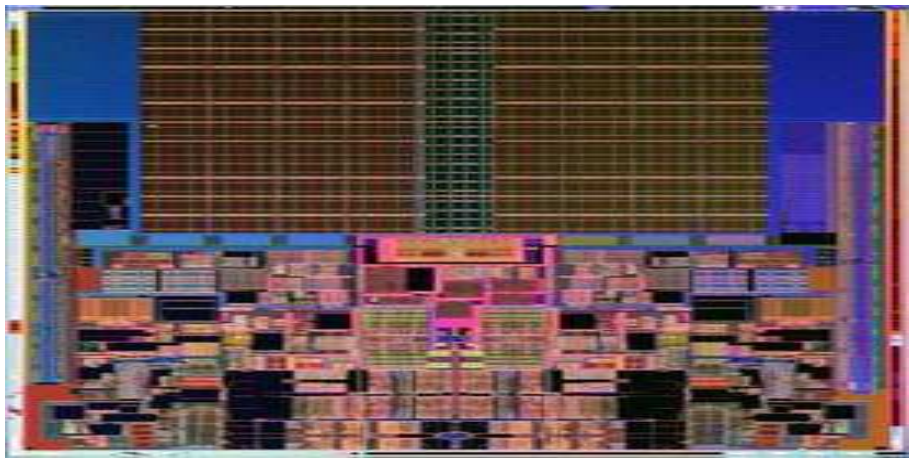
Joint work with Markus Brunk, Stephan Gadau, Stefan Holst

- Introduction
- Device modeling
- Circuit modeling
- Numerical examples

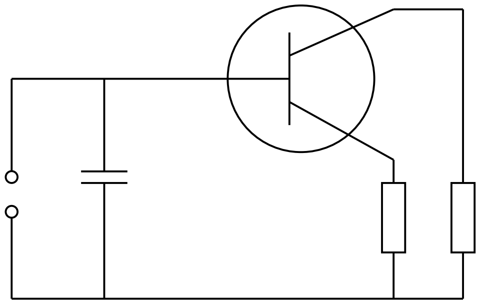


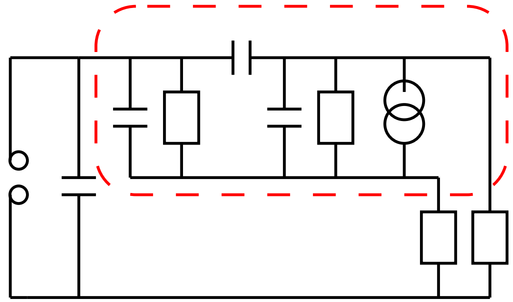
Introduction

Compact modeling approach:

- Describe devices by substituting circuits (compact models)
- Solve compact model using fitting parameters

electric circuit → circuit diagram → compact model

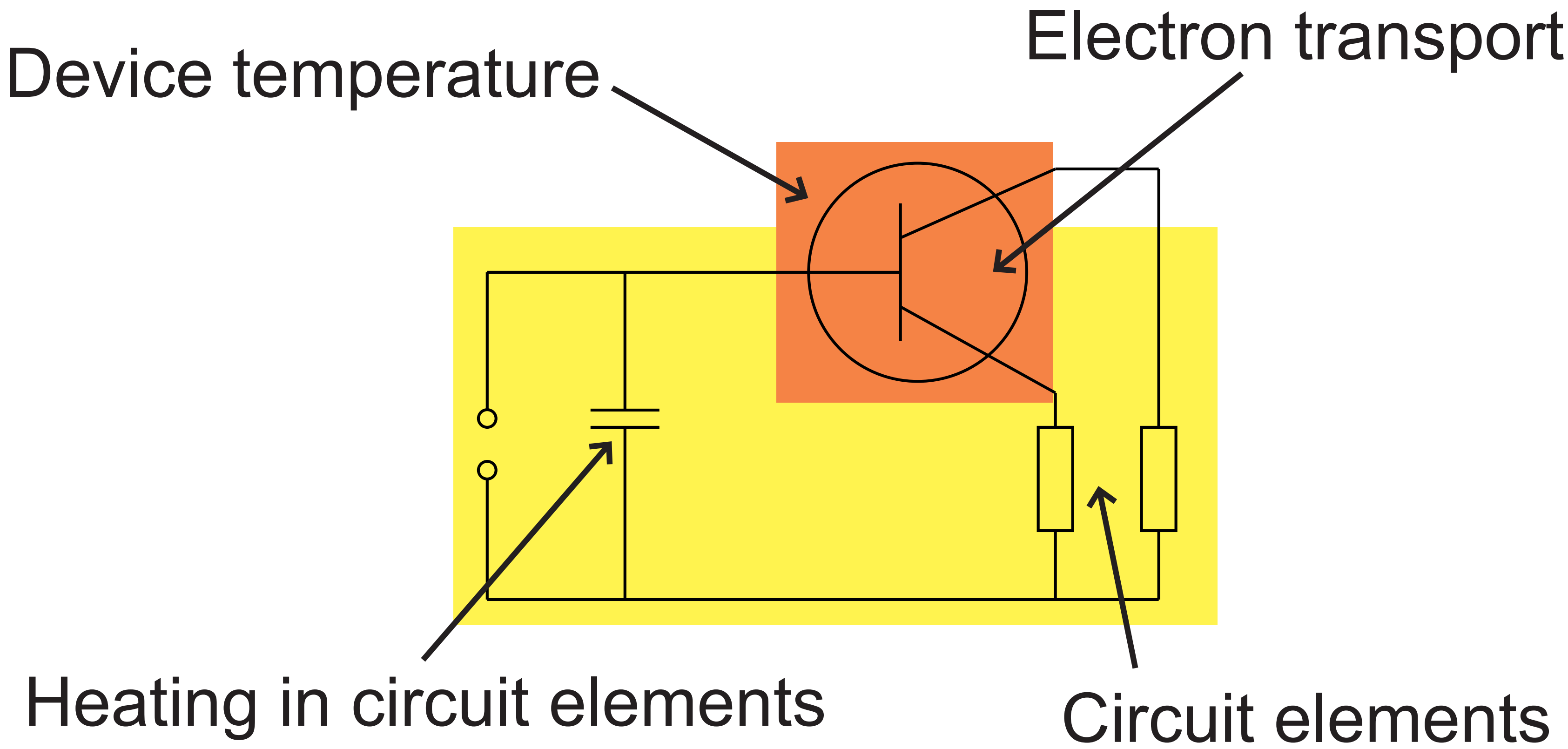




Multiphysics modeling approach:

- Combine semiconductor and circuit models (Tischendorf 2003, Bartel/Günther 2003)
- Here: include also lattice temperature effects

Coupled device-circuit model:



1. Electron transport: energy-transport model
2. Device temperature: heat equation
3. Circuit elements: circuit model
4. Heating in circuit elements: thermal network model

Energy-transport equations

Derivation from Boltzmann eq. with dominant elastic scattering (Stratton 1962, Ben Abdallah/Degond 1996)

Balance equations:

$$\partial_t n - \operatorname{div} J_n = 0$$
$$\tfrac{3}{2}\partial_t(nT) - \operatorname{div} S_n = -J_n \cdot \nabla V + \tfrac{3}{2}\frac{n(T-T_0)}{\tau}$$

Current equations:

$$J_n = \nabla(\mu_n(T_L)T_L n) - \mu_n(T_L)T_L \tfrac{n}{T}\nabla V)$$
$$S_n = \tfrac{3}{2}(\nabla(\mu_n(T_L)T_L nT) - \mu_n(T_L)T_L n\nabla V)$$

- T electron temperature, T_L lattice temperature
- Mobility $\mu_n(T_L) = \mu_0(T_0/T_L)^\alpha$: α depends on elastic scattering model (Degond/A.J./Pietra 2000)

① Device modeling

Energy-transport equations: electrons and holes

$$\partial_t n - \operatorname{div} J_n = R(n,p), \quad J_n = \nabla n - \frac{n}{T}\nabla V$$

$$\frac{3}{2}\partial_t(nT) - \operatorname{div} S = -J_n \cdot \nabla V + \frac{3}{2}\frac{n(T-T_0)}{\tau}$$

$$S = \frac{3}{2}(\nabla(nT) - n\nabla V)$$

$$\lambda^2 \Delta V = n - p - C(x)$$

$$\partial_t p - \operatorname{div} J_p = R(n,p), \quad J_p = \nabla p + p\nabla V$$

Numerical difficulties:

- Convection dominance due to high electric fields
- Positivity preservation of n, p and T, current conservation
- Iterative scheme for nonlinear discrete system

Discretization of stationary equations

$$-\operatorname{div} J + cn = f, \quad J = \nabla n - \frac{n}{T}\nabla V \quad \text{in } \Omega = \bigcup_i K_i$$

Formulation in extensive variables n, $\frac{3}{2}nT$:

- Slotboom var. $z = e^{-V/T}n$, $T|_{K_i} = \text{const.} \Rightarrow J = e^{V/T}\nabla z$
- Mixed finite elements: $z_h \in L^2$ piecewise constant, $J_h \in H_{\text{loc}}(\text{div})$ Marini-Pietra (P_2 with 3 DOF) $\longrightarrow$ M-matrix
- Decoupling Gummel method (Newton in Poisson equation)

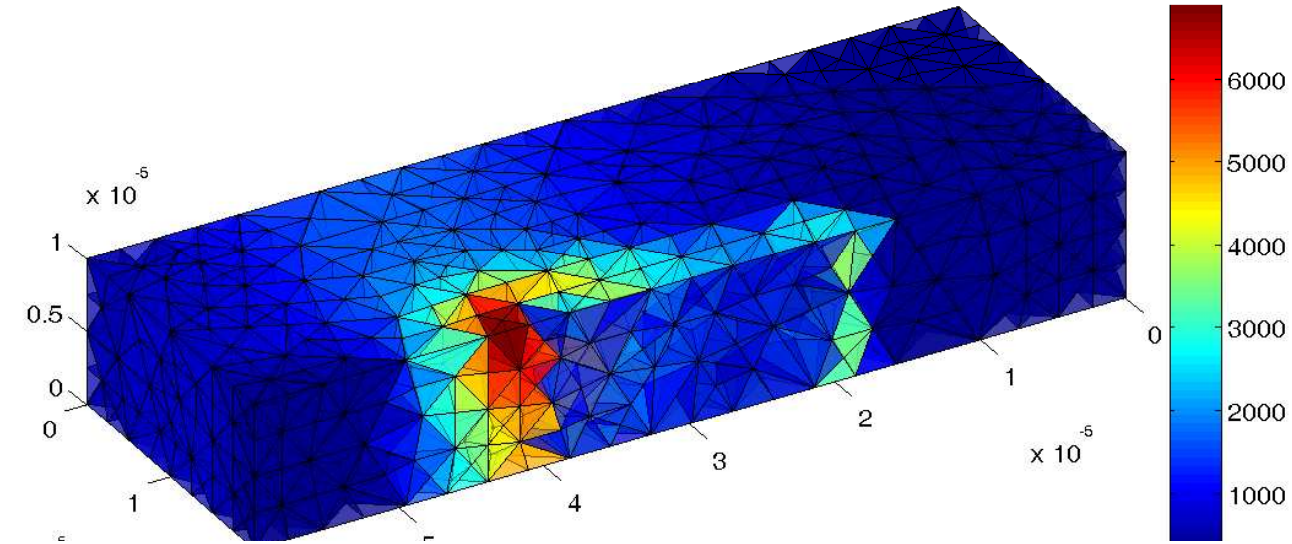
Formulation in dual entropy variables $u_1 = \frac{\mu - V}{T}$, $u_2 = -\frac{1}{T}$:

- Chem. potential μ: $n = T^{3/2}e^{\mu/T} \Rightarrow$ eliminates convection
- Mixed finite elements: $z_h \in L^2$ piecewise constant, $J_h \in H_{\text{loc}}(\text{div})$ Raviart-Thomas $\longrightarrow$ M-matrix
- Full Newton or Gummel with vector extrapolation

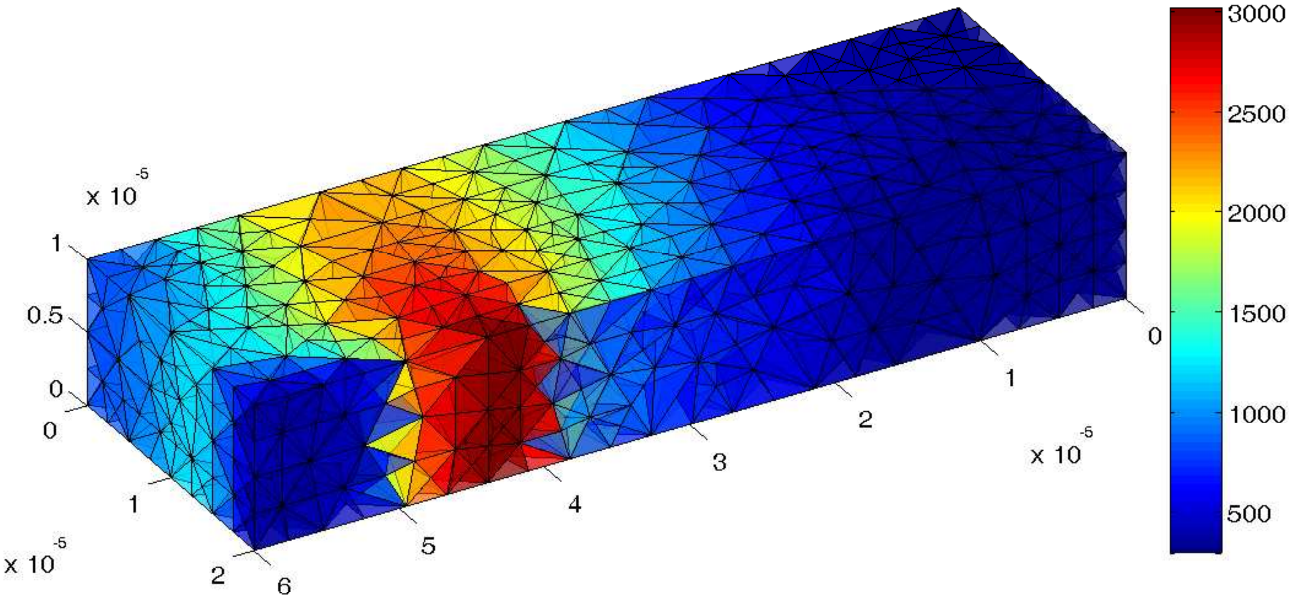
① Device modeling

3D Single-Gate MESFET (Gadau/A.J. 2008)

Closed state

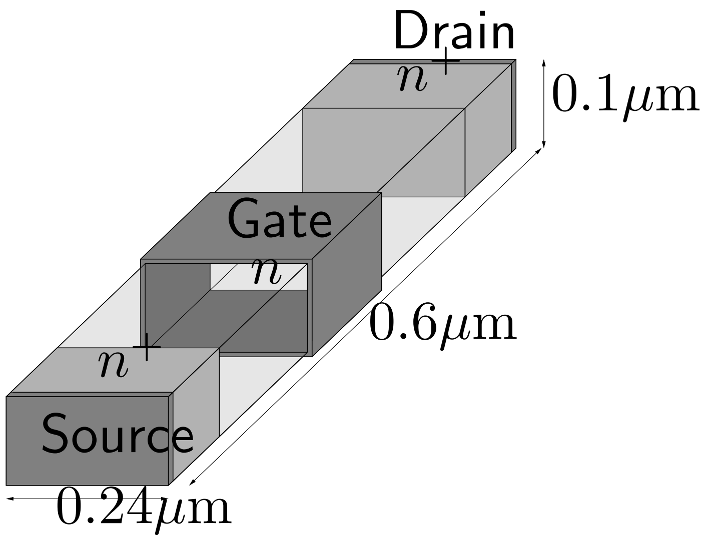


Open state

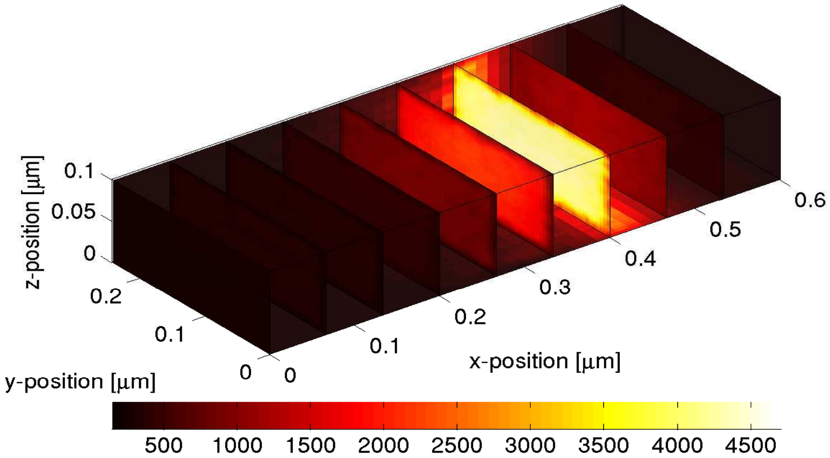


① Device modeling

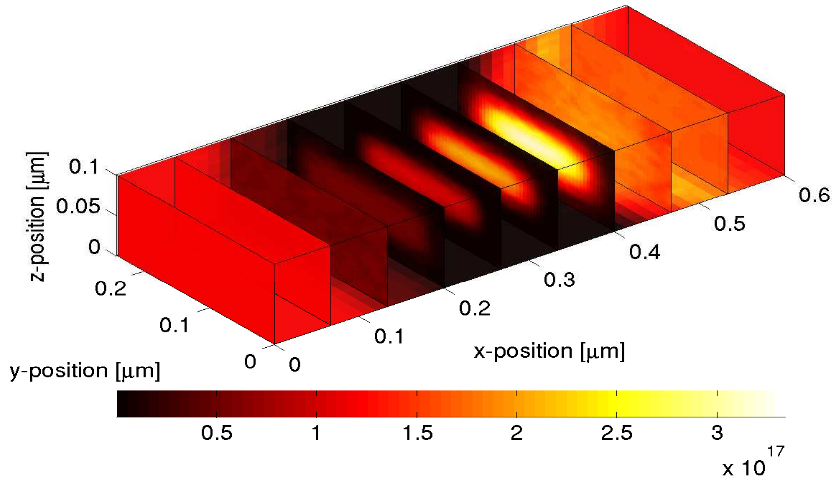
3D Gate-All-Around MESFET (Gadau/A.J. 2008)



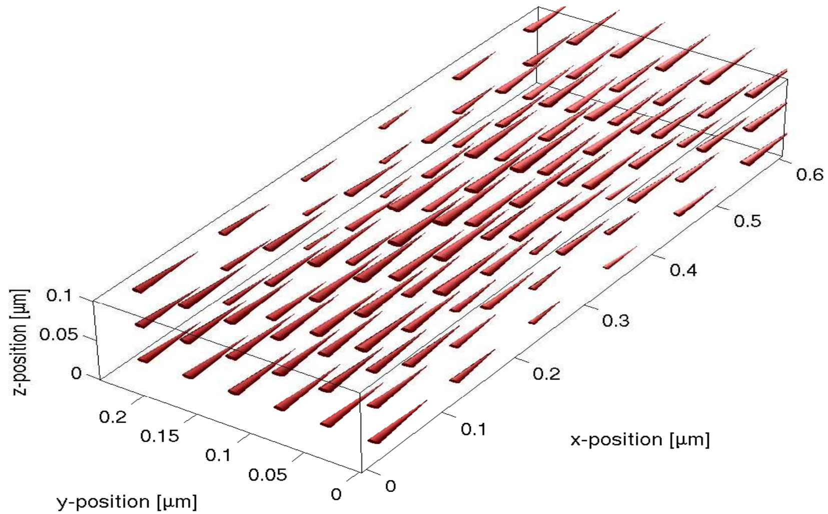
Geometry



Temperature



Thermal energy



Current density

② Lattice heat modeling

- Derivation from first thermodynamic principles (Wachutka 1990, Albinus et al. 2002, Brunk/A.J. 2008)
- Define internal energy u and energy flux J_u:

$$u = \text{electric} + \text{lattice} + \text{conduction-band} + \text{thermal energies}$$

$$J_u = \text{displacement} + \text{Fourier} + \text{particle} + \text{dissipated-power} + \text{band-energy fluxes}$$

- Conservation equation $\partial_t u - \operatorname{div} J_u = 0 \Rightarrow$ heat equation

$$\rho_L c_L \partial_t T_L - \operatorname{div}(\kappa_L \nabla T_L) = H$$

$$H = -\frac{3}{2}\frac{n(T-T_L)}{\tau} + R(n,p)(E_c + \tfrac{3}{2}T) - S_L(T_L - T_0)$$

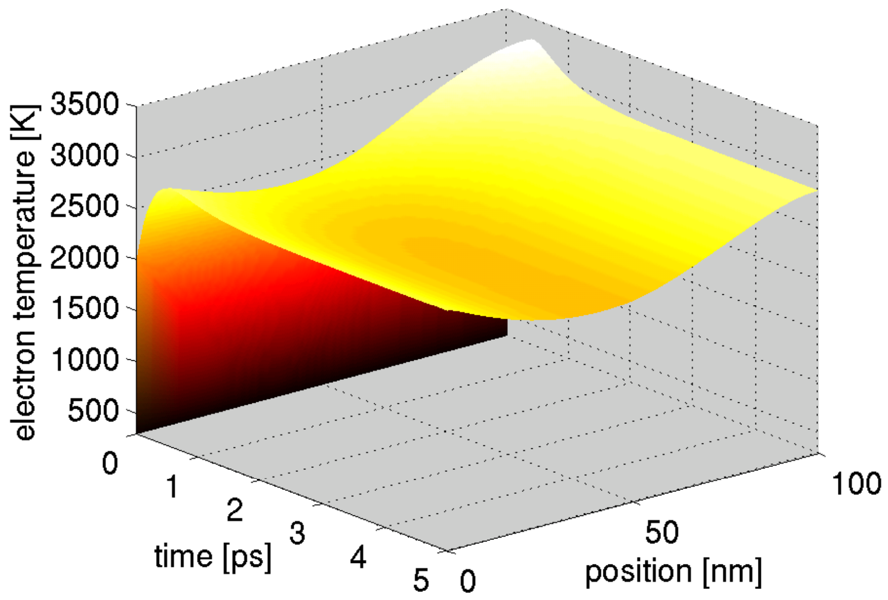
material density ρ_L, lattice heat capacity c_L, radiation $S_L(T_L - T_0)$, conduction-band energy E_c

- Coupling to energy-transport through energy relaxation

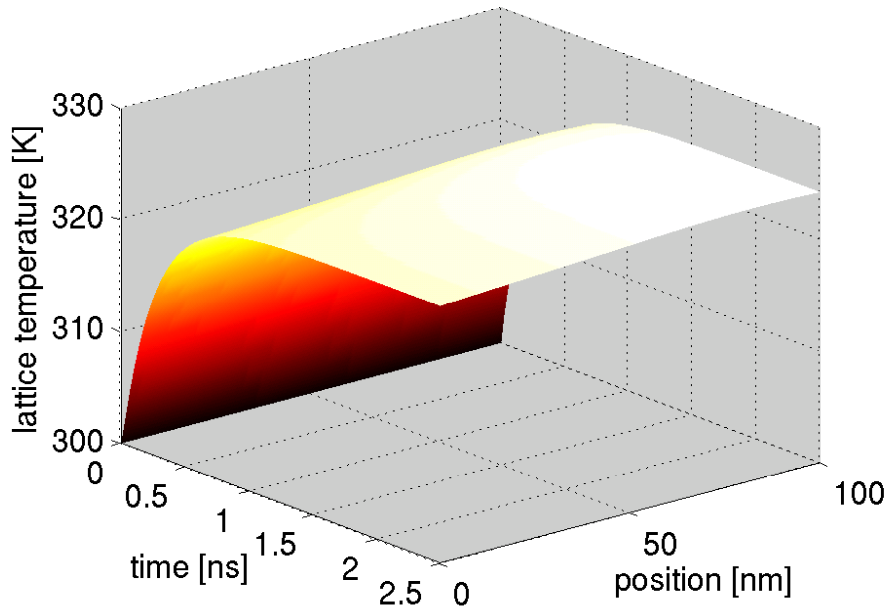
② Lattice heat modeling

1D bipolar diode: forward bias 1.5 V (Brunk/A.J. 2008)

Electron temperature



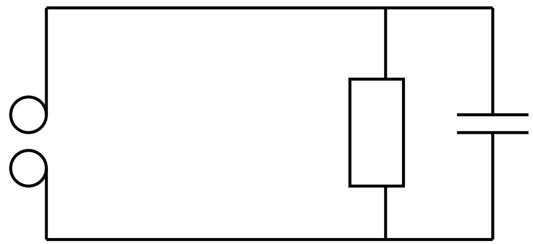
Lattice temperature



- Temperature minimum around junction
- Almost constant lattice temperature due to high heat conductivity
- Lattice heating decreases current density

③ Circuit modeling

- Kirchhoff's current law: $Ai = 0$
  A: matrix of node-to-branch relations, i: branch currents
- Kirchhoff's voltage law: $v = A^\top e$
  v: branch voltages, e: node potentials



- Current-voltage characteristics:

$$v_R = R i_R, \quad i_C = C \frac{dv_C}{dt}, \quad v_L = L \frac{di_L}{dt}$$

Modified nodal analysis: source current i_s, voltage v_s

$$A_C C A_C^\top \frac{de}{dt} + A_R R^{-1} A_R^\top e + A_L i_L + A_V i_V + A_I i_s = 0$$

$$L \frac{di_L}{dt} - A_L^\top e = 0, \quad A_V^\top e = v_s$$

- Differential-algebraic equations of index 1 or 2
- Numerical methods: Rosenbrock-Wanner (Günther), BDF

④ Thermal network modeling

Heating of thermal nodes:

- Thermal nodes $=$ resistors, contact nodes...
- ODE for node temperature $\widehat{T}$:

  $$\widehat{M}\frac{d}{dt}\widehat{T} = \text{source terms} + \text{radiation} + \text{dissipation power}$$

- Dissipation power $=$ current $\times$ electric potential

Heating of distributed elements:

- Distributed elements $=$ electric lines, 1D devices...
- PDE for distributed temperature T^d:

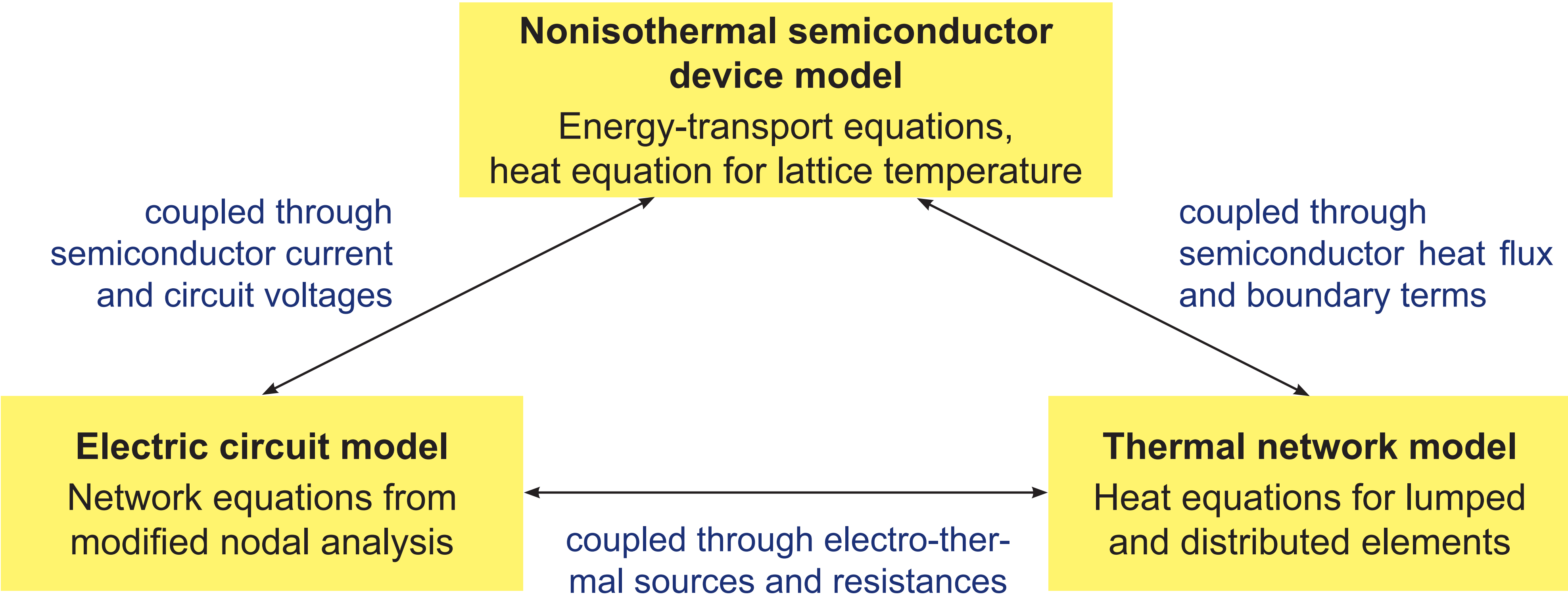
  $$M\partial_t T^d = \partial_x(\kappa_L \partial_x T^d) + \text{radiation} + \text{dissipation power}$$

Coupling to circuit and device models:

- Electro-thermal coupling: dissipation powers
- Thermal-device coupling: through boundary conditions

Model overview

Complete electro-thermal circuit-device model

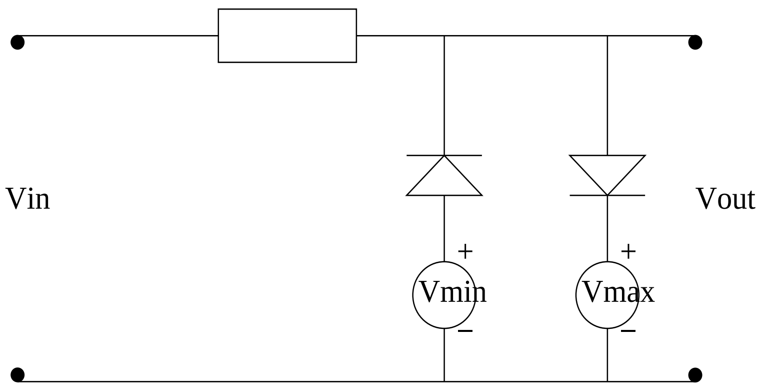


$\rightarrow$ Nonlinear partial differential-algebraic equations (PDAE)

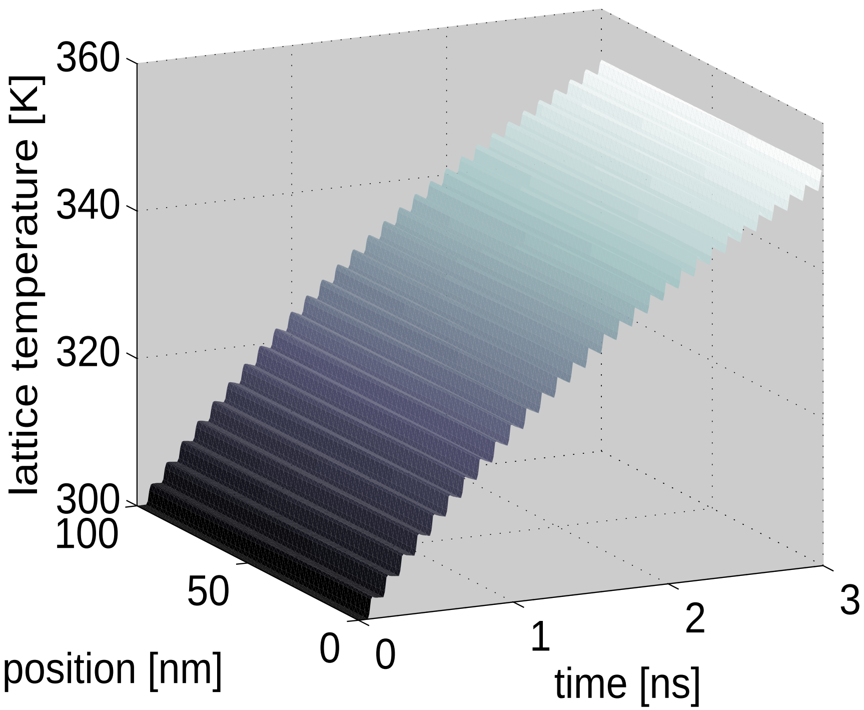
- Discretization: BDF-2 and Marini-Pietra finite elements
- Iteration: Gummel-type inner loop for nonisothermal semiconductor model, fixed-point-type outer loop

Numerical example

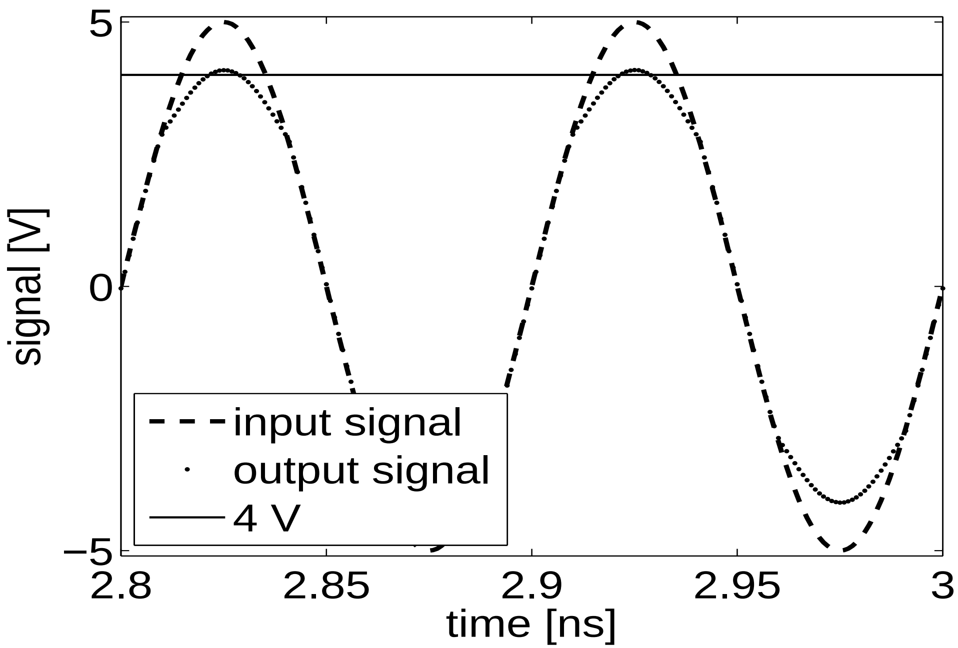
Clipper: (Brunk/A.J. 2008)



- Used for entrance protection
- Output voltage should be less than 4 V
- Input frequency of 10 GHz



Lattice temperature



Output signal

Summary

Novelties:

- Modeling of nonisothermal semiconductor-circuit system
- Numerical solution of complete nonlinear system

Final result:

Lattice heating may lead to undesired large output signal in clipper circuit