RECENT ADVANCES IN HODGE THEORY: PERIOD DOMAINS, ALGEBRAIC CYCLES, AND ARITHMETIC

JAMES D. LEWIS, MATT KERR, AND GREGORY PEARLSTEIN

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1. Conference overview and layout

In its simplest form, Hodge theory is the study of periods – integrals of algebraic differential forms which arise in the study of complex geometry, number theory and physics. Its difficulty and richness arises in part from the non-algebraicity of these integrals. According to the beautiful conjectures of Hodge, Bloch and Beilinson, what algebraic structure they have should be explained by (generalized) algebraic cycles. There has been much recent progress on these conjectures and on classifying spaces for periods, as well as their asymptotics and arithmetic; this conference will bring together leading scholars and students of these topics.

Given that our desire is to create a number of new collaborations, we would aim at a small number of high-quality talks per day (perhaps four) with ample breaks between each. In addition to bringing together leading scholars on the different aspects of the asymptotics, symmetries, and arithmetic of periods together for a research conference, the organizers will also arrange for a prior workshop for graduate students and recent Ph.D.'s on major themes of the conference. We expect this expository part to run approximately 4 days, with lectures by the organizers and some of the invited speakers.

As for the formatting of talks for the conference portion, each day would be focused on a single major theme:

1. Algebraic cycles and the Hodge Conjecture
2. Arithmetic aspects of cycles and period maps
3. Period domains and their compactifications
4. Mumford-Tate groups and representation theory
5. Automorphic forms and automorphic cohomology
6. Relative Completion of π_1

2. Discussion of the themes

The birth of modern Hodge theory began with the work of P. Griffiths, who devised an extension of Lefschetz's original proof of the Hodge conjecture using normal functions and variations of Hodge structure. Despite the fact that this program did not lead to the desired fruition in higher codimension, it has had a lasting impact on the subject. The last 40 years have seen the development of rich theories of "Hodge theory at the boundary" and "symmetries of Hodge structures"

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which are themes of this conference. In particular, by the recent work of Griffiths et. al., the Hodge conjecture itself can now be stated in terms of the asymptotics of such period integrals; while Mumford-Tate (symmetry) groups of Hodge structures have led to proofs of the Hodge and Beilinson-Hodge conjectures in special cases. The other closely related theme of the conference is "the arithmetic of periods", of which Euler's work on relations among multiple zeta values can be interpreted as an early example. Again, Hodge theory at the boundary plays a critical role. In what follows we elaborate on these new developments and the connections between the main themes which we hope will foster new collaborations between participants.

At the heart of current thought on the Hodge conjecture in the last decade, two intertwined programs have emerged. The approach referred to above, proposed by Griffiths and Green (and influenced by work of Clemens and Thomas) reduces the conjecture to the existence of singularities for certain several-variable admissible normal functions obtained from Hodge classes. While this criterion pertains a priori to degenerations of normal functions, a result of Schnell reveals the importance of estimates on the dimension of its zero loci, which have recently been proven to be algebraic by three different groups of researchers. Another approach, championed by Voisin (building on work of Cattani, Deligne and Kaplan), is to break the Hodge conjecture into two pieces: first, to show that the locus of Hodge classes in a variation is defined over a number field; and second, to prove the Hodge conjecture on arithmetic varieties. Key to this approach is showing that a given family of Hodge classes is absolute, extending Deligne's theorem for abelian varieties.

The geometry, arithmetic, and representation theory of generalized period domains is intimately tied to both of these programs, as well as to aspects of the Bloch-Beilinson conjectures on algebraic cycles. In each case, the problem can be summed up in terms of how some period map meets *subdomains* and *boundary components*, which are themselves orbits of $\mathbb{Q}$ -algebraic (Mumford-Tate) groups.[1] Until recently, both were mysterious in higher weight (and for mixed Hodge structure), but a fundamental work of Kato and Usui (and subsequent work with Nakayama) has elucidated the boundary by means of logarithmic geometry. At the same time, work of Griffiths, Green and Kerr has classified possible Mumford-Tate groups and given a path toward classifying all subdomains of individual period domains; the Hermitian symmetric subdomains which parametrize variations of Calabi-Yau type have recently been classified by Friedman and Laza. Moreover, the study of automorphic and characteristic cohomology of period domains (begun by Griffiths, Schmid, Williams and others) has reached a point where it may be feasible to look for universal Hodge classes which pull back to Hodge classes in the intersection cohomology of a variation. Meanwhile, Abdulali's theorem that all CM Hodge structures are motivated by abelian varieties suggests that CM points may play some heretofore unimagined role in proving absolute Hodge.

In the last two decades, conferences on algebraic cycles have done a superb job of bringing together mathematicians working in certain areas of arithmetic and transcendental algebraic geometry, around a common language of regulator maps. The goal of the proposed workshop would be to build a community around period maps, broadening the interface between those applying Hodge-theoretic methods to cycles (including the Hodge conjecture) and those studying pure Hodge theory

[1] In particular, we are thinking of zero-loci as pullbacks of generalized subdomains by normal functions considered as VMHS.

and Griffiths-Schmid varieties. The behavior of period maps with respect to subdomains, boundary components, and special points gives a language in which to recast, in addition to those topics mentioned above, Grothendieck's conjecture on periods, the absolute Hodge conjecture, and generalizations of the Andre-Oort conjecture and Shimura-Taniyama conjecture to higher weight. Accordingly we would seek to involve participants from Shimura varieties, transcendence theory, representation theory and the Langlands program, whose work already has a strong Hodge-theoretic component. In particular one should mention the groundbreaking work of Carayol on arithmetic automorphic cohomology and limits of discrete series.

Another theme which will be explored at the conference is Hain's theory of the relative completion of the fundamental group, which is an object which unifies Chen's theory of iterated integrals with the theory of variations of Hodge structure. In particular, the normal functions arising in the work of Griffiths on the Hodge conjecture are just Hodge representations of the relative completion of a particular kind. On the other hand, many objects of arithmetic interest such as Manin's iterated integrals of modular forms can also be interpreted in terms of the relative completion. Related work (Hain, Matsumoto) on the Galois theory of the relative completion and the non-abelian cohomology theories (Hain) emanating from Kim's proof of Siegel's theorem using the motivic fundamental group will also be included.

3. Sources of funding and cost-sharing support

We have funding from PIMS and plan to request additional support from the NSF, NSA, and CMI. For the speakers, we will at very least be able cover accomodation expenses.

Department of Mathematical and Statistical Sciences, University of Alberta, Edmonton, Alberta, Canada T6G 2G1
E-mail address: lewisjd@ualberta.ca

Department of Mathematics, Washington University, St. Louis, MO, USA
E-mail address: matkerr@math.wustl.edu

Department of Mathematics, Michigan State University, Lansing, MI, USA
E-mail address: gpearl@math.msu.edu