Abstracts
Workshop on Nonlinear Dispersive and Geometric Evolution Problems: Singularities and Asymptotics

August 17-21, 2009

Mini-Course 1

Review on the collision of two solitons for the gKdV and BBM equations

Yvan Martel **(Versailles)**

We will review recent results in collaboration with Frank Merle concerning the collision of two solitons for the nonintegrable gKdV equations and for the BBM equation in different regimes.

Mini-Course 2

On energy critical wave maps to hyperbolic Riemann surfaces

Wilhelm Schlag (University of Chicago)

We will discuss work by Joachim Krieger and the speaker on wave maps from 2+1 dimensions to the hyperbolic plane.
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1 Mini-Course 3

Nonlinear Schrdinger Evolutions from Rough Initial Data

James Collainder (University of Toronto)

This short course will describe some basic ideas in the study of the initial value problem for the nonlinear Schrdinger equation with low regularity initial data. The first lecture will describe Bourgain's bilinear refinement of the Strichartz estimate and survey some of its applications. The second lecture will describe some properties of blowup solutions.
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On the $2D$ Zakharov system with L^2 Schrödinger data

Ioan Bejenaru (Chicago)

We prove local in time well-posedness for the Zakharov system in two space dimensions with large initial data in $L^2 \times H^{-1/2} \times H^{-3/2}$. This is the space of optimal regularity in the sense that the data-to-solution map fails to be smooth at the origin for any rougher pair of spaces in the L^2-based Sobolev scale. Moreover, it is a natural space for the Cauchy problem in view of the subsonic limit equation, namely the focusing cubic nonlinear Schrödinger equation. The existence time we obtain depends only upon the corresponding norms of the initial data – a result which is false for the cubic nonlinear Schrödinger equation in dimension two – and it is optimal because Glangetas–Merle's solutions blow up at that time.

Multi-solitons for the $L2$ supercritical generalized Korteweg-de Vries and non linear Schrödinger equations

Raphaël Côte (École Polytechnique)

This is joint work with Yvan Martel and Frank Merle. We consider the gKdV and NLS equations. It is well known that these equations admit travelling wave solutions called solitons. Our problem is to construct multi-solitons, that is, solutions which behave like a like a sum of soliton as time goes to infinity. In the subcritical case, solitons are stable : earlier results showed existence of multi-solitons, and relied heavily on the stability theory for solitons. In the supercritical case, although solitons are unstable, we are still able to construct multi-solitons.

A Lax pair for the cubic Szegö equation and applications

Patrick Gerard **(University Paris-Sud, Orsay)**

I will introduce the cubic Szeg equation, a simple model of a totally nondispersive Hamiltonian system, and show that this equation admits a Lax pair. As an application of this Lax pair structure, I will discuss various large time features for this equation, and I will present a classification of its traveling waves. This is a jointwork with Sandrine Grellier (Orleans).

Energy exponential decay for the damped NLKG equation with arbitrary growing nonlinearities

Slim Ibrahim **(McMaster University)**

We are interested in proving the exponential decay of the total energy for a damped nonlinear Klein-Gordon equation on R^N, $N \geq 2$. Such a result is already known, in the defocusing case, for energy subcritical nonlinearities. The restrictions to this type of nonlinearity are mainly due to the use of unique continuation and linear approximation arguments. We propose a direct approach based solely on Morawetz-type a priori estimates. It applies to the energy critical case, including exponential nonlinearities in dimension two, and also to the supercritical case, as long as the solution remains regular. We can also treat the focusing case once we have control of the nonlinear part in Morawetz-type estimates. In particular this can be achieved when we know the scattering for the undamped equation.

This is joint work with Lassaad Aloui and Kenji Nakanishi.

Second order corrections for weakly interacting Bosons

Manoussos Grillakis **(Maryland)**

An Interaction Functional Approach to Null Form Estimates

Markus Keel (University of Minnesota)

This is joint work with P. LeFloch and T. Tao. We investigate whether interaction functionals similar to those developed in the context of semilinear Schroedinger equations can say anything for hyperbolic equations in more than one space dimension. One example of such a result is a null form estimate for the massless Dirac equation in higher dimensions.

Quantum many-body systems and the nonlinear Schrödinger equation

Kay Kirkpatrick (MIT)

We will discuss joint work with Benjamin Schlein and Gigliola Staffilani on the persistence of the extremely low-temperature phenomenon of Bose-Einstein condensation in both the planar and toroidal cases. The scaling limit for the microscopic quantum many-body systems with localized repulsive interactions is described macroscopically by the cubic NLS–and the periodic case is especially interesting, as it involves some techniques from analytic number theory.

Asymptotic stability and eternal oscillation of harmonic maps in Schrodinger and heat flows

Kenji Nakanishi (Kyoto)

We study asymptotic behavior for large time of solutions for the Schrodinger map, the harmonic map heat flow and the Landau-Lifshitz equation from the whole plane to the sphere, for small initial perturbation from the harmonic maps of the form z^m under the equivariant symmetry restriction in the energy space. The question is asymptotic stability of the family of the harmonic maps with the two parameters of rotation and scaling. This problem becomes harder as the degree m gets lower, due to slow convergence at spatial infinity of the harmonic map. We prove that when the degree m is bigger than 2, each solution is global and converges to one harmonic map, and that if the degree is 2 and there is no dispersion (i.e. the heat flow) or rotational disturbance, then each solution is global and converges to the family of harmonic maps, but does not necessarily converges to a fixed harmonic map. In fact we have a very explicit formula for the asymptotic behavior of the scaling parameter in terms of the initial data, by which we can construct solutions which repeat (almost) concentration and dispersion forever. This is joint work with Stephen Gustafson and Tai-Peng Tsai.

Almost sure global solutions of the periodic cubic NLS below L^2

Tadahiro Oh **(Toronto)**

We consider the Cauchy problem for the 1-d cubic NLS with the Gaussian measures on the negative Sobolev spaces as initial data. The nonlinear smoothing under randomization is the main ingredient for establishing the LWP almost surely on the support of the Gaussians. In exploiting such nonlinear smoothing under randomization, we use the hypercontractivity of the Ornstein-Uhlenbeck process and its implication on the 3rd order homogeneous Wiener chaos. After establishing the LWP, we extend the local solutions to the global ones by applying the so-called Bourgain's high-low method. In iterating the local steps, we successively apply the deterministic $L2$ local theory to the "low-frequency" part and the probabilistic local theory to the high frequency part, which yields the nonlinear smoothing required for the high-low argument. This is a joint work with J. Colliander.

Global Cauchy Problem for $2D$ Klein-Gordon equations

Tohru Ozawa **(Waseda University)**

This talk is based on my recent joint work with Jun Kato, Nagoya University. We consider the global existence of small amplitude solutions to the Cauchy problem for the quadratic NLKG in two space dimensions. We are interested in the weight conditions on the Cauchy data.

Resonant tunneling of fast solitons through large potential barriers

Walid Abou Salem (Toronto)

I discuss recent results on resonant tunneling of fast solitons through large potential barriers for the nonlinear Schroedinger equation in one dimension.

A Regularity Theory for Wave-Maps

Jacob Sterbenz (UCSD)

I'll discuss recent work, joint with Daniel Tataru, concerning the blowup vs. global regularity and scattering question for energy critical wave-maps.

Concentration and Dispersion for Large Data Wave Maps

Daniel Tataru (Berkeley)

The goal of this talk is to describe recent joint work with Jacob Sterbenz on the dynamics of large data wave maps, precisely the dichotomy between soliton-like concentration and dispersion.

Existence of Cavity Soliton for the nonlinear Schrodinger equation with damping and homogeneous forcing terms

Yoshio Tsutsumi **(Kyoto)**

We consider the existence of stationary solution for the Lugiato-Lefever equation on Euclidean spaces of dimensions less than or equal to three, to which is referred as (LL). The (LL) equation is a nonlinear Schrodinger equation with damping and homogeneous forcing terms, which describes a physical model of a unidirectional ring or Fabry-Perot cavity with plane mirrors containing a Kerr medium driven by a coherent plane-wave field. The stationary solution of (LL) is called a "Cavity Sliton". There are two difficulties in this problem. One is that a forcing term is spatially homogeneous, which implies it deos not belong to the square-integrable function sapce. Another is that (LL) has no variational structure because of the presence of damping term.