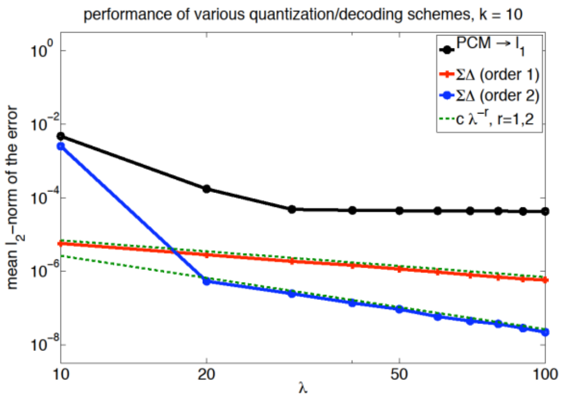
Efficient quantization methods for compressed sensing



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Digital computers and their efficiency at processing data is one of the main driving forces behind much of our modern technology. On the other hand, various important classes of signals are inherently analog, e.g., audio, images, and video. Therefore, an important problem in signal processing is to acquire---or sense---such analog signals, "convert" them to digital, and encode them such that they can be transmitted and stored in digital form reliably and efficiently. One approach to this problem is by decoupling the sensing and coding stages: First one acquires the data (including the analog-to-digital (A/D) conversion), where the accuracy of the sensing relies on an appropriate sampling theorem, and then one uses well-established methods from coding theory (over binary fields) for compression and error correction. This is essentially in line with the practice of, for example, digital cameras. Alternatively, one can attempt to combine the sensing stage with the coding stage, as in the case of "compressed sensing" (CS), a novel paradigm in mathematical signal processing that was spearheaded by the seminal works of Candes, Romberg, Tao, and of Donoho. CS is based on the observation that various classes of signals such as audio and images admit a (nearly) sparse representation with respect to a known basis or a frame---a structured redundant spanning set---and shows that (nearly) sparse signals can be recovered (or approximated well) from a small number of appropriate, non-adaptive "linear measurements". Furthermore, this "recovery" is by means of computationally tractable algorithms, such as one-norm minimization.

The bulk of the CS literature assumes the signals are in Euclidean space, and the CS measurements are also real or complex valued. Accordingly, CS has been shown to be an effective dimension reduction method; however a comprehensive quantization theory has been missing. In recent work, we showed that we can successfully employ noise-shaping Sigma-Delta quantizers for compressed sensing. We prove that, by adopting a two-stage approach involving the use of appropriate reconstruction methods that rely on the use of Sobolev dual frames, Sigma-Delta quantizers utilize the inherent redundancy of compressed sensing more efficiently than ”any” round-off type quantization algorithm, at least in the case of Gaussian measurement matrices. This work develops a novel quantization theory in the setting of CS that yields provably superior accuracy with respect to the bit rate when compared to the round-off type quantization methods.

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