Mathematics of Crime

Andrea L. Bertozzi, *University of California Los Angeles*

Abstract:
There is an extensive applied mathematics literature developed for problems in the biological and physical sciences. Our understanding of social science problems from a mathematical standpoint is less developed, but also presents some very interesting problems. This lecture uses crime as a case study for using applied mathematical techniques in a social science application and covers a variety of mathematical methods that are applicable to such problems. We will review recent work on agent based models, differential equations, variational methods for inverse problems and statistical point process models. From an application standpoint we will look at problems in residential burglaries and gang crimes.

Examples will consider both "bottom up" and "top down" approaches to understanding the mathematics of crime, and how the two approaches could converge to a unifying theory.

The Causes of Crime and the Practical Limits of Crime Control

Jeff Brantingham, *UCLA*

Abstract:
Within criminology there continues to be wide disagreement over the importance the individual, formal and informal social structure and the environment in driving crime patterns. In person-based theories, individuals are assumed to either innately possess the capacity to commit crime, or learn such capacities from their interactions with others. In structural theories, it is generally assumed that individuals are constrained by static social, economic or political organization, which makes crime a necessary or acceptable alternative to non-crime activities. In environmental theories, the built environment creates abundant, if unevenly distributed opportunities for crime that are easily exploited. While each of these theoretical perspectives finds some justification in empirical studies, they are not equal practical from the point of view of crime control. This talk will review several key ideas underlying crime and crime pattern formation and argue in favor of modeling of short-term, local crime processes because it is these processes that are most easily disrupted and are likely to yield practical results.

Predicting Criminal Incidents Using Geographic, Demographic, and Twitter-derived Information

Donald E. Brown, *University of Virginia*

Abstract:
Predictive policing seeks to anticipate the times and locations of crimes to better allocate law enforcement resources to combat these crimes. The key to predictive policing is modeling that

combines available data to forecast or estimate the areas most threatened by crimes at different times. We have developed models that integrate geographic, demographic, and social media information from a specific area of interest to produce the needed predictions. In this presentation, I describe our approach to this predictive modeling, which combines spatial-temporal generalized additive models (STGAM) with a new approach to text mining. We use the STGAM to predict the probability of criminal activity at a given location and time within the area of interest. Our new approach to text mining combines Latent Dirichlet Allocation (LDA) with Latent Semantic Indexing (LSI) to identify and use key topics in social media relevant to criminal activity. We use social media since these data provide a rich, event-based context for criminal incidents. I present our application of this approach to actual criminal incidents in Charlottesville, Virginia. Our results indicate that this combined modeling approach outperforms models that only use geographic and demographic data.

The shape of data

Gunnar Carlsson, *Stanford*

Abstract:
The notion of higher dimensional shape has turned out to be an important feature of data. It encodes the qualitative structure of data, and allows one to find useful distinct groups in data sets. Topology is the branch of mathematics which deals with shape, and in recent years methods from topology have been adapted for the study of data. This talk will survey these developments, with examples.

How criminal defectors may lead the way to a peaceful society

Maria-Rita R. D'Orsogna, *California State University at Northridge*

Abstract:
Traditional models of human cooperation are usually cast in the form of a prisoner's dilemma, where although cooperation may be beneficial, players may choose to "defect" and pursue selfish goals. In this talk we consider an adversarial evolutionary game developed for criminal activity where players not only choose whether or not to cooperate for the common good but also whether or not to actively harm others by committing crimes. The introduction of this new choice gives rise to four possible strategies among which the so called "informant", a player who cooperates while still committing crimes. We find two possible equilibration regimes, a defection-dominated and an ideal, cooperation-dominated one and show that the number of informants is crucial in determining which of these two regimes is achieved. Since large numbers of informants lead to the ideal cooperative society we also study their active recruitment from the overall society, by considering differential recruitment costs and benefits, via an optimal control problem where finite resources are included. We discuss our results in the

context of extreme adversarial societies, such as those marred by wars, insurgencies and organized crime.

A general class of probability density functions with positive support, with applications in survival analysis

Rolando de la Cruz, *Pontificia Universidad Catolica de Chile*

Abstract:
In this talk, we introduce a new class of distributions with positive support called epsilon{positive which are generated on the basis of the distributions with positive support. This new class has as special cases the exponential, Weibull, log-normal, etc. distributions, and is an alternative to analyze survival data. For this new class of distributions we present the stochastic representation, which is very useful for generating random numbers, the cumulative distribution function, and the hazard and survival functions. We study basic properties such as moments and moment generating function of a member of this new class of distributions which is called epsilon-exponential density since this is generated using the exponential distribution. For this particular density, statistical inference performed by the method of moments and maximum likelihood via an EM{type algorithm is carried out. We use the epsilon-exponential density in modeling censored data with the di_erence that this density does not show a constant hazard function as in the case of exponential density. An application with recidivism data is presented. Work partially funded by grant ANILLO ACT{87. Joint work with Perla Celis and H_ector G_omez from University of Antofagasta, Chile.

Title: Crime hot-spots with or without Levi Flights

Theodore Kolokolnikov, *Dalhousie*

Abstract:
In the first part of the talk, we consider the Short et.al. model of crime. This model exhibits hot-spots of crime -- localized areas of high criminal activity. In a certain asymptotic limit, we use singular perturbation theory to construct the profile of these hot-spots and then study their stability.

In the second part of the talk, we extend the original model to incorporate biased Levi Flights for the criminal's motion. Such motion is considered to be more realistic than the biased diffusion that was originally proposed. This generalization leads to fractional Laplacians. We then investigate the effect of introducing the Levi Flights on the formation of hot-spots using linear stability and full numerics.

Joint works with Jonah Breslau, Tum Chaturapruek, Daniel Yazdi, Scott McCalla, Michael Ward and Juncheng Wei.

Quasilinear systems and residential burglary

Raul Manasevich, *CEAMOS & U Chile*

Abstract:
In this talk we will present some results for systems of equations modeling residential burglary.

For the parabolic system model proposed by Andrea Bertozzi et-al, we study the equilibrium case. By using bifurcation theory we show that this system does support pattern formation. We also give some results concerning stability of the bifurcating patterns. These results correspond to a joint work with Chris Cosner and Steve Cantrel from the University of Miami.

The model has been recently modified by Pitcher giving rise to a new parabolic system of equations. We show some results for this system that contain a condition for existence of global solutions.
This work corresponds to a collaboration with Philippe Souplet and Quoc Hung Phan from Paris 13.

Population dynamics and cellular automata for the description of criminality

M. Primicerio, *Universita degli Studi di Firenze*

Abstract:
In this paper we study the dynamics of a population where the individuals can either be contributors (tax payers) or no contributors (tax evaders or cheaters). We introduce a 2D cellular automaton on which the probability of transition from one of the above states to the other is the sum of the local effect and of the global field effect. The model also includes the policy that allocates a fraction of the budget to fight tax evasion. This scheme allowed us to simulate the cases in which inhomogeneous strategies in contrasting tax evasion is applied in a region and the case in which cooperative policies are adopted by neighbor societies.

Joint work with L. Meacci, Juan C. Nuno (Universidad Politecnica de Madrid).

Point Process Methods for Crime Hotspots

George Mohler, *Santa Clara University*

Abstract:
This talk focuses on the application of point process methods to crime and security data. We will discuss semi- and non- parametric models, as well as their estimation using Expectation-Maximization algorithms. We conclude the talk with some results from a randomized controlled trial in Los Angeles where police patrols are determined each day using a semi-parametric self-exciting point process.

Development of a Social-Behavior Modeling and Simulation Framework for Assessing Strategies in Response to Crisis AH1N1 in Chile

Jaime H. Ortega, *Universidad de Chile*

Abstract:
The occurrence of catastrophic phenomena such as pandemic, natural disasters or social crisis, detrimentally affects a nation's population and its environment. If the authorities responsible for managing national infrastructure, such as public health, transportation, educational and communications networks, defense forces and law enforcement agencies, among others, are not sufficiently prepared to face these kinds of events, their responses may not be timely sufficient nor appropriate to mitigate their effects. Hence, it is vitally important to have a system capable for simulating the social and economical consequences of natural and unnatural disasters. Such a system can provide criteria for strategic decision-making and helps to mitigate the impact on health and safety of the population and the national economy.

In this work we present some suitable quantitative tools, based on mathematical modeling, computational algorithm and social analysis to analyze the social behavior under an emergency. We consider particularly the occurrence of a known pandemic (e.g. H1N1) in Chile in during 2009. We take into account the capacities of public and private organizations to face the disaster, their possible actions and the people's reactions to the emergency. This system can simulate scenarios considering different actions as a vaccines program, transport regulations, among others, allowing us to measure the social behavior and population response. We note that this analysis can be extended to another kind of emergencies with a suitable source of data, for instance, natural disasters, social crisis among others.

Joint work with J. Amaya, F. Padilla and M. Escobar (Universidad de Chile).

Exploring the Relationship between Commuter Criminals and Integro-Differential Equations

Nancy Rodriguez, *Stanford U*

Abstract:
Recently, there has been increased interest in the use of mathematical tools to obtain insight into the governing laws of criminal activity. In this talk I will discuss two methods to model criminal activity committed by “commuter" criminals. This has important applications in cities, such as Paris, where much of the crime is committed in the city by criminal agents that commute from the outskirts of Paris. The first method I will discuss is an agent based model where the motion of the criminal agents is a biased Lévy walk (a random walk with step length that have a thick-tailed distribution). Second, I will talk about the use of integro-differential equations to model such phenomena.

Security and Game Theory: Key Algorithmic Principles, Deployed Applications, Lessons Learned

Milind Tambe, *University of Southern California*

Abstract:
Security is a critical concern around the world, whether it's the challenge of protecting ports, airports and other critical national infrastructure, or protecting wildlife and forests, or suppressing crime in urban areas. In many of these cases, limited security resources prevent full security coverage at all times; instead, these limited resources must be scheduled, avoiding schedule predictability, while simultaneously taking into account different target priorities, the responses of the adversaries to the security posture and potential uncertainty over adversary types.

Computational game theory can help design such unpredictable security schedules. Indeed, casting the problem as a Bayesian Stackelberg game, we have developed new algorithms that are now deployed over multiple years in multiple applications for security scheduling: for the US coast guard in Boston and New York (and potentially other ports), for the Federal Air Marshals(FAMS), for the Los Angeles Airport Police, with the Los Angeles Sheriff's Department for patrolling metro trains, with further applications under evaluation for the TSA and other agencies. These applications are leading to real-world use-inspired research in the emerging research area of security games; specifically, the research challenges posed by these applications include scaling up security games to large-scale problems, handling significant adversarial uncertainty, dealing with bounded rationality of human adversaries, and other interdisciplinary challenges. This lecture will provide an overview of my research's group's work in this area, outlining key algorithmic principles, research results, as well as a discussion of our deployed systems and lessons learned.

Community detection and interactions among Hollenbeck street gangs

Yves van Gennip, *UCLA*

Abstract:
Using LAPD field interview card data from the Los Angeles policing district Hollenbeck we study community formation among gang members and interactions between gangs.

Unsupervised data clustering techniques, such as spectral clustering and multiplex modularity optimization, are used to group gang members, based on where and with whom they are seen in the data set. Preliminary investigations of the different types of gang interaction also are performed.

Despite the sparsity of interactions in the data set, interesting observations are made. One surprising result is the presence of many non-violent meetings between members from different gangs.

The Stability of Steady-State Hot-Spot Patterns for Reaction-Diffusion Models of Urban Crime

Michael Ward, *UBC*

Abstract:
The existence and stability of localized patterns of criminal activity is studied for the two-component reaction-diffusion model of urban crime that was introduced by Short et.~al.~[Math. Models. Meth. Appl. Sci., **18**, Suppl. (2008), pp.~1249--1267]. Such patterns, characterized by the concentration of criminal activity in localized spatial regions, are referred to as hot-spot patterns and they occur in a parameter regime far from the Turing point associated with the bifurcation of spatially uniform solutions. Singular perturbation techniques are used to construct steady-state hot-spot patterns in one and two-dimensional spatial domains, and new types of nonlocal eigenvalue problems (NLEP's) are derived that determine the stability of these hot-spot patterns to **O(1)** time-scale instabilities. From an analysis of these NLEP's, and a further analysis of the spectrum associated with the slow translational instabilities, an explicit threshold for the minimum spacing between stable hot-spots is derived. The theory is confirmed via detailed numerical simulations of the full PDE system. Moreover, the parameter regime where localized hot-spots occur is compared with the parameter regime, studied in previous works, where Turing instabilities from a spatially uniform steady-state occur. Finally, in the 1-D context, we show how the existence and stability of hot-spot patterns is altered from the inclusion of a third component to the reaction-diffusion system that incorporates the effect of police. In the context of this extended model, the optimal strategy for the police is discussed.

Joint Work with Theodore Kolokolnikov (Dalhousie) and Juncheng Wei (Chinese
U. of Hong Kong and UBC).

Game Theory and Agent-Based Modeling: Tools for Efficient Use of Policing Resources in Public Security

Richard Weber, *CEAMOS and Universidad de Chile, Santiago, Chile*

Abstract:
We present tools that have been developed at CEAMOS in order to study and understand crime in public places.

For a static situation we developed tools based on Game Theory to model the interaction between police forces and delinquents in public places. Whereas in the well-known Stackelberg game a leader is faced with one follower in our application police has to deal with many offenders simultaneously which can be organized or act independently of each other. This motivates the development of two games: a classical leader-follower interaction between police and organized criminals on the one hand and a novel approach between the leader and selfishly acting offenders on the other.

In order to support similar developments for a more realistic, dynamic situation, we developed an agent-based simulation tool that generates artificial street-crime data which can be used to test different policing strategies in a virtual environment. The tool can thus evaluate the strategies' effectiveness and collateral effects before putting them into practice and provide support for the policing decision-making process.