

Applied Harmonic Analysis Conference

Calgary, August 26-30, 2013

Titles and Abstracts (alphabetic)

Enrico Au-Yeung (University of British Columbia)

Title: On a new class of random matrices with applications to signal recovery

Abstract: Random matrices in which the entries of the matrix are independent standard Gaussian random variables have the property that with high probability, columns are nearly orthogonal to each other. Matrices that enjoy this property are said to satisfy the restricted isometry property. They have been used in sparse signal recovery in recent years, and it is of great interest to have different ways to generate such matrices, especially when the matrices enjoy some random structure. In this talk, I will discuss a new class of structured random matrices. No previous background in non-asymptotic random matrix theory will be assumed. Part of this talk will be semi-expository so that enough mathematical background will be provided.

Almaz Butaev (Concordia University, Montreal, Canada)

Title: On continuous wavelet transforms of distributions

Abstract: Wavelet analysis is a universal and promising tool with very rich mathematical content and great potential for applications in various scientific fields, in particular, signal (image) processing and the theory of differential equations. Since distributions are widely used in these areas, it is important to define and investigate wavelet transforms of distributions. In this talk we introduce continuous wavelet transforms of distributions and study convergence properties of these transforms.

Jian Deng (University of Alberta)

Title: Symplectic numerical schemes for stochastic systems preserving Hamiltonian functions

Abstract: We construct high order symplectic schemes for stochastic Hamiltonian systems preserving Hamiltonian functions. We follow an approach based on the generating function method, and we prove that for this type of systems the coefficients of the generating function are invariant under permutations. As a consequence the high-order symplectic methods have simpler forms than the explicit Taylor expansion schemes with the same order. Moreover, we illustrate numerically that the symplectic schemes are more suitable for long term simulations.

Tom Duchamp (University of Washington)

Title: Necessary and sufficient conditions for smoothness equivalence of nonlinear subdivision schemes

Abstract: A nonlinear subdivision scheme is said to satisfy "smoothness equivalence"

if it has the same smoothness properties as the underlying linear scheme. Sufficient conditions for smoothness equivalence based on certain "proximity conditions" have been given by Wallner-Dyn, Xie-Yu, and Grohs. In this talk, I will discuss a new "differential proximity condition" that is both necessary and sufficient for smoothness equivalence.

The proof relies on three technical innovations:

(i) We show that the breakdown of smoothness equivalence is related to the presence of certain "resonance terms" in a discrete dynamical system derived from the nonlinear scheme, which slow the decay of high order differences in the subdivision scheme.

(ii) To conclude that the slowed decay causes a breakdown of smoothness equivalence, we introduce a "super-convergence property" which replaces the usual stability condition in linear subdivision theory.

(iii) We observe an unexpected algebraic structure in the proximity conditions that implies that the previously known sufficient conditions for smoothness equivalence are implied by weaker proximity conditions than previously known.

This is joint work with Gang Xie and Thomas Yu.

Hartmut Führ (RWTH Aachen)

Title: Wavelet coorbit theory in higher dimensions

Abstract: Coorbit theory provides a functional-analytic framework for the construction and study of Banach frames arising from the action of an integrable representation. This talk is concerned with existence and basic properties of coorbit spaces associated to wavelet transforms arising from an irreducible, square-integrable representation of a semidirect product of the type $G = \mathbb{R}^d \rtimes H$ acting naturally on $L^2(\mathbb{R}^d)$. Here H is a suitably chosen, closed matrix group.

The talk provides a unified and rather general approach to a setting that so far has only been studied for very special choices of affine group actions (such as the similitude group, or the shearlet group). It establishes the well-definedness of a scale of Besov-type coorbit spaces, and provides the existence of atomic decompositions for these spaces in terms of suitably chosen band-limited Schwartz functions. Under suitable assumptions on the dual action of H I establish easily verified concrete conditions for frame atoms, in terms of vanishing moments, smoothness and decay. In particular, these results imply the existence of compactly supported smooth atoms.

Navid Ghadermarzy (University of British Columbia)

Title: Using prior support information in compressed sensing

Abstract: Consider the standard compressed sensing problem. We want to recover a sparse—or compressible—signal from few linear measurements. In this talk we investigate sufficient recovery conditions when we have prior information about the support, i.e., the indices of the non-zero entries, of the signal to be recovered. First we address the "weighted ℓ_p minimization algorithm with $0 < p < 1$ " and derive sufficient conditions for stable and robust recovery and show that these conditions are better than those for recovery by regular ℓ_p and weighted ℓ_1 . Then we derive a weighted approximate message passing (AMP)

algorithm which incorporates prior support information into the AMP algorithm. We empirically show that this algorithm recovers sparse signals significantly faster than weighted ℓ_1 minimization. We also introduce a reweighting scheme for AMP and weighted AMP which, we observe, substantially improves the recovery conditions of these algorithms. This is joint work with Ozgur Yilmaz and Hassan Mansour.

Bin Han (University of Alberta)

Title: Directional tensor product complex tight framelets

Abstract: For 2D data such as images, edge singularities are ubiquitous and play a fundamental role in image processing. Due to simple implementation and construction of 1D wavelets, real-valued tensor product wavelets have been commonly used in many applications. However, real-valued tensor product wavelets lack directionality and can only capture edge singularities along the horizontal and vertical direction. In this talk, we present a comprehensive theory and construction of directional tensor product complex tight framelets. While keeping the simple tensor product structure, our approach has the advantages of improved directionality and uses finitely supported filter banks. In particular, we propose a family of directional tensor product complex tight framelets with increasing directions.

Rong-Qing Jia (University of Alberta)

Title: Fast numerical computation based on multilevel approximation

Abstract: The multigrid method and the wavelet method are two efficient schemes for numerical solutions of partial differential equations and partial difference equations arising from mechanics and physics, image processing, and financial mathematics. In this talk we use the theory of multilevel approximation to give new insights into these efficient numerical methods.

Palle Jorgensen (University of Iowa)

Title: Applications of spectral theory to wavelets and fractals

Abstract: Recently, there has been an enormous amount of interest in the theory and applications of wavelets. In the talk I will cover some topics going beyond some of the more familiar applications: signal processing, data compression, turning fingerprints into digital data files, subdivision algorithms for graphics, and the JPEG 2000 encoding of images. I will be using tools from functional and harmonic analysis, numerical analysis, mathematics of computation, and the theory of operator in Hilbert space. Multiresolutions (including wavelets) now serve as alternatives to classical Fourier methods, Fourier series and integrals. The reasons for this are their versatility; plus that they are better localized than Fourier bases: They are better adapted to discontinuities; and they have a certain form of self-similarity, which makes the theory suited for the analysis of fractals and non-linear dynamical systems. The self-similarity properties of the scaling functions connect them to fractals and non-linear dynamical systems. Moreover the multiresolutions offer fast algorithms. The feature of localization for wavelets is shared by related recursive basis constructions from multi-resolutions in Hilbert spaces, for example for fractals and iterated function systems in dynamics. With multiresolutions one gets much better pointwise approximations than is

possible for traditional Fourier bases; fewer terms, and better control on error estimates.

Chun-Kit Lai (McMaster University)

Title: Fourier frames for absolutely continuous measures and its extension

Abstract: We will prove that a necessary and sufficient condition for the L^2 space of an absolutely continuous measure with compact support is that its density is bounded above and away from 0 on its support. We will discuss several proofs and some of its consequences and generalizations.

Michael P. Lamoureux (University of Calgary)

Title: Gabor methods for imaging

Abstract: The Gabor transform of a 1D signal is a windowed Fourier transform which creates a time-frequency representation of physical data which can be directly manipulated through multiplication by symbols. This is analogous to the action of pseudodifferential operators and their representation through symbols in the time-frequency domain.

This talk is structured as a brief tutorial on the basics of Gabor transforms and Gabor multipliers, including windowing considerations, representation of linear operators including differential operators, and the use of Gabor multipliers in nonstationary filtering, deconvolution, and numerical wave propagation. Some particular imaging applications are described.

Joshua MacArthur (Dalhousie University)

Title: On all Haar type wavelets from crystallographic shifts

Abstract: A general construction for Haar type MRAs and their associated wavelets from crystallographic shifts is presented for any dimension. The scaling function arises from the generalized notion of self replicating multi-tiles as explored by Grochenig et al. The corresponding wavelet is constructed by following Bagget et al in their Abstract Approach, explicitly constructing the intertwining map giving the equivalence between a specific finite number of copies of the regular representation, and the core wavelet space. Particular examples and open questions will conclude.

Kateryna Melnykova (University of British Columbia)

Title: Memoryless scalar quantization (MSQ) for random frames and application to compressed sensing

Abstract: In the error analysis for memoryless scalar quantization of frame expansions, perhaps the most common tool for error analysis in the engineering literature is the "White Noise Hypothesis" (WNH) that assumes that, for a given deterministic frame and when the vector to be quantized is random (with an appropriate probability distribution), the quantization errors for individual frame coefficients are independent, and identically distributed. However, the WNH is not rigorous and is not completely valid. Moreover, recently random frames are of interest, especially because of connection to compressed sensing. In this talk, we consider Gaussian random frames and obtain rigorous quantization error estimates when the random frame coefficients of a fixed deterministic vector are quantized via MSQ. Our

results essentially agree with what we would get using the WNH and identify some subtleties that disappear when we make the WNH. This is joint work with Ozgur Yilmaz.

Yi Shen (University of Alberta, University of Calgary)

Title: Image denoising using block matching and low rank tensor approximation

Abstract: We aim to remove the additive independent and identically distributed (i.i.d) Gaussian noise from corrupted images. The proposed method combines the idea of block matching and low rank tensor approximation. For the given noisy image, similar image patches are stacked to be a tensor which is assumed to be sparse in some sense. The image denoising problem is formulated as a low rank tensor approximation problem. Two iterative thresholding algorithms are proposed to reduce the noise. Experimental results show that the proposed algorithms are alternative image denoising approaches.

Qiyu Sun (Department of Mathematics, University of Central Florida)

Title: The abc-problem for Gabor systems

Abstract: One of fundamental problems in Gabor analysis is to identify window functions and time-frequency shift lattices such that the corresponding Gabor system is a Gabor frame for the space of all square-integrable functions on the real line. In this talk, we introduce invariant sets of a discontinuous transformation on a circle, and we provide a full classification of triples (a, b, c) for which the Gabor system generated by the ideal window function on an interval of length c on the time-frequency lattice $aZ \times bZ$ is a Gabor frame.

Kun Wang (University of Alberta)

Title: New finite difference schemes for the Helmholtz equation

Abstract: In this talk, we develop pollution-free finite difference schemes for solving the Helmholtz equation. A family of high-order algorithms is derived by applying the Taylor expansion and imposing the conditions that the resulting finite difference schemes satisfied the original equation and the boundary conditions to certain degrees. The most attractive features of the proposed schemes are: first, the new difference schemes are pollution-free. Hence, the error is bounded even for the equation at high wave numbers. Secondly, the resulting difference scheme is simple, namely it has the same structure as the standard three-point central differencing regardless the order of accuracy. Numerical simulations are reported to confirm the superior performance of the proposed schemes.

Ozgur Yilmaz (University of British Columbia)

Title: Quantization of random frame expansions

Abstract: In this talk, we will review the literature on frame quantization and present a number of recent results. Motivated by compressed sensing we will focus on random frames and explain why classical quantization methods are bound to be substantially sub-optimal. We will then show that by using sigma-delta quantizers along with reconstruction via "Sobolev duals", we can improve the quantization error substantially when we quantize sub-Gaussian frame expansions. Specifically, we prove that using an r -th order sigma-delta scheme, we get an accuracy of order $(-r)$ -th power of the aspect ratio of the frame. Fur-

thermore, if we optimize the order of the scheme depending on the aspect ratio, this yields root-exponential accuracy.

Zhenpeng Zhao (University of Alberta)

Title: Tensor product approach to dual-tree complex wavelet transform

Abstract: In this talk, we address a new approach to dual-tree complex wavelet transform based on the construction of tensor product filters in the frequency domain. The dual-tree complex wavelet transform will be explained firstly, then followed by the construction of tensor product filters that used in this approach. Lastly we expand this tensor product method to more directions and give some numerical results on the application in image denoising.

Xiaosheng Zhuang (City University of Hong Kong)

Title: Directional multi-scale representation systems and smooth affine shear tight frames

Abstract: Finding efficient representations is one of the most challenging and heavily sought problems in mathematics. Representation using directional multi-scale representation systems recently receives a lot of attention due to their desirable properties in both theory and applications.

In this talk, we shall discuss the construction of directional multi-scale representation systems under the framework of “Smooth Affine Shear Frames”. Based on a simple characterization of a frequency-based affine shear frames to be a tight frame in $L_2(\mathbb{R}^2)$, we shall present the construction of two types of smooth affine shear tight frames. One is the non-stationary construction and the other is the quasi-stationary construction. The connection of our construction of frequency-based affine shear systems to other directional multi-scale representation systems shall also be addressed. Finally, we shall show that a frequency-based affine shear tight frame can be regarded as a subsystem of a cone-adapted directional framelet and therefore has also an inherited filter bank.