G2S3 :: July 2011 :: Waves and Imaging :: Radar, July 5 version

Today we'll use the following convention:

$$S(\omega) = \int e^{i\omega t} s(t)\, dt = \int e^{2\pi i\nu t} s(t)\, dt.$$

$$s(t) = \frac{1}{2\pi} \int e^{-i\omega t} S(\omega)\, d\omega = \int e^{-2\pi i \nu t} S(2\pi\nu)\, d\nu.$$

(the convention we use yesterday is more common for spatial FT.)

Ambiguity functions:

$$|\chi(0,\nu)| = |\int |s(t)|^2 e^{2\pi i\nu t}\, dt|,$$

$$|\chi(\tau,0)| = |\int |S(2\pi\nu)|^2 e^{2\pi i \nu\tau}\, d\nu|.$$

Relation between frequency ν (Hz) and velocity v (m/s):

$$\nu = 2\frac{v}{c}\omega_c,$$

where ω_c is the carrier (angular) frequency. Note $\omega = 2\pi\nu$. Relation between delay τ and range R:

$$\tau = 2\frac{R}{c}$$

Questions:

1. Plot the ambiguity function for the CW pulse

$$s(t) = \begin{cases} \cos(2\pi\nu t) & \text{if } 0 \le t \le T; \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

   where $\nu = 10$ GHz and $T = 10^{-4}$ sec. What is the range resolution? What is the velocity resolution? Try some other values of ν and T.

2. Plot the ambiguity function for the upchirp

$$s(t) = \begin{cases} \cos(2\pi(\nu t + \gamma t^2)) & \text{if } 0 \le t \le T; \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

   for $\nu = 10$ GHz, $T = 10^{-4}$ sec, and $\gamma = 100$ GHz/sec. What is the bandwidth (roughly)? What is the range resolution? What is the velocity resolution? Try some other values of ν, T, and γ. What happens if you use a downchirp ($\gamma \mapsto -\gamma$) instead?

3. Plot the ambiguity function for the train of pulses

$$p(t) = \sum_{n=0}^{N-1} s(t - nT_R) \tag{3}$$

   where s is one of the waveforms above, $N = 1000$, and $T_R = 3 \cdot 10^{-4}$ sec. Try varying N and T_R, as well as the other parameters in s. What are the range and velocity resolution of the central peak?