Submittee: Cristian Rios
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Title: Western International Workshop on Harmonic Analysis and PDEs
Event Type: Conference-Workshop

Location:
University of British Columbia

Dates:
June 10 to 12, 2015

Topic:
The purpose of this workshop was to promote the communication and interaction, mainly on the western part of the continent, of researchers at all levels in the area of Harmonic Analysis, Partial Differential Equations, and Geometric Measure Theory. While these are three distinct areas within mathematics with vibrant research communities, they also share a lot of overlap that has contributed to and shaped the evolution of these fields. Tools from harmonic analysis have been used with great success to address problems in PDE-s and geometric measure theory. Differential and measure-theoretic questions, in turn, have given rise to interesting and outstanding analytical problems, resulting in the development of extremely sophisticated machinery and emerging connections with many other areas such as additive combinatorics and number theory. Western Canada is the home of many experts in these areas. What is missing is an interactive network and an ongoing dialogue between the researchers of these diverse yet closely related fields.

Methodology:
The workshop offered a mini-course dictated during the mornings. This mini-course was directed to graduate students and postdoctoral fellows. ///

The rest of the mornings and afternoons were dedicated to specialized talks by senior researchers and although they will be on more focused topics, speakers were encouraged to make them as accessible as possible to include advanced graduate students. ///

This event also emphasized the inclusion of advanced graduate students by featuring a series of short contributed talks. ///

These courses and lectures conduced to informal discussions and exchanges by all the participants, motivate scientific exchange, and built a deeper knowledge among participants about each other's activities, thus supporting the main objective of this workshop. ///

At the end of the second day there was a "problem discussion session" where participants had the opportunity to present their favourite math problems in a succinct and motivating fashion. These were problems that would be easy to pose, may or may not be related directly to current research, but were interesting and engaging challenge for the audience.

Objectives Achieved:
The workshop was successful in bringing together experts in analysis and related areas of research from institutions in Canada, USA and other countries. There was plenty of interaction and several possible new collaborations projects were discussed. One future project includes an application to a PIMS CRG in analysis and differential equations.

Scientific Highlights:
During the problem solving session a presenter posed the quest for a proof or a counter-example for a conjecture involving Banach spaces. A participant came up with a counter-example for one of the instance of the conjecture; this result was quite a revelation for the presenter.

Organizers:
Rios, Cristian, Department of Mathematics and Statistics, University of Calgary //
Malabika Pramanik, Mathematics Department, University of British Columbia //
Tatiana Toro, Department of Mathematics, University of Washington

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Speakers:
Arpad Benyi, Department of Mathematics, Western Washington University //
Bilinear compactness, commutators and weights (Invited speaker) ///

We will present several facts connecting bilinear commutators and the notion of compactness, including results in the context of weighted Lebesgue spaces. ///

Mike Kouritzin, Department of Mathematical and Statistical Sciences, University of Alberta //
Simulation and Estimation in Stochastic Differential Equations (Invited speaker) ///

Randomness is prevalent. It appears in images, stock prices, animal movement, packet data, even within living cells. To make valid interpretations and predictions, we must learn how to simulate and process noisy dynamical systems. A most popular way to model noisy dynamical systems is through stochastic differential equations. In this talk, we will introduce new methods of simulation and estimation of stochastic differential equations. In particular, the notions of strong and weak solutions to stochastic differential equations, differential form and commutator conditions for simple explicit solutions to stochastic differential equations, the stochastic partial differential equations of filtering theory and a novel branching particle filter will all be introduced. The efficacy of our methods will be validated by simulation and intuition. References will be given for the rather-involved mathematical proofs. ///

Michael Lamoureux, Department of Mathematics and Statistics, University of Calgary //
Gabor Multipliers (Invited speaker) ///

Introduced as a non-stationary generalization of Fourier multipliers, the Gabor multipliers share many of the characteristics of the familiar pseudodifferential operators that arise in the analysis of partial differential equations. We will demonstrate a functional calculus for these linear integral operators, and demonstrate their use in physical modelling as well as their computational advantages in numerical work. Applications to simulation of wave propagation and non-stationary deconvolution in seismic imaging with be presented. ///

Diego Maldonado, Department of Mathematics, Kansas State University //
Analysis on the Mongeâ€“AmpÃ¨re quasi-metric space and applications (Invited speaker) ///

We will start by reviewing the construction of a doubling quasi-metric structure associated to convex solutions of the Monge-Ampere equation det D^2 u = f. Then we will report on the existence of Sobolev and PoincarÃ©-type inequalities adapted to such quasi-metric structure. Finally, we will go over some applications of the resulting first-order calculus, including intrinsic Harnack inequalities for the linearized elliptic and parabolic Monge-Ampere equations. ///

Virginia Naibo, Department of Mathematics, Kansas State University //
Recent developments on mapping properties of certain bilinear pseudodifferential operators (Invited speaker) ///

The study of bilinear pseudodifferential operators is motivated by topics in analysis and partial differential equations such as commutators, paraproducts and fractional Leibniz-type rules. In this lecture, we will present an overview of recent results and techniques on boundedness properties of operators with symbols in the bilinear Hormander classes and in the settings of Lebesgue spaces, Besov and Triebel-Lizorkin spaces, and BMO. ///

Joonil Kim, Yonsei University at Korea (Department of Mathematics, University of California at Irvine) //
Oscillatory integrals over global domains (Invited speaker) ///

We discuss about the asymptotic behaviors for the 2D oscillatory integrals with polynomial phase functions. Especially, we study the cases that (1) the domain of integral is unbounded, or (2) the phase function involves a vector polynomial. Similar problem is measuring a sublevel-set which can be an unbounded set, or an intersection of various different sublevel sets. Our results are described in terms of generalized notions of Newton polyhedra. ///

Eric Sawyer, Department of Mathematics and Statistics, McMaster University //
Local boundedness and continuity of weak solutions to infinitely degenerate elliptic equations, with applications to hypoellipticity of certain smooth quasilinear equations with infinite degeneracy (Invited feature three-lecture mini-course) //

The classical DeGiorgi-Nash-Moser theory of local boundedness and continuity of weak solutions to elliptic equations with bounded measurable coefficients has seen significant extension over the decades to the case of subelliptic degeneracy where the appropriate eigenvalues vanish to finite order. In the case where the coefficients are in addition smooth, this theory has been extended even to the infinitely degenerate regime in special cases by Strook, Kusouka, Morimoto, Christ and others. In these three talks we will briefly review these rough subelliptic and smooth infinitely degenerate theories, and introduce a new modification of these ideas that can be used to further extend the theory to the rough coefficient infinitely degenerate regime. We concentrate on model cases where Orlicz spaces play a critical role with submultiplicative Young functions, and a careful determination of properties of infinitely degenerate geometries can be computed. This is ongoing joint work with Ludmila Korobenko, Cristian Rios and Ruipeng Shen. ///

Deniz Sezer, Department of Mathematics and Statistics, University of Calgary //
X-harmonic functions, conditioning, and the Martin boundary of Super-Brownian motion (Invited speaker) ///

In this talk I am going to describe a measure valued stochastic process called Super Brownian Motion (SBM) and how one can obtain certain conditional distributions of this process via X-harmonic functions. I will show an explicit construction of a Super-Brownian Motion conditioned on

its exit measure, and discuss the connections of this conditioning to the Martin boundary of Super-Brownian motion. ///

Xinwei Yu, Department of Mathematics, University of Alberta //
On Stationary Solutions to Doi-Onsager Models (Invited speaker) ///

We study the Doi--Onsager models with general potential kernel in dimensions two and higher, with special emphasis on the classical Onsager kernel. We obtain the uniqueness of the trivial solution for low temperatures as well as the local bifurcation structure of the solutions through application of topological degree methods. This is joint work with Dr. Mohammad Ali Niksirat. ///

Matthew Coles, Mathematics Department, University of British Columbia //
Resonance in the Nonlinear Schrodinger Equation (Contributed short talk) ///

The focusing Nonlinear Schrodinger Equation exhibits solitary wave solutions (solitons) whose stability is related to the spectrum of a linearized operator. In one space dimension with a cubic nonlinearity this operator has a resonance eigenvalue on the edge of its essential spectrum. When the power in the nonlinearity is perturbed from 3 we see the resonance pop out of the essential spectrum and become a true eigenvalue. We discuss an analytic study of this bifurcation. ///

Robert Fraser, Mathematics Department, University of British Columbia //
Kakeya-Type Sets in Local Fields with Finite Residue Field (Contributed short talk) ///

In a 2013 paper, Evan Dummit and MÃ¡rton Hablicsek published a paper describing a Kakeya set of measure zero in the ring of formal power series over a finite field. I describe how a construction appearing in a 1987 paper of Eric Sawyer, and a modification appearing in a 2004 paper of Laura Wisewell, can be modified to describe measure-zero Kakeya sets in other discrete valuation rings and local fields. ///

Senthil Raani, Indian Institute of Science //
L^p-Fourier Asymptotics of Fractal Measures ///

One of the basic questions in harmonic analysis is to study the decay properties of the Fourier transform of measures or distributions supported on thin sets in R^n. If dS denotes the surface measure on the sphere and if any compactly supported function on R^n, then using the properties of Bessel functions, the behavior of the Fourier transform of fdS are known at infinity. Similar results are known for measures supported in lower dimensional manifolds in R^n under appropriate curvature conditions. In this talk we discuss L^p-asymptotics of the Fourier transform of fractal measures supported on a set E of finite packing measure by studying the behavior of the Fourier transform of such measures at infinity. ///

Dimitrios Roxanas, Mathematics Department, University of British Columbia //
Global large energy solutions of the equivariant heat-flow (Contributed short talk) ///

We consider m-corotational solutions to the harmonic map heat flow from $\mathbb{R}^2$ to $\mathbb{S}^2, m \geq 2.$. For maps with topological degree zero and energy of the initial data below two times the energy of the stationary harmonic map solutions, we establish global existence and decay. The proof is based on the ``concentration-compactness plus rigidity'' approach of Kenig and Merle and relies on a profile decomposition and the dissipation of energy. If time permits, we will discuss extensions to some higher energy and non-trivial topology cases. (This is based on joint work with S.Gustafson) ///

Tatchai Titichetrakun, Mathematics Department, University of British Columbia //

Weighted Removal Lemma and A Multi-dimensional Szemeredi's Theorem in the Primes (Contributed short talk) ///

Multi-dimension Szemeredi theorem, first proved by Furstenburg-Katnelson, states that any given subset of Z^d with positive upper density must contain affine copies of any finite subset of Z^d . In 2004, Green and Tao extended this theorem in case d=1 to primes (with relative density in place of density), using a pseudo-randomness property of primes. Questions of extending Green-Tao result to higher dimensions left open for some time due to some correlations between elements of the cartesian product of primes, making it hard to satisfiy a randomness condition. This problem is resolved by three methods in 2013 (also due to Tao-Zeigler and Fox-Zhao). In this talk, I will describe our approach using weighted hypergraph. By transferring hypergraph regularity and energy increment arguments to our weighted setting. Then we can use it to prove a simplex removal lemma on the corresponding weighted hypergraph which would imply multi-dimensional Green-Tao Theorem. This is a joint work with B. Cook and A. Magyar. ///

Kazuo Yamazaki,Department of Mathematics, Washington State University //
On the three-dimensional magnetohydrodynamics system in scaling-invariant spaces (Contributed short talk) ///

Sufficient and necessary conditions for a solution to fluid mechanics PDEs such as the Navier-Stokes equations, magnetohydrodynamics system and surface quasi-geostrophic equations has caught much attention over many decades. We discuss component reduction results of such conditions that require in their proofs key identities due to divergence-free properties of the solutions and how anisotropic Littlewood-Paley theory has also become useful in the recent years. In connection to the rescaling of the solutions to these PDEs, we also discuss some challenging open problems.

Links:
