Abstracts

The PDE workshop on Analysis of Nonlinear PDEs and Free Boundary Problems: Applications to Homogenization

July 20-24, 2009

Mini-course 1

L. Caffarelli (University of Texas)

Harnack inequalities for non local equations and the non linear regularity theory

Mini-course 2

P.E. Souganidis, (University of Chicago)

Athanasopoulos, Ioannis (University of Crete)

Continuity in the Fractional Stefan Problem, Anomalous Diffusions, and related problems

In a joint work with L. Caffarelli, we prove the continuity of the solution for these problems. We first treat the 1/2 fraction case. This case can be viewed as a a boundary non-linear Neumann condition of a boundary value problem for the heat equation in a plus one dimension. Boundary value problems of this type are related to boundary and/or heat control problems (see Duvaut and Lions /"Les inequations en mechanique et en physique" /Dunod, Paris 1972). We then extend the result to the fractional order other than 1/2.

Bardi, Martino (University of Padua, Italy)

Multiscale problems for parabolic Bellman-Isaacs equations

We consider control systems involving some random effects modeled by a dependence on an ergodic diffusion process evolving on a fast time scale. For a given cost functional, the value function solves a degenerate parabolic Bellman-Isaacs equation with a small parameter. We study the asymptotics as such parameter tends to 0 and prove convergence to an effective PDE associated to an averaged system and cost functional. The correct averages depend on the whole optimization problem and may differ from the historical means appearing in the classical probabilistic results of Kushner (1990).

An example is the Merton portfolio optimization problem with fast mean reverting stochastic volatility. Fouque, Papanicolaou, and Sircar (2000) studied this model by asymptotic expansions and showed that the limit has the same form, but the effective volatility must be computed as a harmonic mean. We discuss other examples of economic models where the averaging procedure changes also the form of the original system.

Capuzzo Dolcetta Italo (Universit di Roma, Italy)

A numerical method for the homogenization of periodic Hamilton-Jacobi equations

We study approximation strategies for the limit problem arising in the homogenization of periodic Hamilton-Jacobi equations. They involve first an approximation of the effective Hamiltonian then a discretization of the Hamilton-Jacobi equation with the approximate effective Hamiltonian. We give a global error estimate which takes into account all the parameters involved in the approximation.

Cardaliaguet, Pierre (Universite de Brest)

Long time behavior of HJ equations in the plane

We investigate the long time average of the solutions of planar, periodic Hamilton-Jacobi equations with a non-coercive, non convex Hamiltonian. We give non-resonnance conditions under which the long-time average converges to a constant. In the resonnant case, we show that the limit still exists, although it is non constant in general. We compute the limit at points where it is not locally constant.

Da Lio, Francesca (Universita di Padova)

Analysis of Half-Harmonic Maps into Manifolds

We present some recent progress in the analysis of 1/2 harmonic maps into manifolds. These nonlocal systems of equations should be relevant in the asymptotic of fractional reaction-diffusion equations.

Davini, Andrea

Qualitative analysis of critical Hamilton-Jacobi equations in the stationary ergodic setting

We present some results in collaboration with Antonio Siconolfi about Hamilton-Jacobi equations with stationary ergodic Hamiltonians. The key idea of our analysis is to observe that the stationary ergodic structure of the Hamiltonian induces a stochastic geometry in the space of the state variable where the fundamental objects are the random closed stationary sets which, somehow, play the same role as the points in the deterministic case. In this context we give a suitably adapted Lax formula, then we propose a generalization of the notion of Aubry set and establish some existence and nonexistence results for exact and approximate correctors.

Dirr, Nicolas (University of Bath)

Effective velocity for interfaces in heterogeneous media

I consider parabolic PDEs for the evolution of an interface with an additive periodic or random perturbation (modeling the interaction with a heterogeneous environment) and a constant forcing (driving field). I will present some results on the effective velocity of the interface on large scales for weak periodic forcing, and some results in the case of random forcing, with particular emphasis on the difference between the periodic and the random case.

Imbert, Cyril (Universite Paris-Dauphine)

Level set approach for fractional mean curvature flows

This paper is concerned with the study of a geometric flow whose law involves a singular integral operator. This operator is used to define a non-local mean curvature of a set. Moreover the associated flow appears in two important applications: dislocation dynamics and phase field theory for fractional reaction-diffusion equations. It is defined by using the level set method. The main results of this paper are: on one hand, the proper level set formulation of the geometric flow; on the other hand, stability and comparison results for the geometric equation associated with the flow.

Ishii, Hitoshi (Waseda University, Tokyo, Japan)

Nonlinear singular integral equations and approximation of p-Laplace equations

I will discuss the well-posedness of the Dirichlet problem for a class of nonlinear singular integral equations in a domain of n-dimensional Euclidean space. The integral operator has the monotone, power nonlinearity of order p-1, with p¿1, and the kernel function of the Euclidean distance with negative power of order $n + s$, where $s < p$. I will then discuss the uniform convergence on the domain, as s tends to p, of the solution u_s of the Dirichlet problem to the solution u of the Dirichlet problem for the p-Laplace equation with the same Dirichlet condition. The talk will be based on a recent joint work with Gou Nakamura.

Kim, Inwon (UCLA)

Homogenization of free boundary velocities

Nolen, Jim (Stanford University)

Fluctuations for traveling waves in an inhomogeneous medium

For generalized traveling waves in a statistically stationary medium, the position of the wave is a stochastic process which has a well-defined asymptotic speed (law of large numbers). Under suitable mixing conditions on the environment, this process also satisfies a functional central limit theorem.

Petrosyan, Arshak (Purdue University)

Almost Monotonicity Formulas for Elliptic and Parabolic Equations with Variable Coefficient

We prove almost monotonicity formulas for pairs of continuous sub-solutions of uniformly elliptic and parabolic equations in divergence form with nonzero bounded right-hand side, provided the coefficients are double Dini (e.g. Holder) continuous. This generalizes the original monotonicity formulas of Alt-Caffarelli-Friedman (elliptic case) and Caffarelli (parabolic case) as well as the almost monotonicity estimates of Caffarelli-Kenig, Caffarelli-Jerison-Kenig, and Edquist-Petrosyan.

Finally, we give an application of the new almost monotonicity formula (in the elliptic case) to prove the optimal regularity in a quasilinear obstacle-type problem related to superconductivity.

Ryzhik, Lenya (University of Chicago)

Waves and particles in random media with slowly decaying correlations

Weakly random media affect propagation of waves and particles over long distances in a non-trivial way. When medium correlations decay rapidly, the long time behavior is reasonably well understood: both waves and particles behave, loosely speaking, diffusively, and most non-trivial phenomena happen on the same time scale. Slow decay of media correlations seems to produce an interesting effect when single particle behavior becomes non-trivial on a much shorter time scale than the multi-particle behavior. Similarly, the wave phase becomes random long before wave energy randomizes. Moreover, while some quantities acquire fractional Brownian limits because of the slow correlations decay, others keep their diffusive limits. I will describe some rigorous as well as formal results in these directions. This is a joint work in progress with T.Komorowski.

Schwab, Russell (University of Texas)

Periodic Homogenization for Fully Nonlinear Nonlocal Elliptic Equations

Building upon ideas of Caffarelli-Souganidis-Wang, we present the periodic homogenization for fully nonlinear nonlocal elliptic equations; such equations include optimal control and differential games with a large class of pure jump Levy Processes. The main issues are to identify the appropriate notion of a nonlocal "corrector" equation, characterize the right value of the limiting operator through a "corrector" decay criterion combined with an auxiliary obstacle problem, and adapt the proofs to account for technical difficulties arising from the nonlocal nature of the operators.

Shahgholian, Henrik (KTH-Stockholm)

Obstacle problem, the unstable case

In this lecture I will present recent developments around the unstable case of a free boundary problem of obstacle type

$\Delta u = -\chi_{u>0}$.

The simple introduction of a sign in front of the nonlinearity introduces a substantial difficulty in studying this problem.

This is a joint work with John Andersson and Georg S. Weiss

Stefania Patrizi

On the Rate of Convergence in Homogenization of First Order Equations With u/ϵ-Periodic Hamiltonians

We consider the homogenization problem of first order Hamilton-Jacobi equations with u/ϵ-periodic Hamiltonians. We give an estimate of the rate of convergence of the solutions of the small scale problems to the solution of the homogenized problem. Moreover, numerical approximation strategies for the limit problem are presented.

This is a joint work with Yves Achdou (Universitè Paris 6).

Ki-Ahm Lee

Homogenizations in Perforated domains

In this talk, we are going to consider viscosity methond for Homogenizations in Perforated domains. When Neuman conditions are imposed at the boundary of the periodically distributed perforated domains, we are going to find the effective equation satisfied by the limit solution as the perodicity goes to zero through viscosity method. We found discrete gradient estimate and almost flatness in each cell. And we also found the correctors which will correct the solution of the effective equation to be solutions of ϵ problems.