Abstracts

New Connections Between Differential and Random Turn Games, PDE's and Image Processing

July 27-31, 2009

Mini-Course 1

Tutorial on the p-Laplacian

Bernd Kawohl (University of Cologne)

In the first lecture I shall treat the Dirichlet problem for p-harmonic functions and for $p \in [1, \infty]$. The cases $p = 1$ and $p = \infty$ present surprises. When $p = 1$ there can be many viscosity solutions, only one of which minimizes the associated energy, while for $p = \infty$ the can be many energy minimizers, only one of which is a viscosity solution. I shall also comment on the normalized (or game-theoretic) p-Laplacian, which I denote by A_p, and show that it has nice geometric interpretations.

In the second lecture I discuss several variational and PDE-approaches to mathematical image processing and try to convince the audience that on a discretized level they are more or less equivalent to each other. Moreover I point out why it is only natural to consider evolution equations of type $u_t - A_p u = 0$ in this context.

In the third lecture I present results on the eigenvalue problem $\Delta_p u + \lambda |u|^{p-2} u = 0$, in particular as $p \to 1$, where the first eigenfunction often becomes discontinuous.

Mini-Course 2

Deterministic games for curvature flows and fully nonlinear PDEs

Sylvia Serfaty (New York University)

In this mini-course I will present some results obtained with Robert V Kohn, where we give some deterministic control interpretations (via some simple repeated two-person games) first for curvature flows, and then for a general class of second order elliptic and parabolic fully nonlinear PDEs. I will first review the classical link between optimal control and first order PDEs, then present our games and the convergence results, and compare them with the first order situation and the stochastic control approaches.

Mini-Course 3

TBA

Scott Sheffield (MIT)

Regularity for simple 1-Laplacian problems

Antonin Chambolle (CMAP - Ecole Polytechnique)

I will discuss the "Rudin-Osher-Fatemi" problem of minimizing the Total Variation (TV) with a quadratic penalization (or computing the "prox" operator of the TV), and particularly of the solutions: I will give conditions under which the solution inherits regularity properties of the initial data, locally either globally, and also information on the jump set of solutions, when the initial data is BV.

Level Lines Shortening

Adina Ciomaga (CMLA)

We set forth a new image processing algorithm simulating a contrast invariant evolution of an image via mean curvature motion. We show that Curve Shortening and Mean Curvature Motion are equivalent. We give an explicit method to construct a viscosity solution for the scalar mean curvature motion, provided we are given the curve shortening evolutions of the level lines of the initial data. Applications to subpixel computation of image curvatures are presented. This is a joint work with Jean-Michel Morel and Pascal Monasse.

A variational model for denoising surfaces in computer graphics applications

Selim Esedoglu (University of Michigan)

Geometry denoising, also known as surface fairing, is an important problem in computer graphics. It is needed whenever 3D objects are digitized and represented as triangulated surfaces, typically by sampling points from the surface of an object using a 3D scanner. Measurement errors (noise) in coordinates of the sampled points inevitably lead to oscillations in the digitized surface, giving it a rough and unnatural appearance. The goal of surface fairing is to smooth out the oscillations that are due to noise without also rounding out "legitimate" singularities such as creases and corners that are commonly found on manmade objects (such as machine parts). We will describe a new variational model for surface fairing that is the natural geometric analogue of an image processing model due to Rudin, Osher, and Fatemi. Gradient descent for the energy that we propose, which is defined on surfaces, leads to an interesting new geometric motion that removes the noise but keeps creases and corners intact as it evolves a surface. Its numerical implementation (discretization) using triangulated surfaces turns out to be quite tricky; to accomplish this task, we rely on important previous work of geometers who were interested in extending certain classical theorems from smooth to polyhedral surfaces. Joint work with Matt Elsey.

Singular diffusion equations with nonuniform driving force

Yoshikazu Giga (University of Tokyo)

There are several examples of nonlinear diffusion equations whose diffusivities are so strong that the speed of evolution is determined in nonlocal way. Examples include a total variation flow in image processing and a crystalline flow in materials science. If the equation is formulated in a divergence form like total variation flow, the solvability itself follows from the standard theory of subdifferential equations. However, if the equation does not have a good divergence structure like a crystalline flow such a theory does not work.

During 1998-2001 Mi-Ho Giga and the author developed the theory of viscosity solutions so that such problems can be handled when the space dimension is one and the evolution law is invariant under translation. For a crystalline flow this corresponds to the curve evolution by crystalline curvature with spatially homogeneous driving force. The theory technically depends on the property that a flat portion called a facet stays as a facet during the evolution.

However, this assumption itself is not natural if the driving force is spatially not uniform or the problem is a surface evolution not a curve evolution. In the case of a surface evolution, G. Bellettini, M. Novaga and M. Paolini (Italy) developed a variational theory and constructed a unique solution when initial shape is convex.

In this talk we try to develop a viscosity theory for a crystalline curvature flow with nonuniform driving force for a curve evolution. In particular we introduce a notion of viscosity solution so that comparison principle is valid. Although the several steps are still under work in progress, we expect to have a level set flow for this type of curve evolution equations. There are several explicit solutions having facet breaking phenomena which are constructed by solving several free boundary value problems in a series of the author's joint work with Piotr Rybka (Poland). We expect that these solutions are viscosity solutions in our sense.

This is a joint work with Mi-Ho Giga of University of Tokyo.

Curvature flow connections with the infinity-Laplacian

Robert Jensen (Loyola University Chicago)

I will start this talk by exposing the connections between solutions of the infinity-Laplacian in n dimensions and certain types of inverse sectional curvature flow problems. In particular I will demonstrate how (local) solutions of the infinity-Laplacian can be constructed from smooth solutions of these inverse sectional curvature flows. I will conclude my talk by examining the consequences that this connection implies for solutions of the infinity-Laplacian in 2 dimensions.

On the definition and properties of p-harmonious functions

Juan Manfredi (University of Pittsburgh)

We consider functions that satisfy a weighted mean value property on balls of size epsilon, obtained by considering a variant of the tug-of-war games with noise of Peres and Sheffield. By analogy with the definition of harmonius functions of Le Gruyer and Archer we call them p-harmonius functions. While, in general, p-harmonious functions turn out to be discontinuous, we show that the Dirichlet problem for p-harmonic functions is well-posed, that p-harmonious functions satisfy the strong comparison principle, and that they approximate standard p-harmonic functions as we let epsilon go to zero. This is joint work with Mikko Parviainen and Julio Rossi.

Random-Turn Games

Yuval Peres (Microsoft Research)

Any function f of n variables yields a game, where players alternate turns setting variables of f, with player 1 trying to maximize the value of f and player 2 trying to minimize it. A nice example is provided by the game of Hex, where two players take turns placing stones of their colors on the hexagons of a rhombus-shaped hexagonal grid (so the variables are the colors assigned to the hexagons). White (=Player I) wins by completing a crossing between two opposite vertical edges (If he succeeds then $f = 1$), while Black (=Player II) wins by completing a crossing between the two horizontal edges (In this case $f = 0$). Exactly one of them can succeed. Although ordinary Hex is famously difficult to analyze, random-turn Hex, in which players toss a fair coin before each move to decide who gets to place the next stone, has a simple optimal strategy. I will describe the optimal strategy and estimate the expected length of the game under optimal play for several random-turn games; I will also discuss connections with decision-tree complexity of Boolean functions and Percolation theory. Other random turn games relate to difference schemes for PDE, as described in Scott Sheffield's minicourse. (Talk based on joint work with Oded Schramm, Scott Sheffield and David Wilson.)

Randomly terminated and multiobjective optimal control – computational challenges

Alexander Vladimirsky (Cornell University)

I will present numerical methods for two recent optimal control projects.

The first of these (joint work with J. Andrews) deals with deterministic optimal control of processes with probabilistically specified fixed-horizon. Subject to additional technical assumptions on cost & dynamics, this problem can be converted to an infinite-horizon obstacle problem. Despite the occurrence of non-trivial free boundary, we show that causal numerical algorithms (e.g., Fast Marching, Ordered Upwind) are still applicable. We illustrate our method using examples from optimal idle-time processing.

The second project (joint with A. Kumar) deals with multiple criteria for optimality (e.g., fastest versus shortest trajectories) and optimality under integral constraints. We show that an augmented PDE on a higher-dimensional domain describes all Pareto-optimal trajectories. Our numerical method uses the causality of this PDE to approximate its discontinuous viscosity solution efficiently. The method is illustrated by problems in robotic navigation (e.g., minimizing the path length and exposure to an enemy observer simultaneously).

Uniqueness of the running payoff function

Yifeng Yu (University of California at Irvine)

In talk, we will show that the running payoff function is uniquely determined by the value function. This is equivalent to say that the normalized infinity Lapalcian operator is single valued. This gives an affirmative answer to an open problem posed in the paper by Peres, Schramm, Sheffield, Wison.