Stochastic variability of mass flux in a cloud resolving simulation

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References

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- Brenda G. Cohen and George C. Craig, 2006: J. Atmos. Sci. **63** , 1996-2004.
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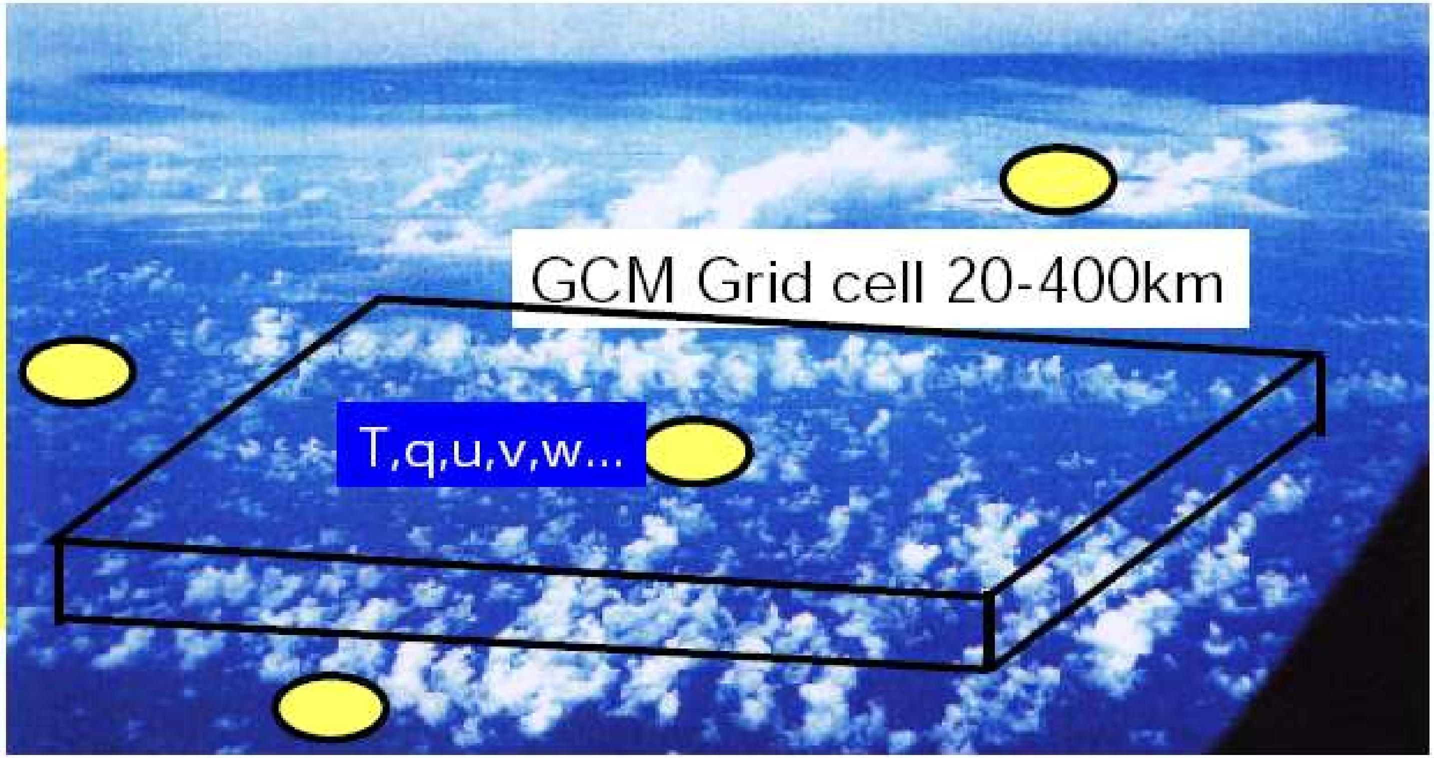
Convective parameterization



Many clouds and especially the processes within them are subgrid-scale size both horizontally and vertically and thus must be parameterized.

This means a mathematical model is constructed that attempts to assess their effects in terms of large scale model resolved quantities.

Parameterization Basics

Arakawa $\&$ Schubert 1974

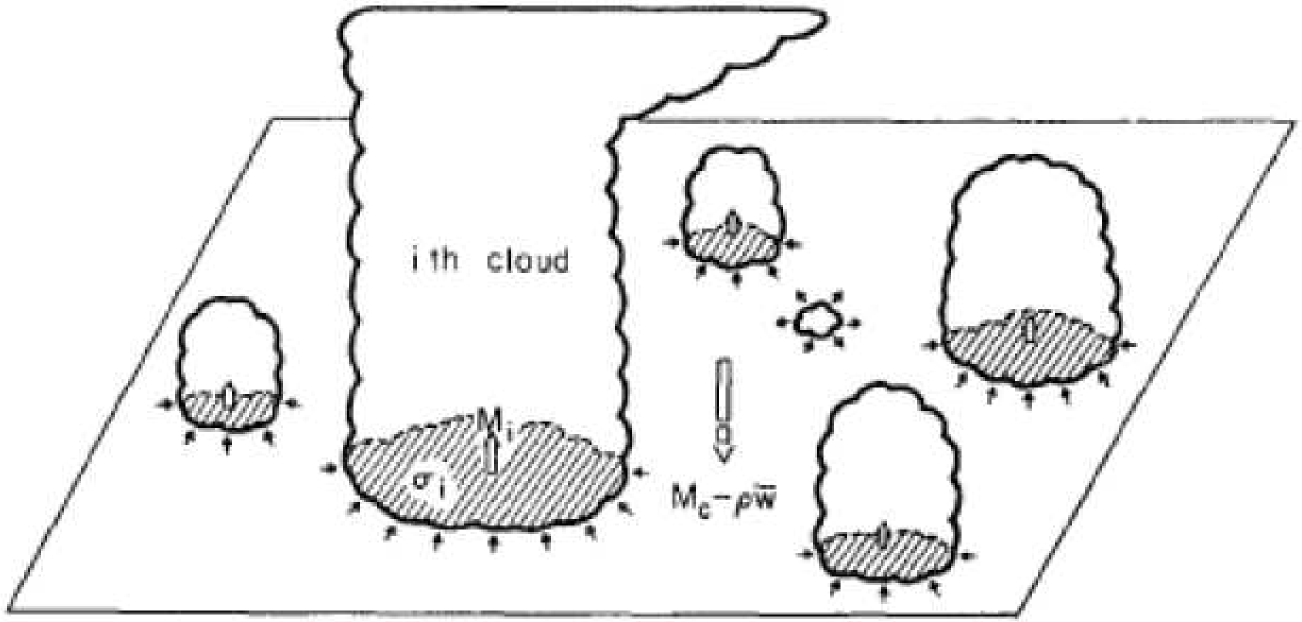
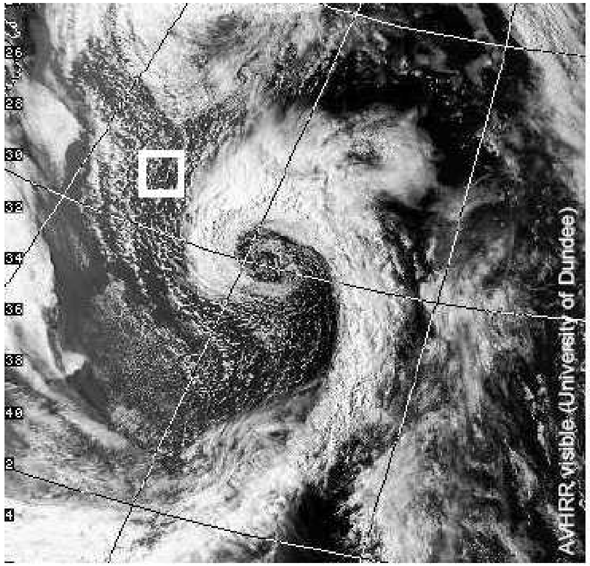


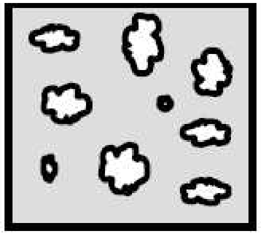
FIG. 1. A unit horizontal area at some level between cloud base and the highest cloud top. The taller clouds are shown penetrating this level and entraining environmental air. A cloud which has lost buoyancy is shown detraining cloud air into the environment.

Key equilibrium assumption:

$$\tau_{adj} \ll \tau_{ls}$$

Fluctuations in radiative-convective equilibrium





- Convective ensemble
- Analogous to the equation of state

  p=ρRT

- For convection in equilibrium with a given forcing, the mean mass flux should be well defined.
- At a particular time, this mean value would only be measured in an infinite domain.
- For a region of finite size:
  - What is the magnitude and distribution of variability?
  - What scale must one average over to reduce it to a desired level?

Main assumptions

Assume:

1. Large-scale constraints- mean mass flux within a region $\langle M \rangle$ is given in terms of large scale resolved conditions

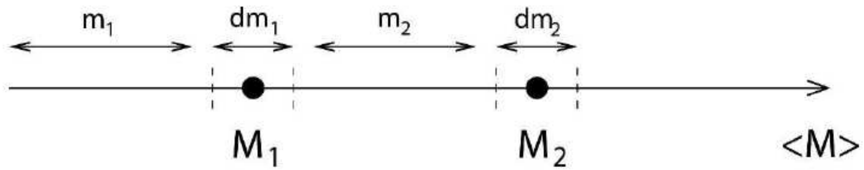
2. Scale separation- environment sufficiently uniform in time and space to average over a large number of clouds

3. Weak interactions- clouds feel only mean effects of total cloud field(no organization)

Find the distribution function subject to these constraints

Large scale constraint

- $\langle M \rangle$ is determined by the requirement that the convection balance the large scale forcing when averaged over a large region.
- $\langle m \rangle$ is not necessarily a function of large scale forcing
- Observations suggest that $\langle m \rangle$ is independent of large scale forcing
- Response to the change in forcing is to change the number of clouds.
- $\langle m \rangle$ might be only sensitive to the initial perturbation triggering it and the dynamical entrainment processes.



- mass flux of individual clouds are statistically un-correlated :

$$P_M(n) = Prob\{N\,[(0,M]) = n\} = \frac{(\lambda M)^n e^{-\lambda M}}{n!} \quad n = 0, 1, \cdots$$

given $\lambda = 1/(\langle m \rangle) = \frac{\langle N \rangle}{\langle M \rangle}$ is fixed.

- Poisson point process implies:

$$P(m) = \frac{1}{\langle m \rangle} e^{-\frac{m}{\langle m \rangle}}$$

- The total Mass flux for a given N Poisson distributed plumes is a Compound point process:

$$M = \sum_{i=0}^{N} m_i$$

Predicted distribution

So the Generating function of M is calculated exactly:

$$\langle e^{tM} \rangle = e^{-\Lambda} e^{\Lambda G(t)} \qquad (1)$$
$$G(t) = \langle e^{tm} \rangle \qquad \Lambda = \langle N \rangle$$

Therefore the probability distribution of the total mass flux is exactly given by:

$$P(M) = P(M) = \left(\frac{\langle N \rangle}{\langle m \rangle}\right)^{1/2} e^{-\langle N \rangle} M^{-1/2} e^{-M/\langle m \rangle} I_1 \left(2\left(\frac{\langle N \rangle}{\langle m \rangle} M\right)^{1/2}\right)$$

All the moments of M are analytically tractable and are functions of $\langle N \rangle$ and $\langle m \rangle$.

$$\frac{\langle (\delta M)^2 \rangle}{\langle M \rangle^2} = \frac{2}{\langle N \rangle}$$
$$\langle M^r | N \rangle = \frac{(N + r - 1)!}{(N - 1)!} \langle m \rangle^r$$

Estimates

- In a region with area A and grid size $\Delta x \gg L$ where L the mean cloud spacing is:

$$L = (A/\langle N \rangle)^{1/2} = (\langle m \rangle A/\langle M \rangle)^{1/2}$$

- Assume latent heat release balance radiative cooling S,
rate of Latent heating $\simeq$ Convective mass flux $\times$ Typical water vapor mass mixing ratio q

$$l_v q \frac{\langle M \rangle}{A} = S$$

- Estimate:
$S = 250 W m^{-2}$, $q = 10 g kg^{-1}$ and $l_v = 2.5 \times 10^6 J kg^{-1}$ gives

$$\langle M \rangle / A = 10^{-2} kg s^{-1} m^{-2}$$

- $\langle m \rangle = w \rho \sigma$ with $w \simeq 10 m s^{-1}$ and $\sigma \simeq 1 km^2$ gives

$$\langle m \rangle \simeq 10^7 kg s^{-1}$$

hence

$$L \simeq 30 km$$

- $\frac{\delta M}{M} = \sqrt{2} \frac{L}{\Delta x}$

Simulations with a 'cloud resolving' model

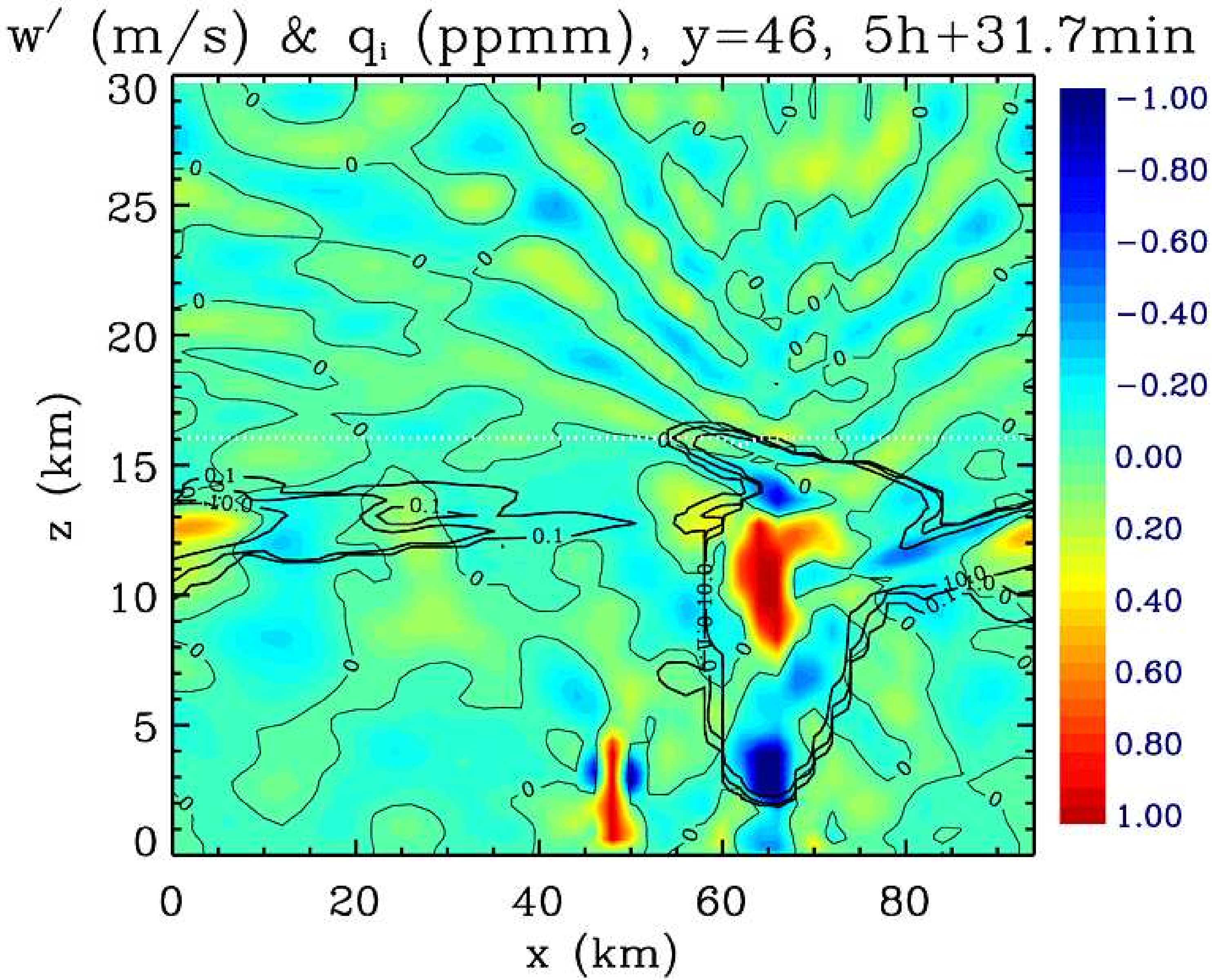
Resolution: $2\text{km} \times 2\text{km} \times$ 90 levels

Domain: 96 km $\times$ 96 km $\times$ 30 km

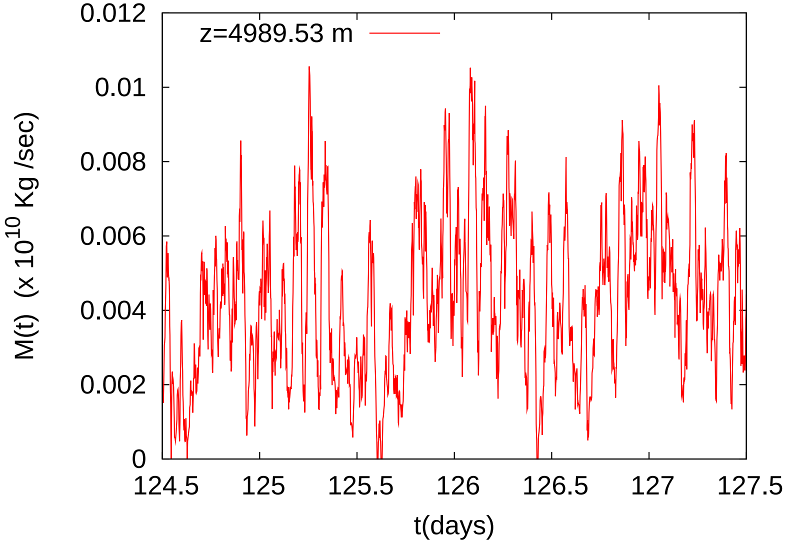
Boundary conditions: doubly periodic, fixed SST of 300 K

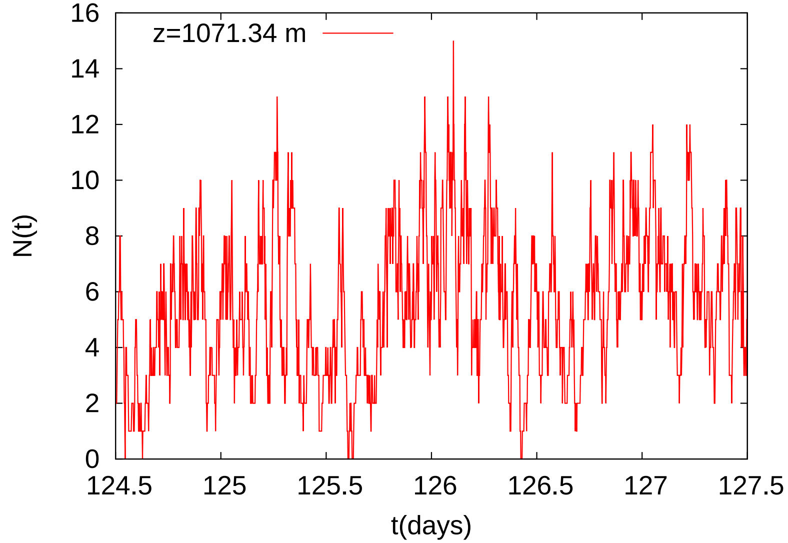
Forcing: An-elastic equations with fully interactive radiation scheme

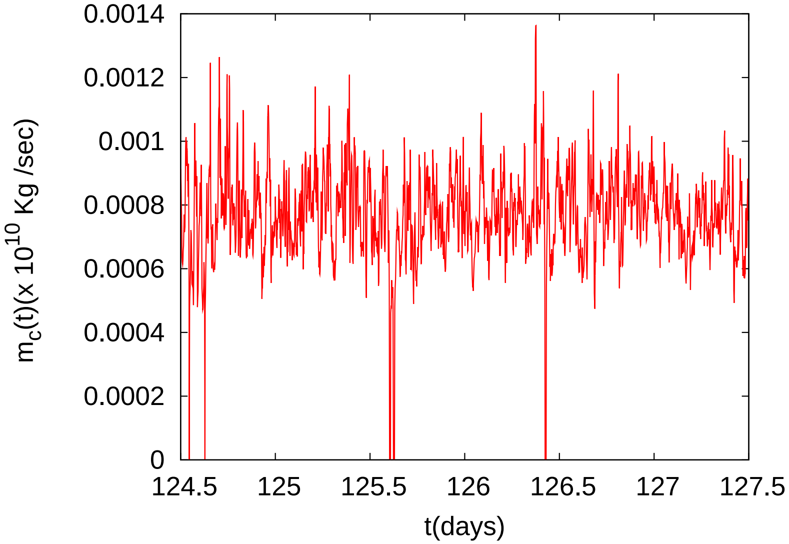
A 2D cut through the convective field



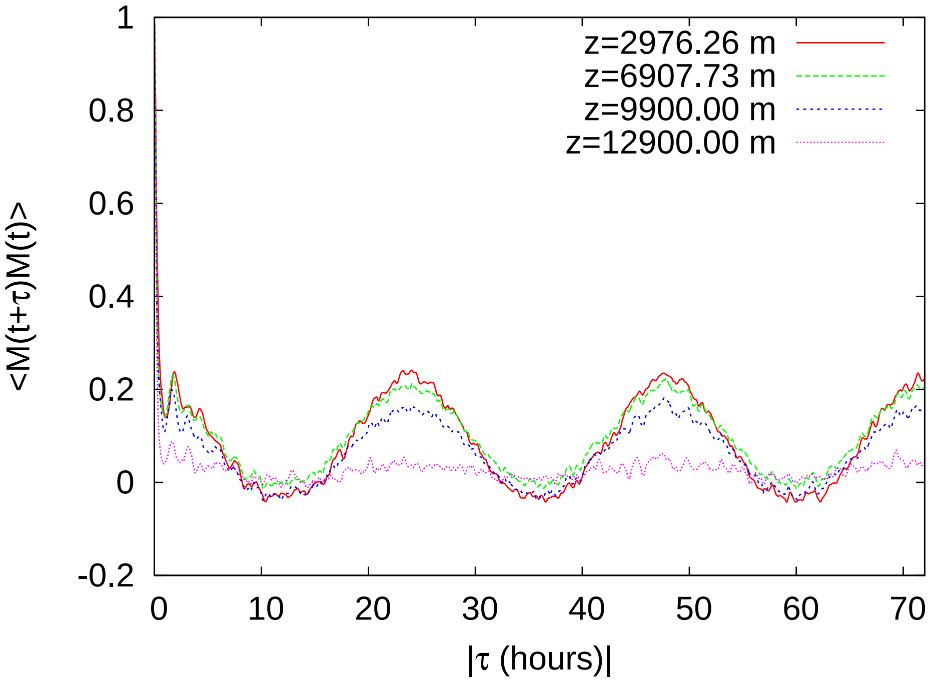
Stochastic variability of up-draught M and N

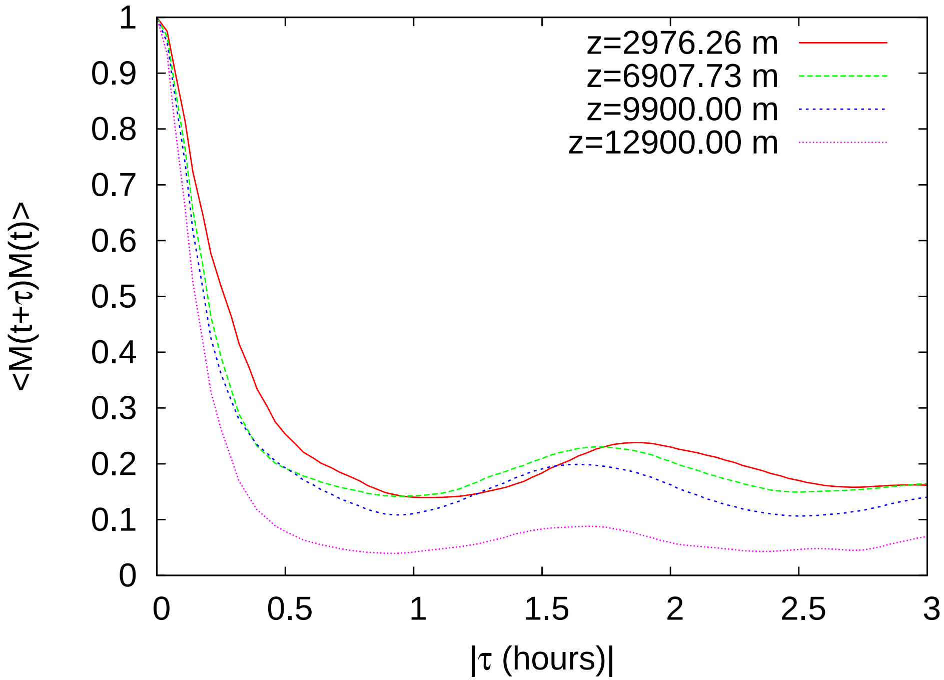




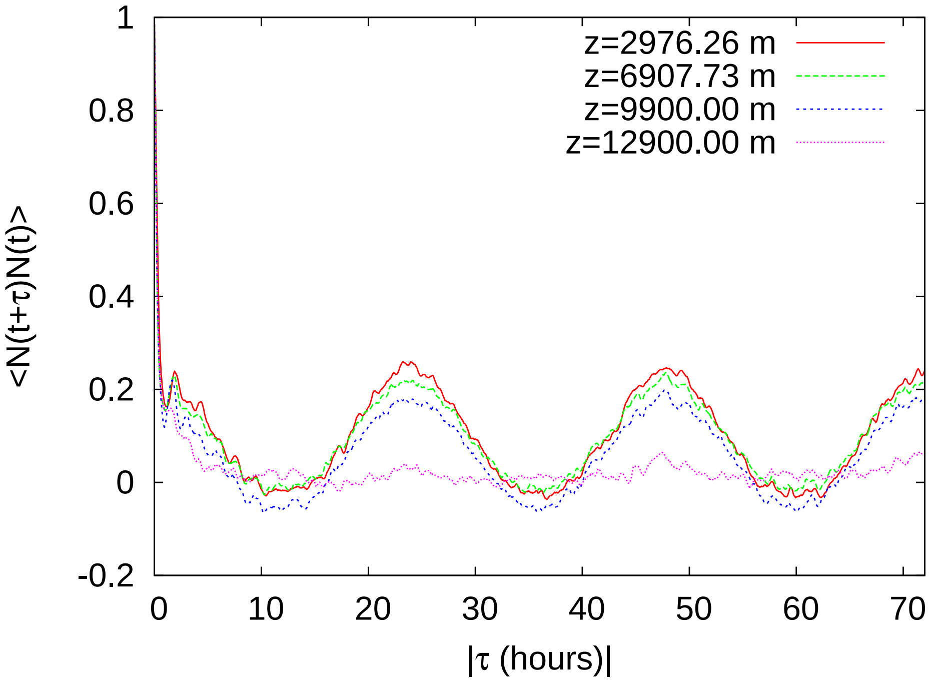


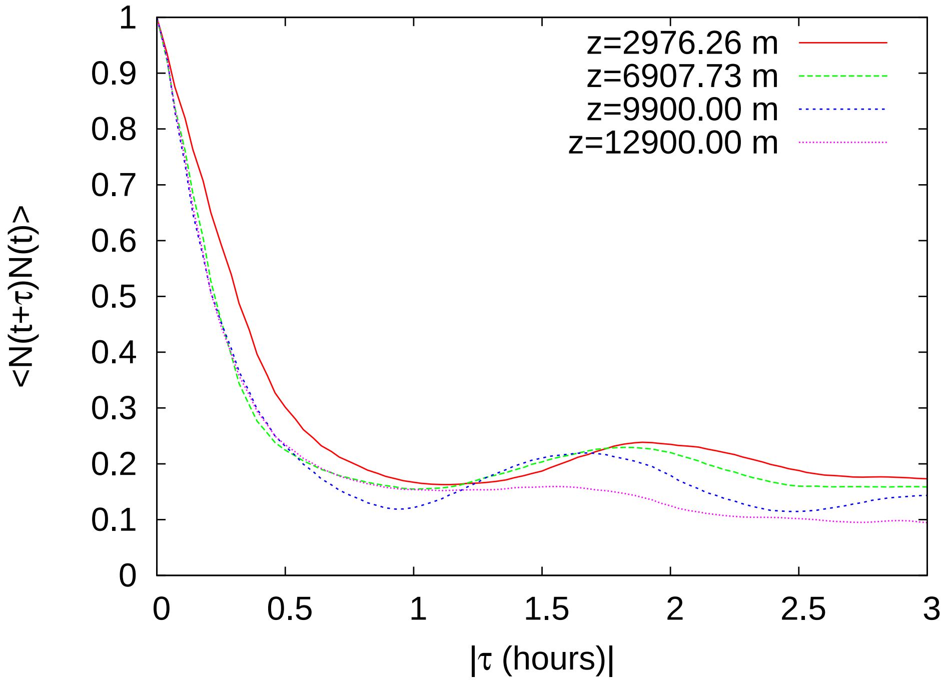
Auto-correlation of up-draught M



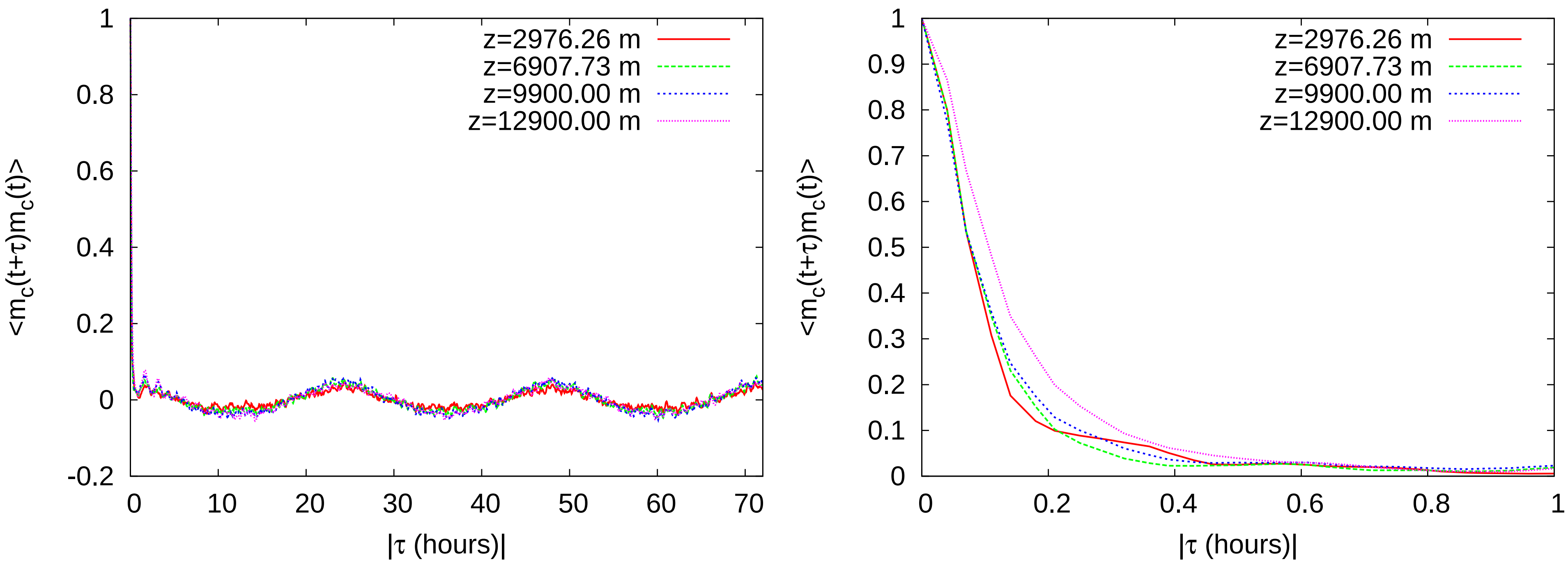


Auto-correlation of up-draught N

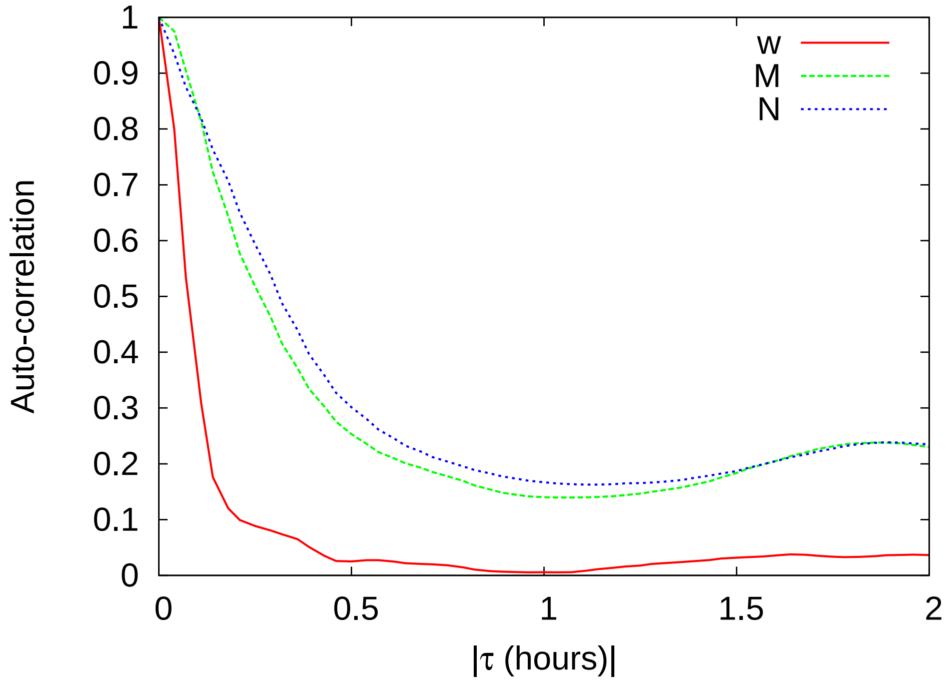


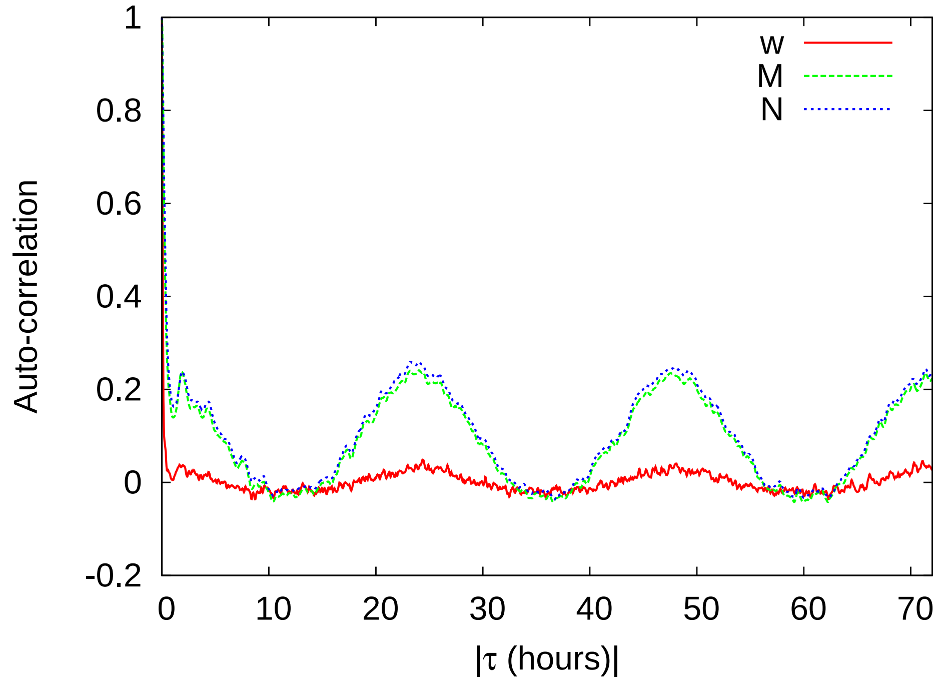


Auto-correlation of up-draught m_c

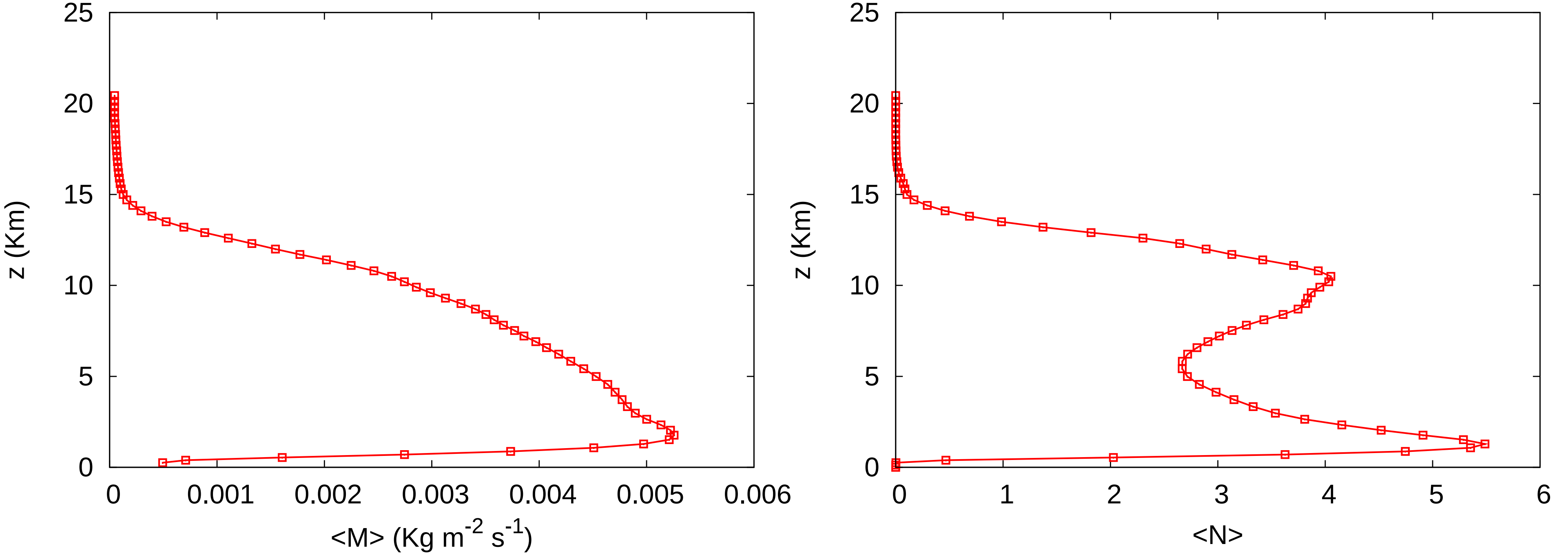


Comparison of short de-correlation time

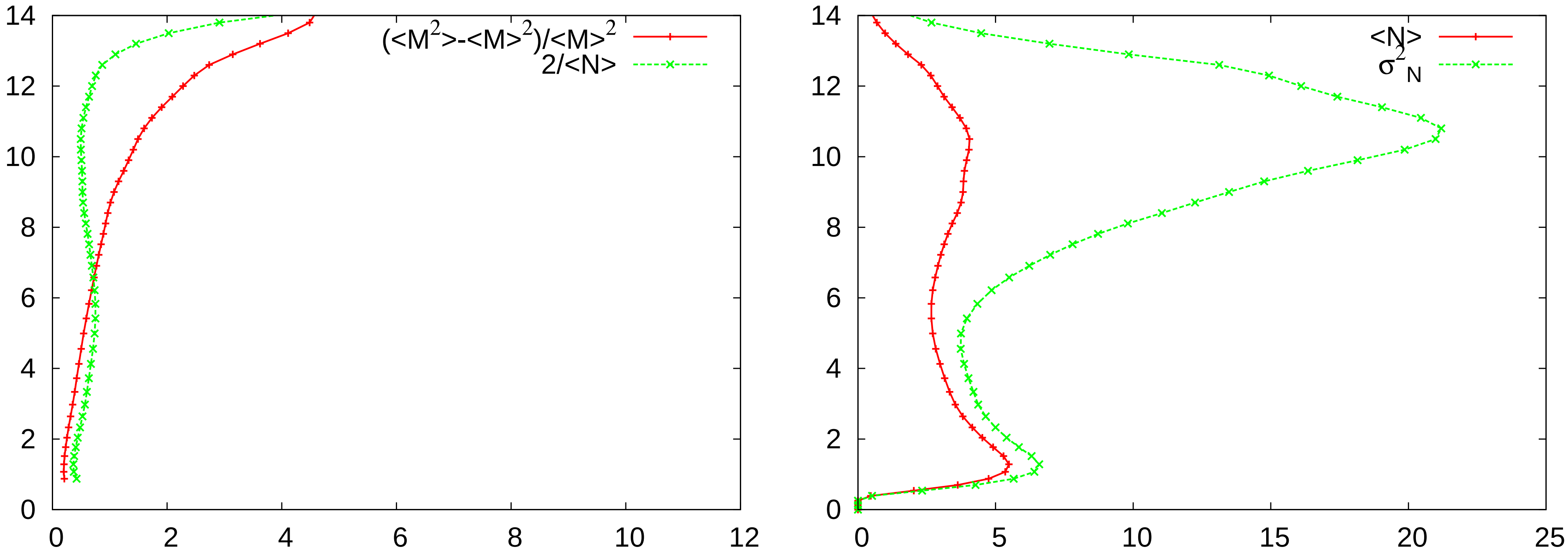




Mean characteristics



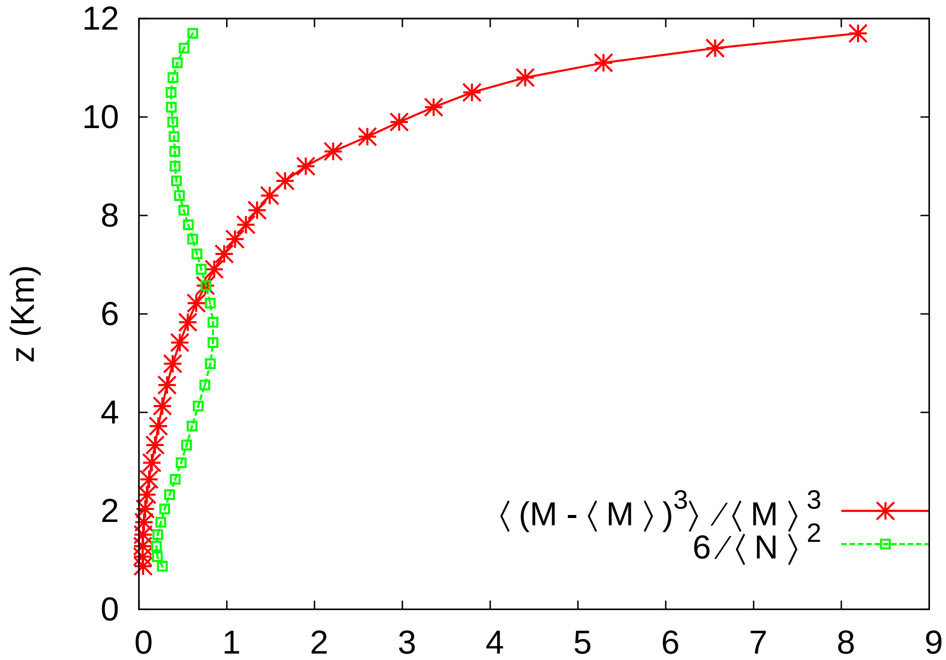
Variance scaling



The scaling of the variance of total mass flux and number of active grids in different heights with the Craig and Cohen prediction:

$$\frac{\langle(\delta M)^2\rangle}{\langle M\rangle^2} = \frac{2}{\langle N\rangle}$$

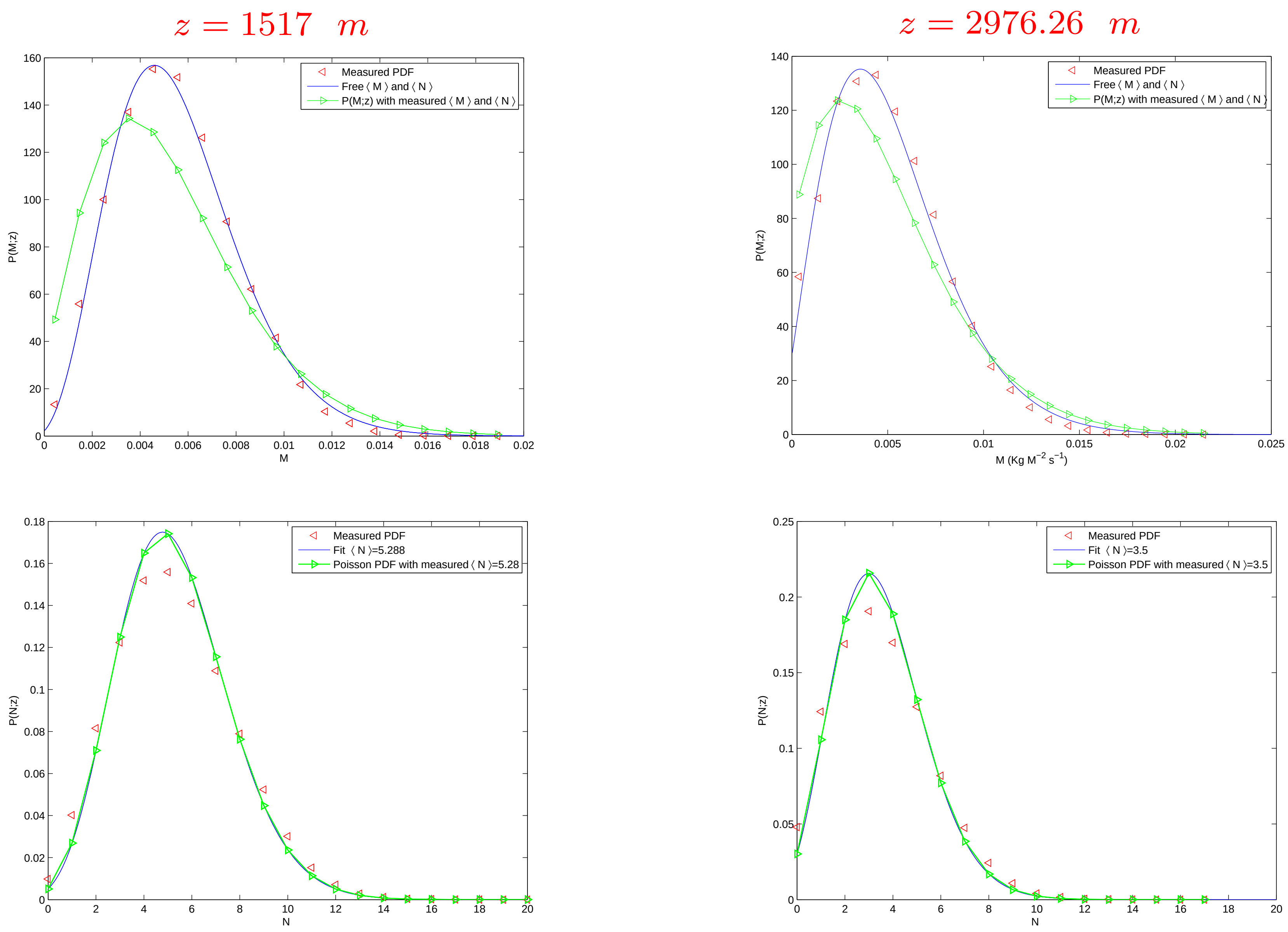
Skewness scaling



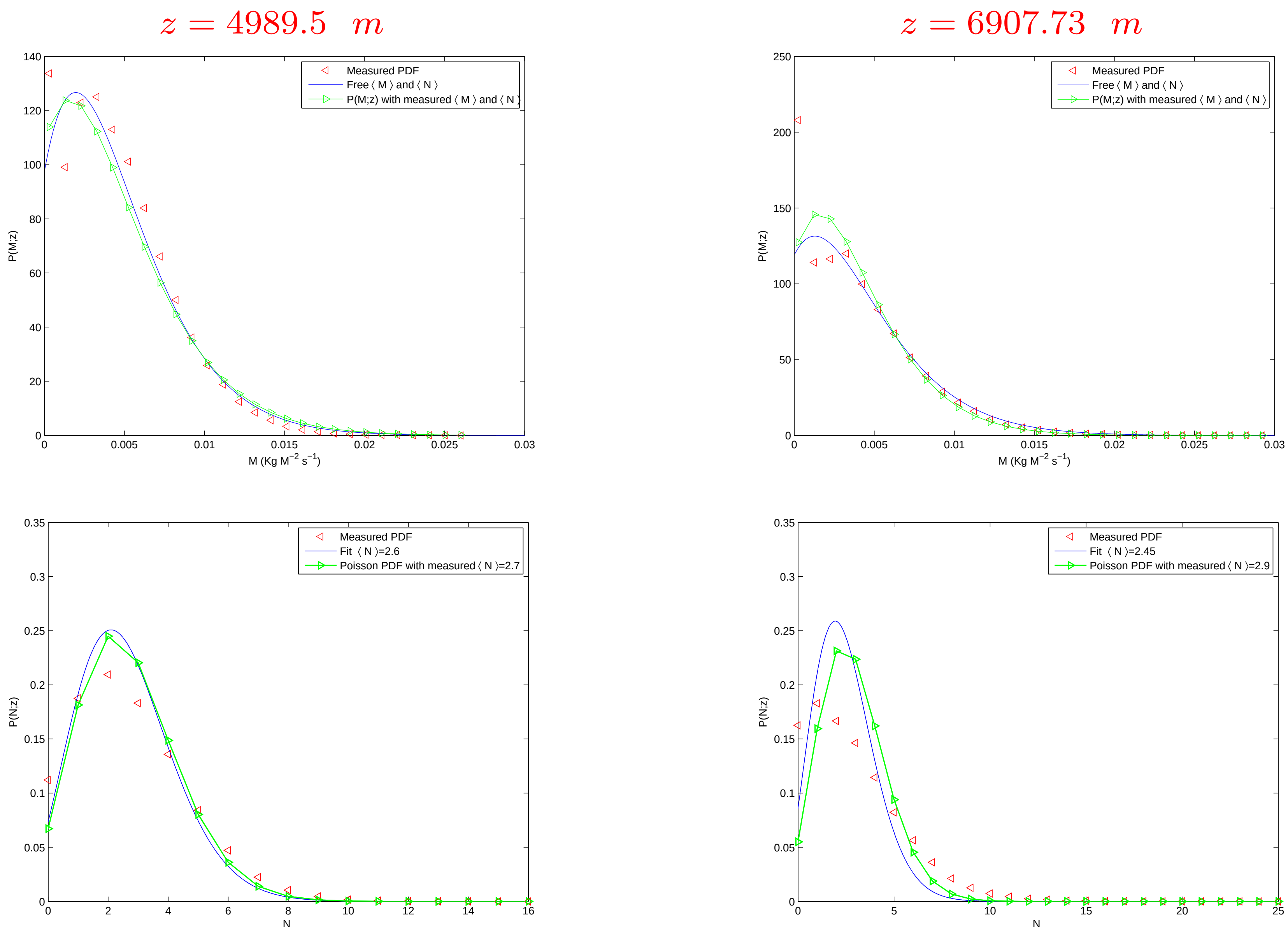
The prediction:

$$\frac{\langle(\delta M)^3\rangle}{\langle M\rangle^3} = \frac{6}{\langle N\rangle^2}$$

Altitude variability of total Mass flux PDF

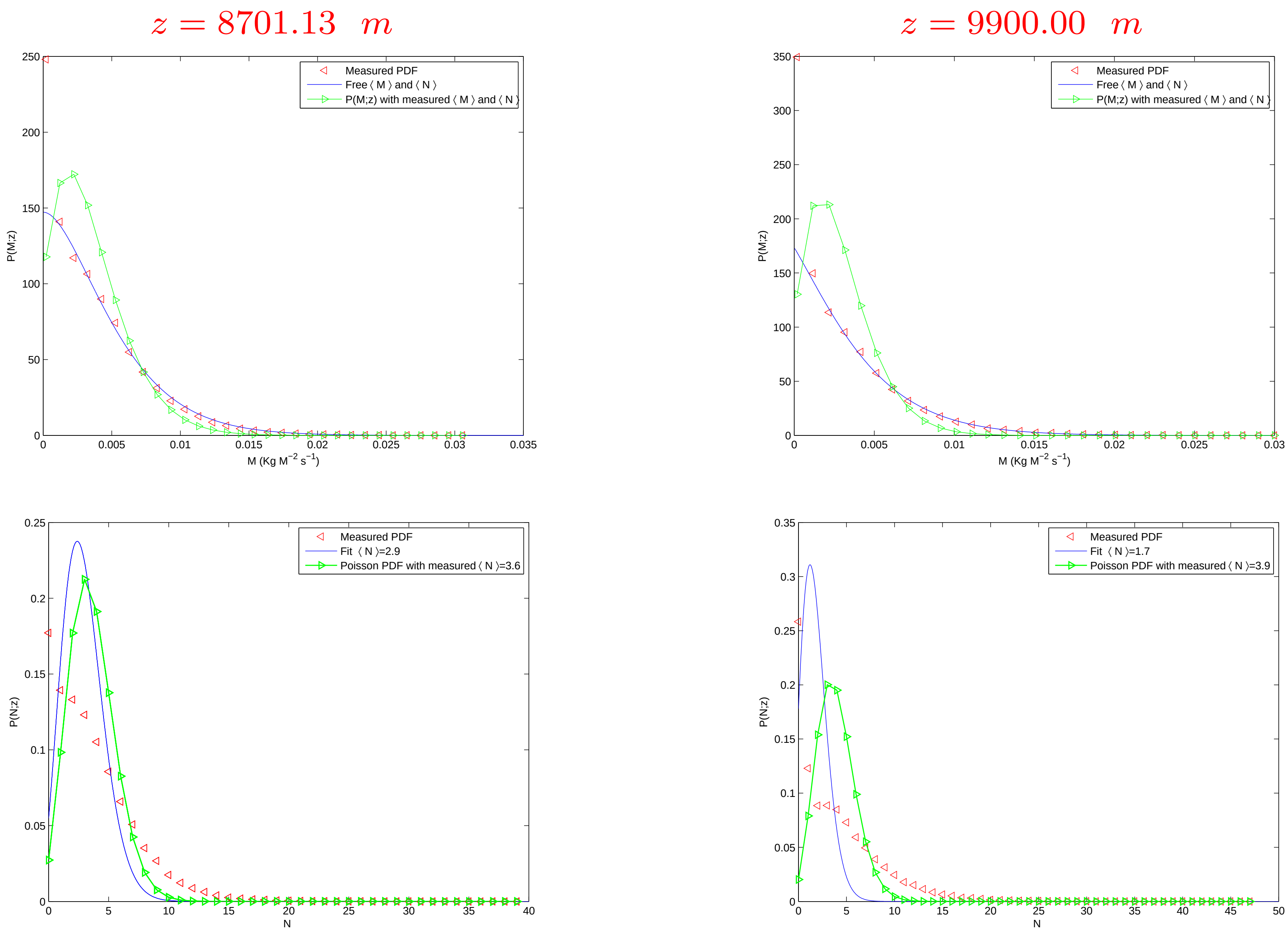


P(M,z) and P(N,z)



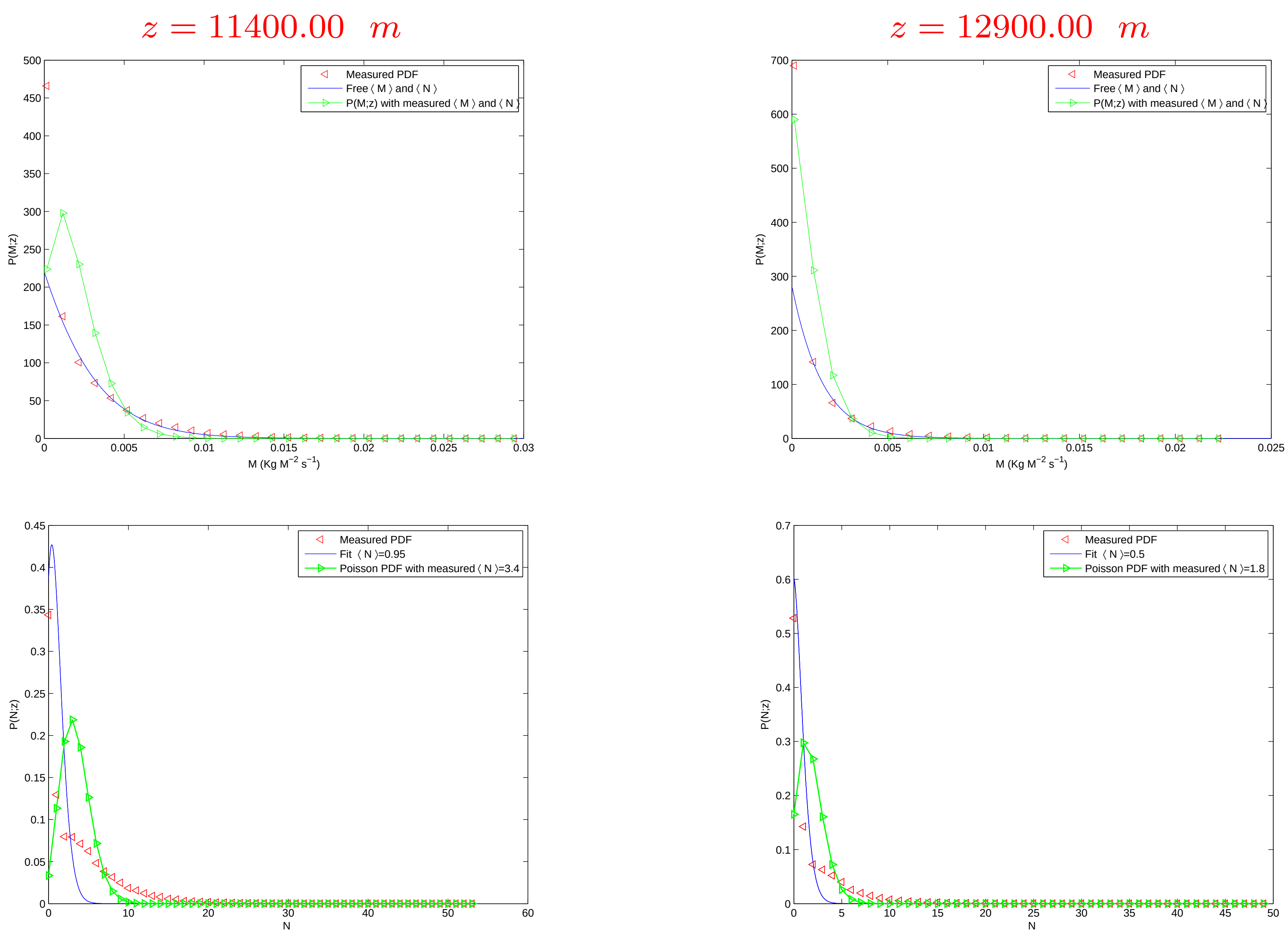
$z = 4989.5 \quad m$
$z = 6907.73 \quad m$
Measured PDF
Free ⟨ M ⟩ and ⟨ N ⟩
P(M;z) with measured ⟨ M ⟩ and ⟨ N ⟩
P(M;z)
M (Kg M^{-2} s^{-1})
Fit ⟨ N ⟩=2.6
Poisson PDF with measured ⟨ N ⟩=2.7
Fit ⟨ N ⟩=2.45
Poisson PDF with measured ⟨ N ⟩=2.9
P(N;z)
N

P(M,z) and P(N,z)



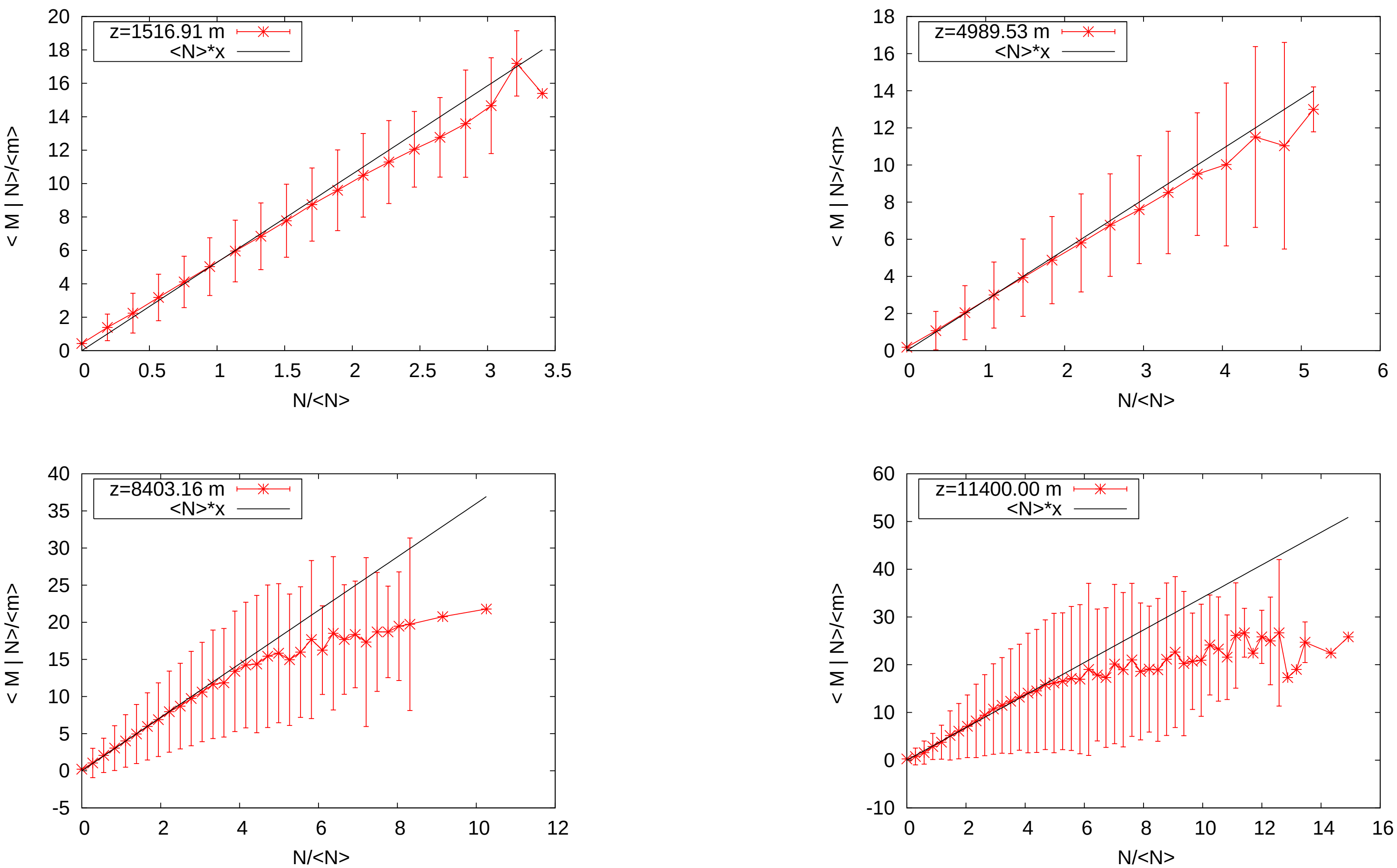
z = 8701.13 m
z = 9900.00 m
Measured PDF
Free ⟨ M ⟩ and ⟨ N ⟩
P(M;z) with measured ⟨ M ⟩ and ⟨ N ⟩
P(M;z)
M (Kg M−2 s−1)
Measured PDF
Fit ⟨ N ⟩=2.9
Poisson PDF with measured ⟨ N ⟩=3.6
Fit ⟨ N ⟩=1.7
Poisson PDF with measured ⟨ N ⟩=3.9
P(N;z)
N

P(M,z) and P(N,z)



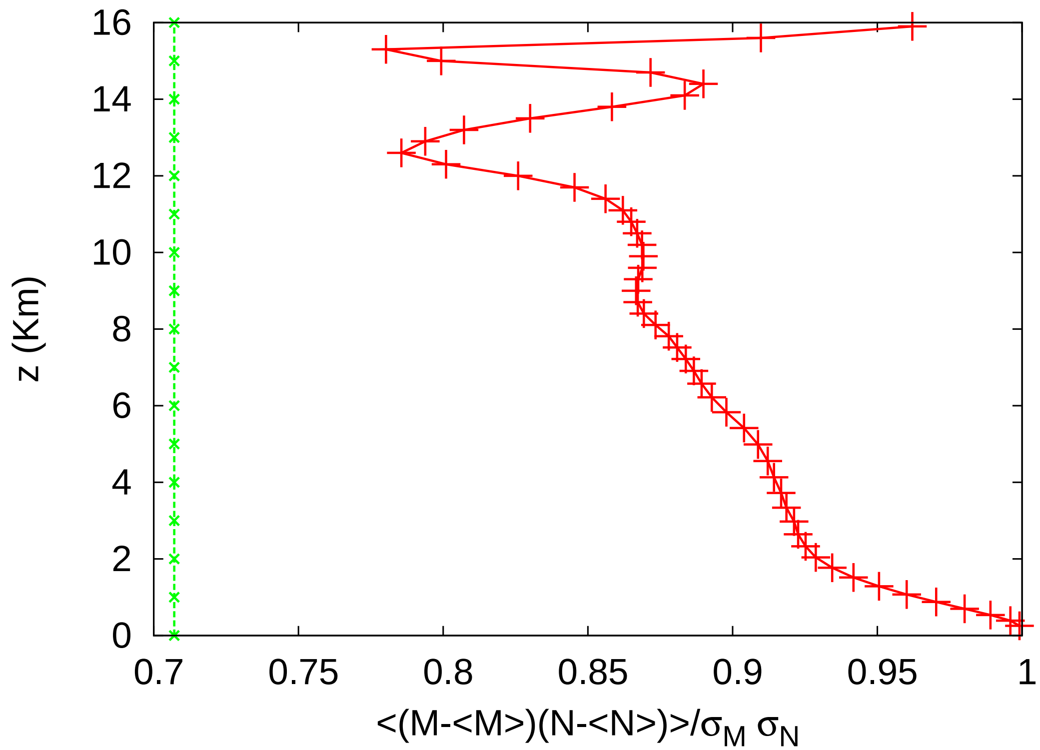
$z = 11400.00 \quad m$
$z = 12900.00 \quad m$
Measured PDF
Free ⟨ M ⟩ and ⟨ N ⟩
P(M;z) with measured ⟨ M ⟩ and ⟨ N ⟩
P(M;z)
M (Kg M^{-2} s^{-1})
Fit ⟨ N ⟩=0.95
Poisson PDF with measured ⟨ N ⟩=3.4
Fit ⟨ N ⟩=0.5
Poisson PDF with measured ⟨ N ⟩=1.8
P(N;z)
N

Conditional average $\langle M|N \rangle$



$$\langle M|N \rangle = \langle m \rangle N$$

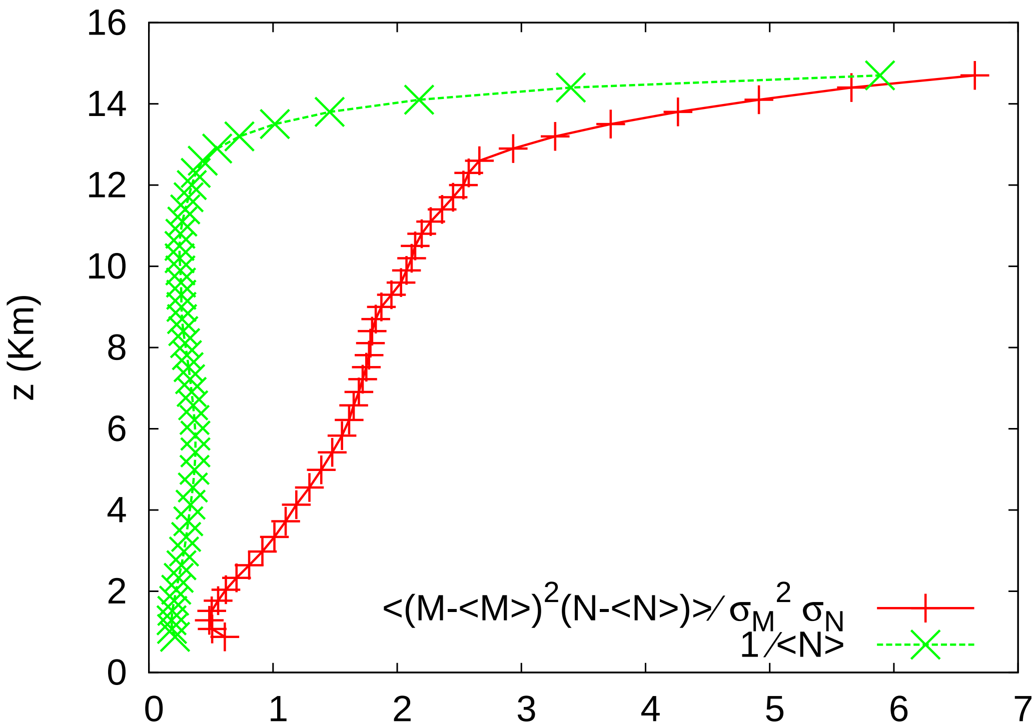
Cross-correlation of M and N



The prediction:

$$\frac{\langle (M - \langle M \rangle)(N - \langle N \rangle) \rangle}{\sigma_M \sigma_N} = \frac{1}{\sqrt{2}}$$

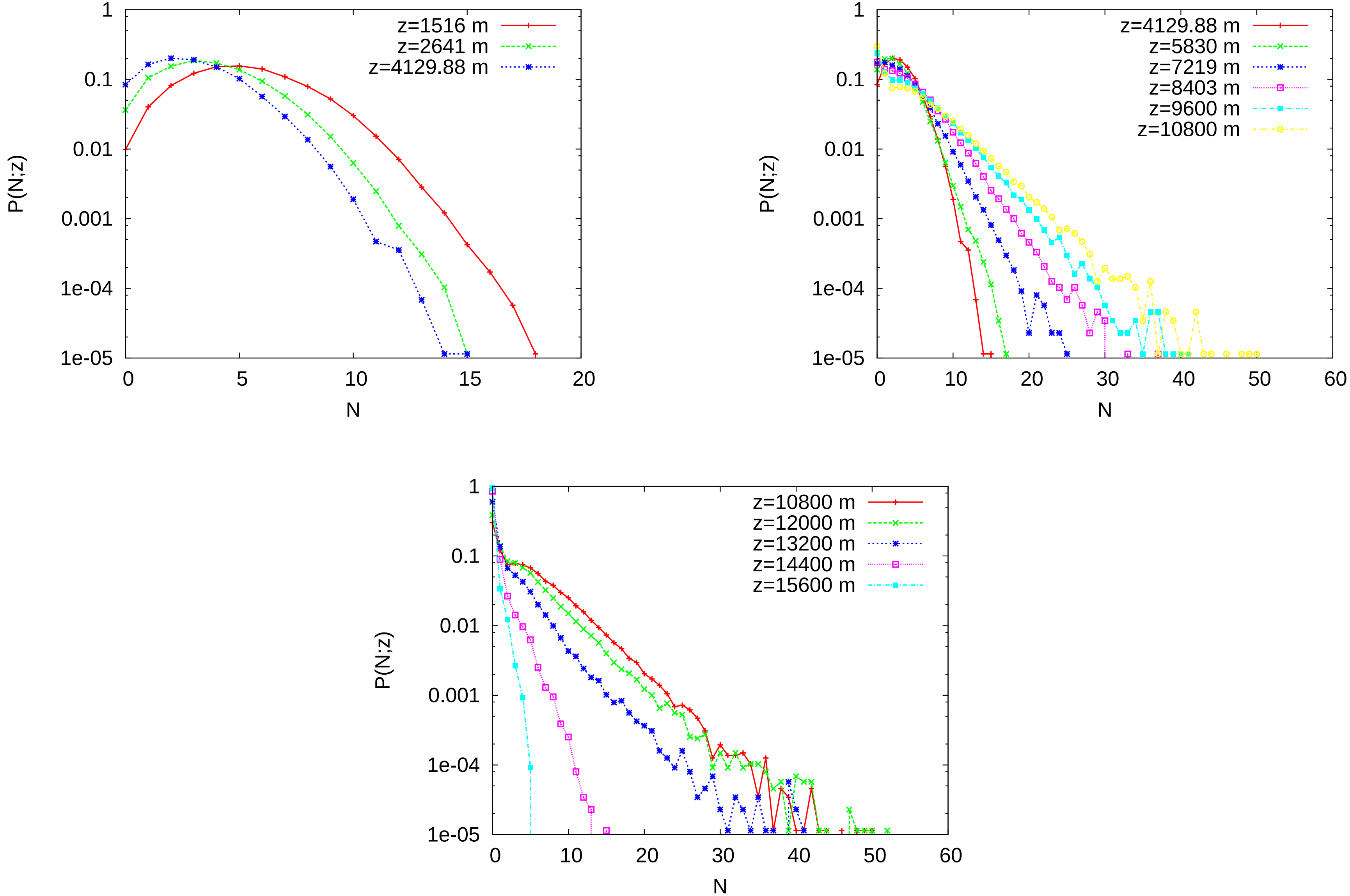
Low order mixed moment



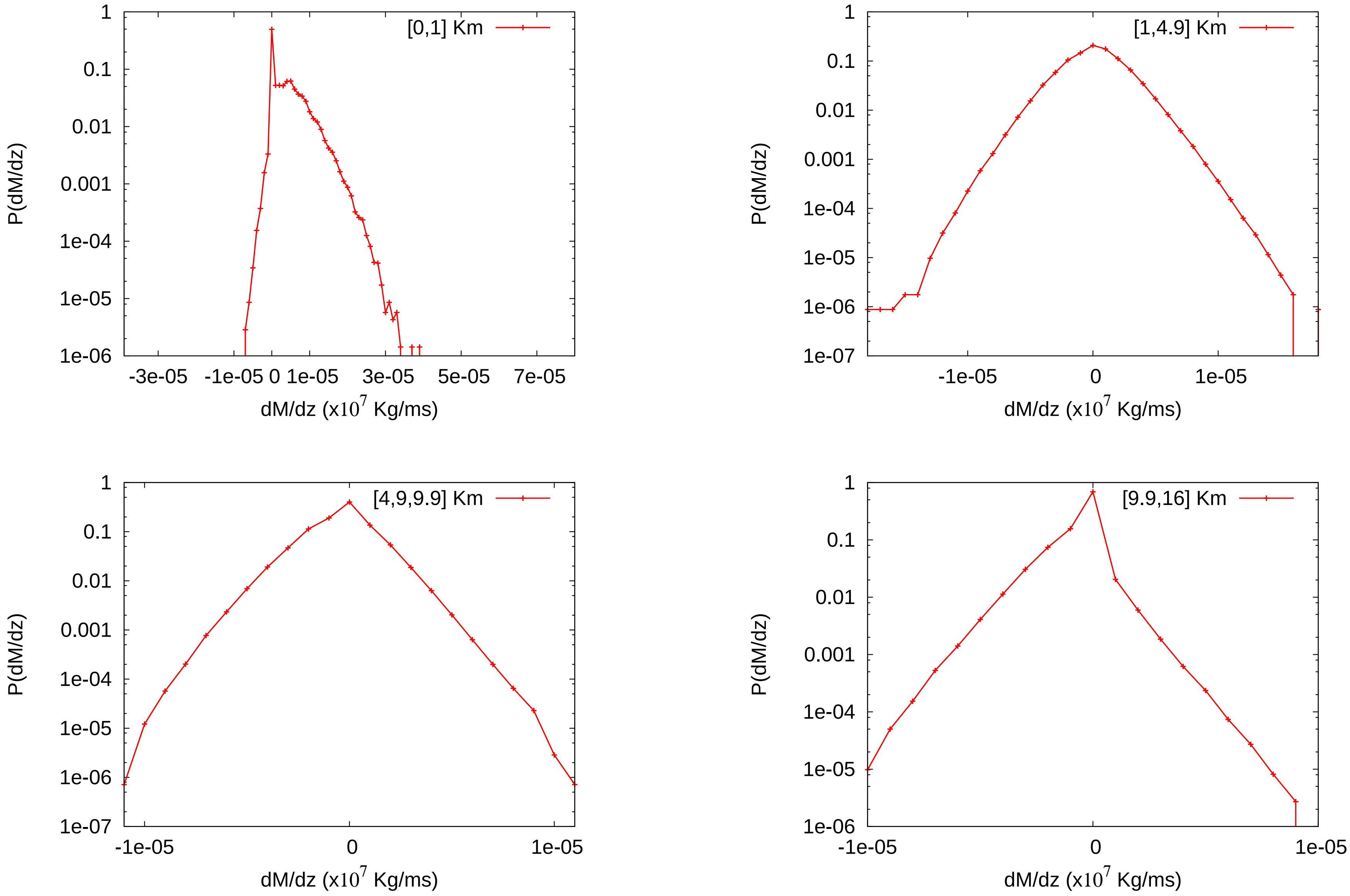
The prediction:

$$\frac{\langle (M - \langle M \rangle)^2 (N - \langle N \rangle) \rangle}{\sigma_M^2 \sigma_N} = \frac{1}{\sqrt{N}}$$

Fat tails of marginal PDF of active grids

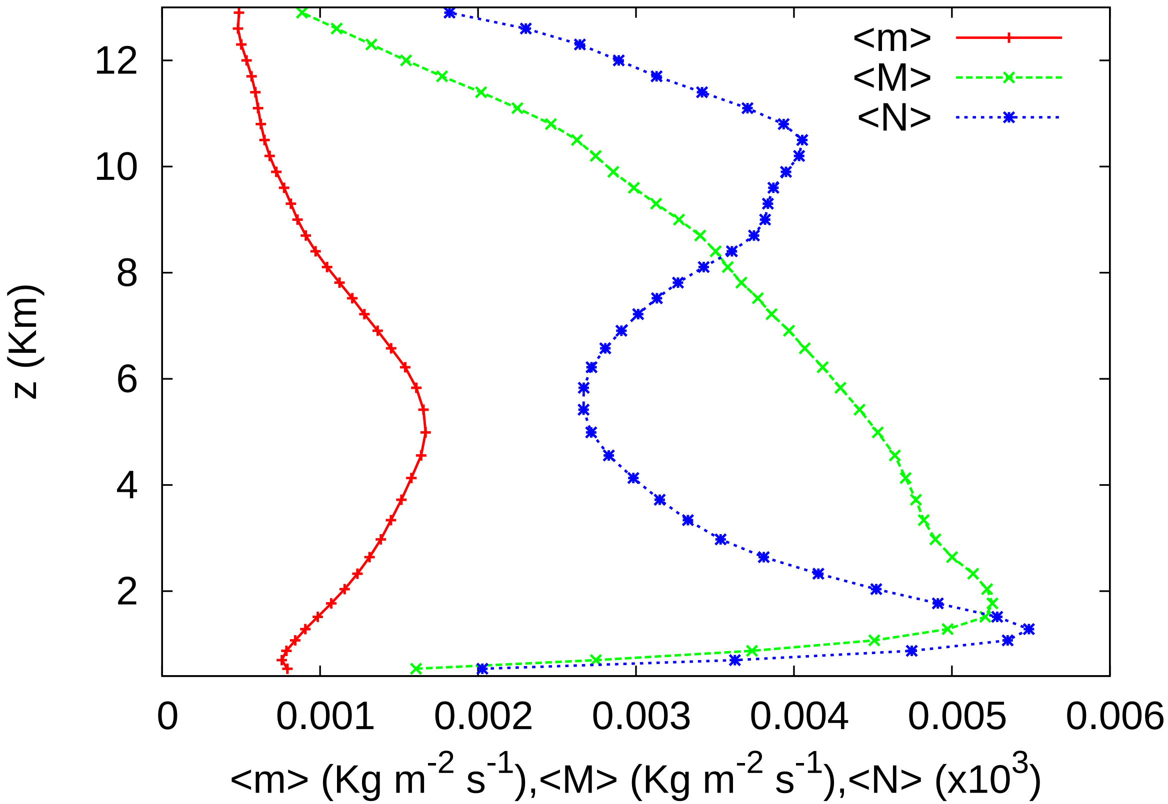


PDF of dM/dz



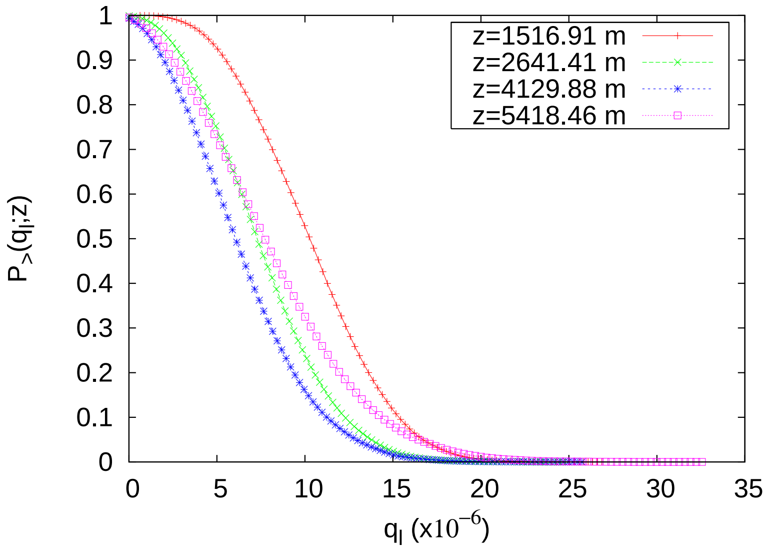
[0,1] Km
P(dM/dz)
1
0.1
0.01
0.001
1e-04
1e-05
1e-06
-3e-05
-1e-05 0 1e-05
3e-05
5e-05
7e-05
dM/dz (x10^7 Kg/ms)
[1,4.9] Km
P(dM/dz)
1
0.1
0.01
0.001
1e-04
1e-05
1e-06
1e-07
-1e-05
0
1e-05
dM/dz (x10^7 Kg/ms)
[4,9,9.9] Km
P(dM/dz)
1
0.1
0.01
0.001
1e-04
1e-05
1e-06
1e-07
-1e-05
0
1e-05
dM/dz (x10^7 Kg/ms)
[9.9,16] Km
P(dM/dz)
1
0.1
0.01
0.001
1e-04
1e-05
1e-06
-1e-05
0
1e-05
dM/dz (x10^7 Kg/ms)

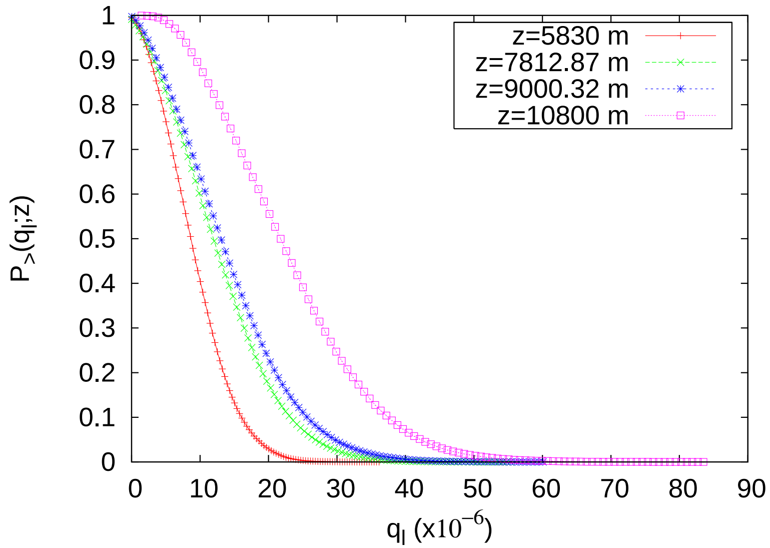
Regime change

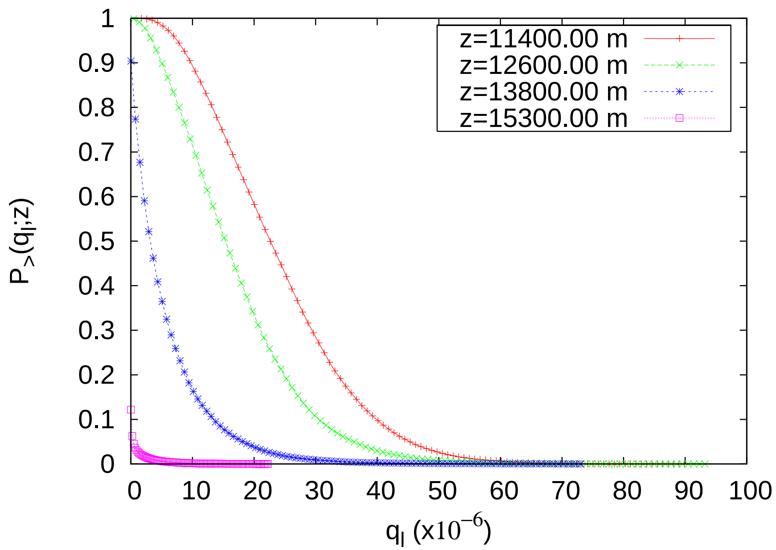


- $z \sim [1, 6]$ Km: The $\langle m \rangle$ is increasing while $\langle N \rangle$ is decreasing.
- $z \sim [6, 11]$ Km:The $\langle m \rangle$ is decreasing while $\langle N \rangle$ is increasing.
- $z \sim [11, 15]$ Km: The $\langle m \rangle$ is increasing while $\langle N \rangle$ is decreasing.

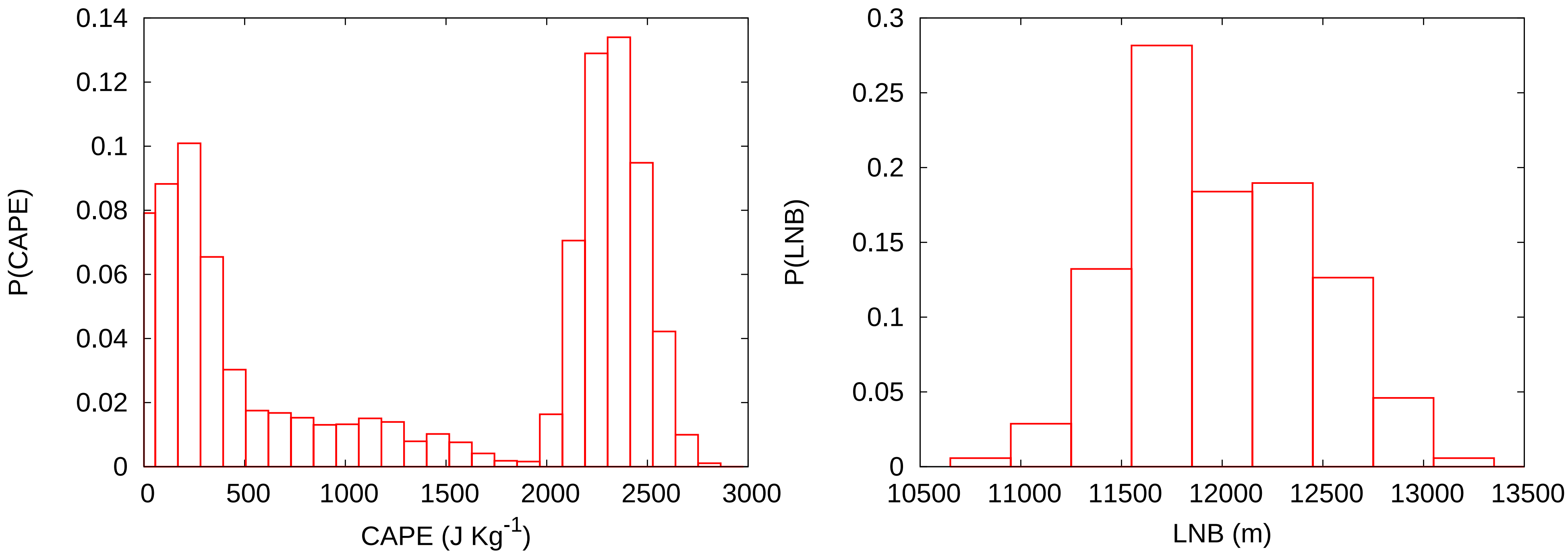
Cumulative Probability of q_l



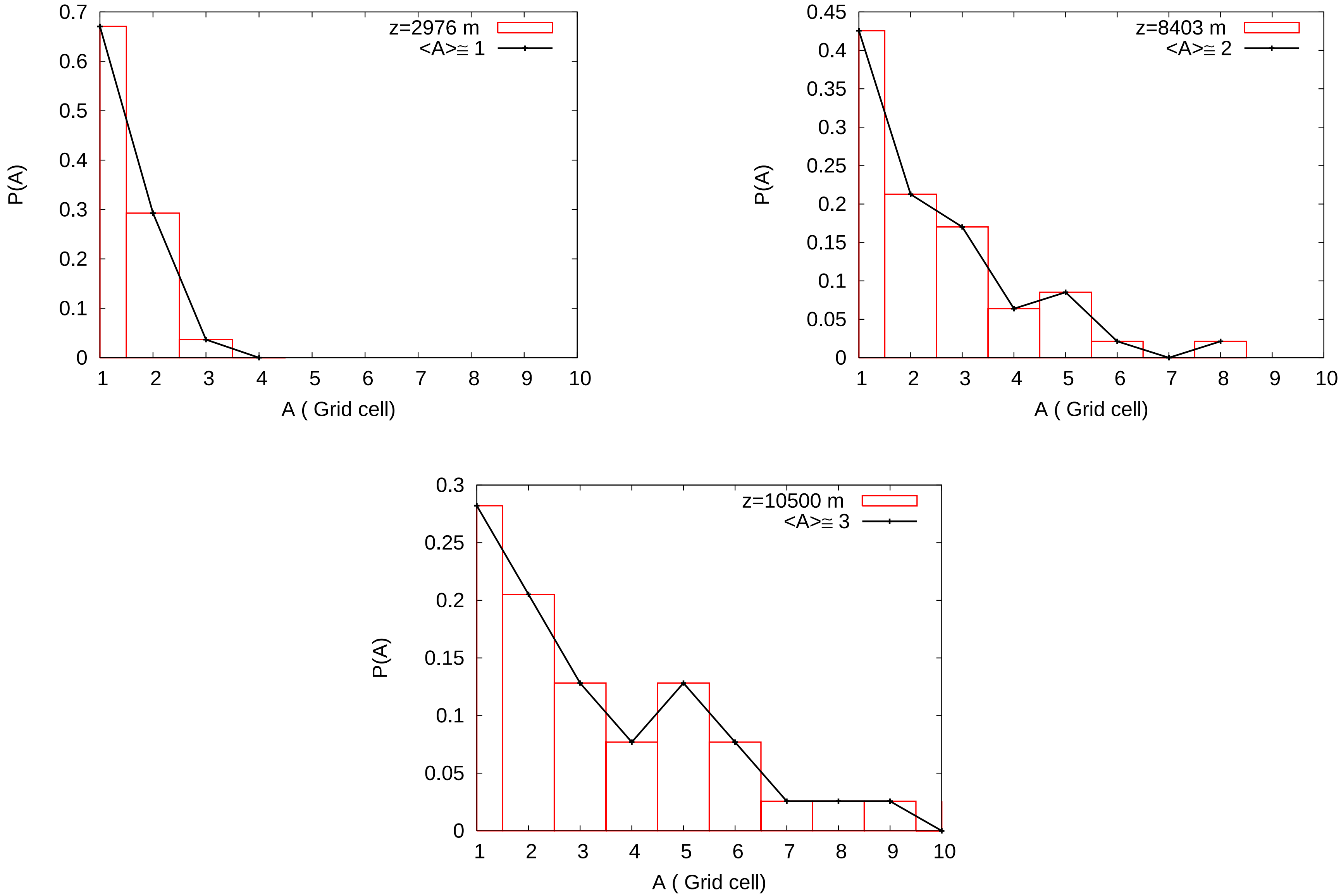




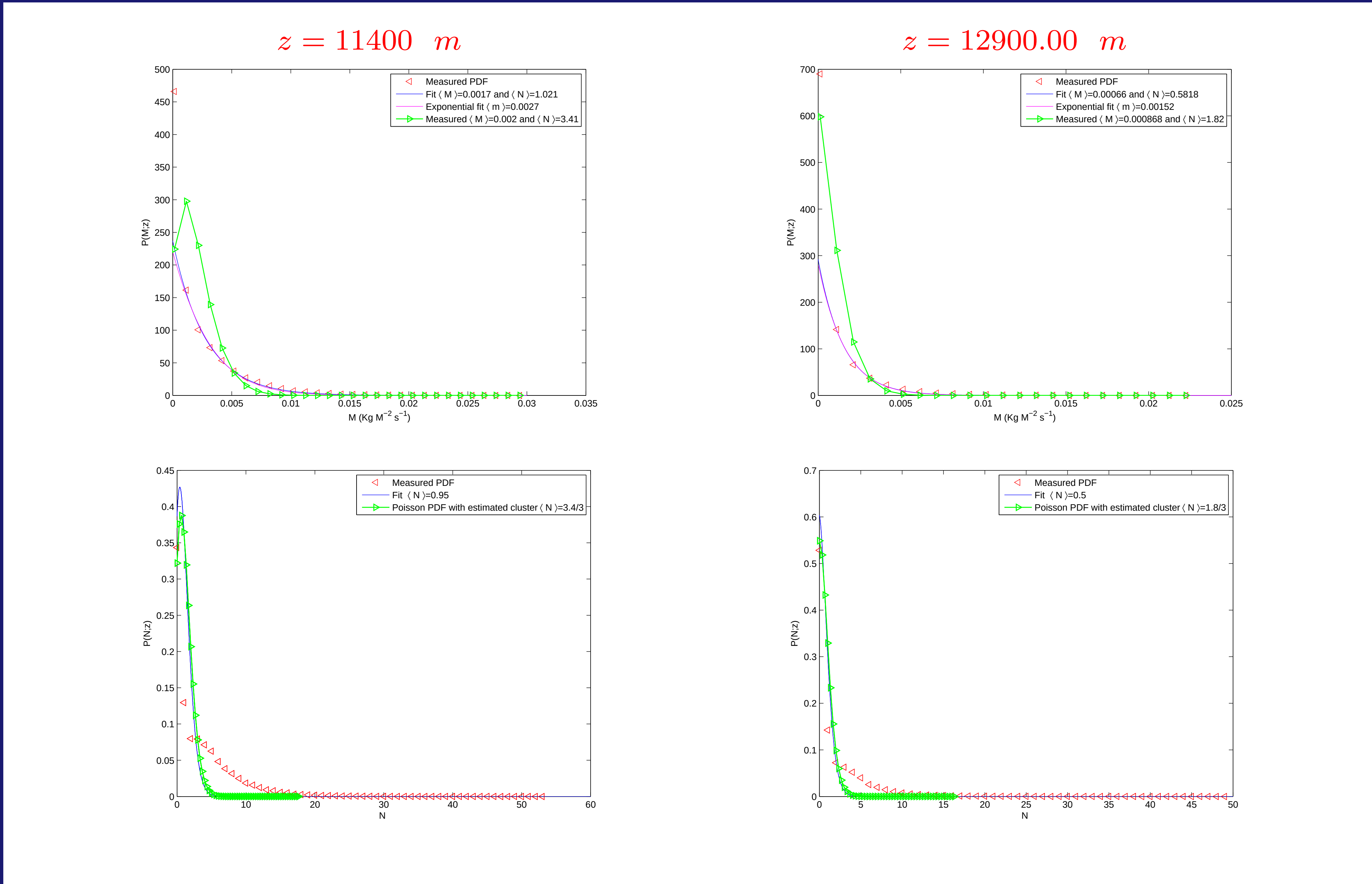
Distribution of CAPE and Level of Neutral Buoyancy



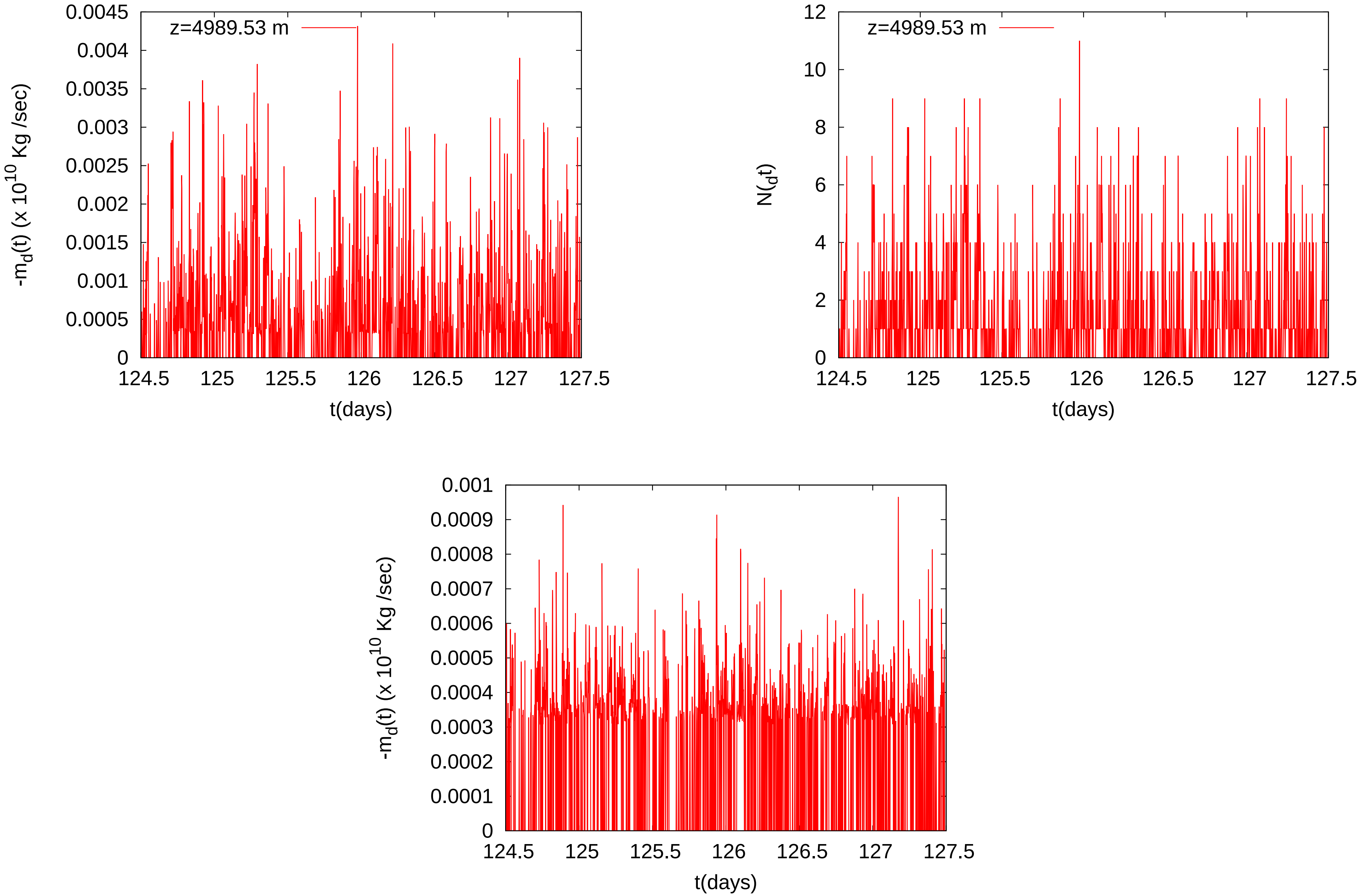
Size distribution of clouds



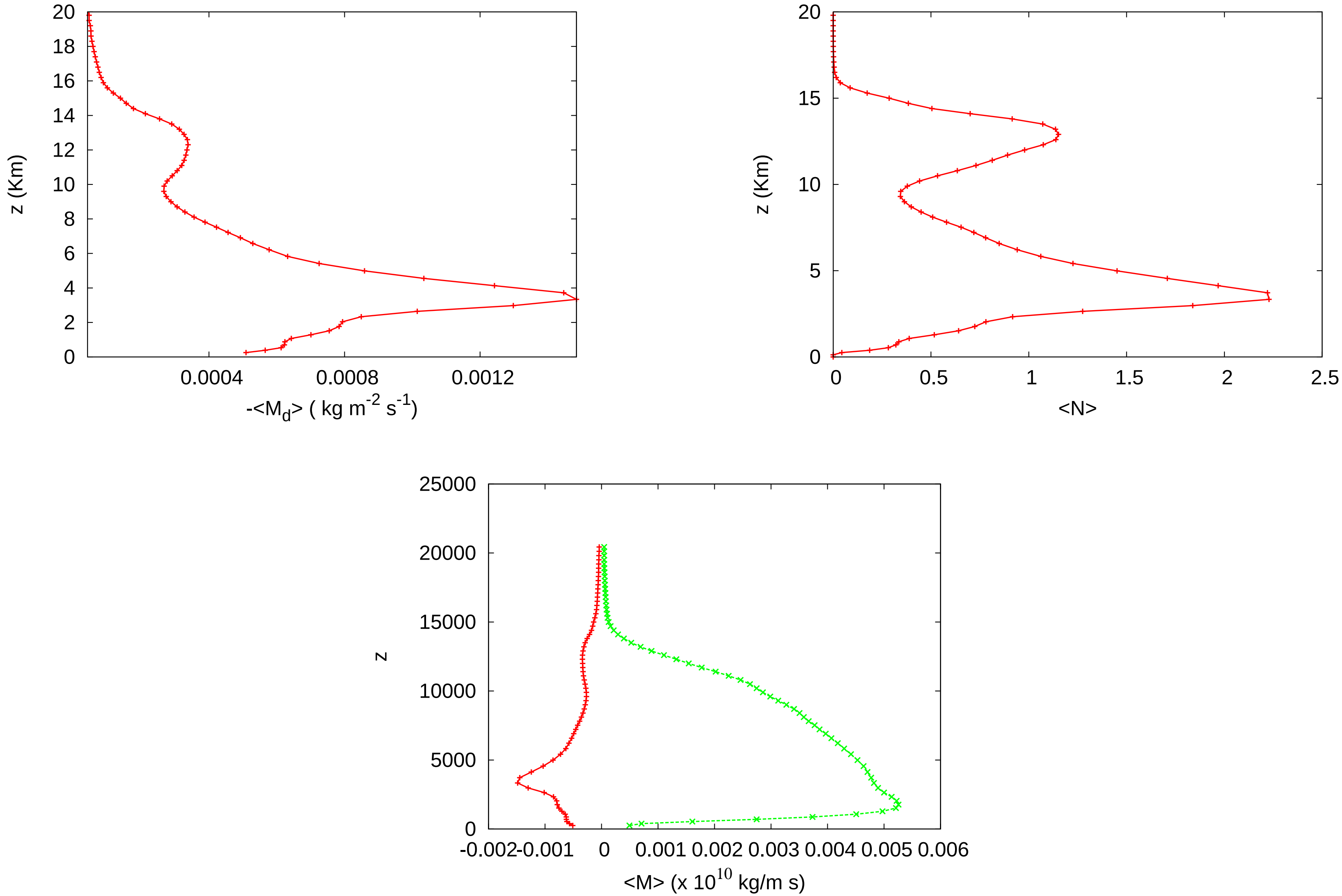
Exponential distribution P(M,z) above LNB



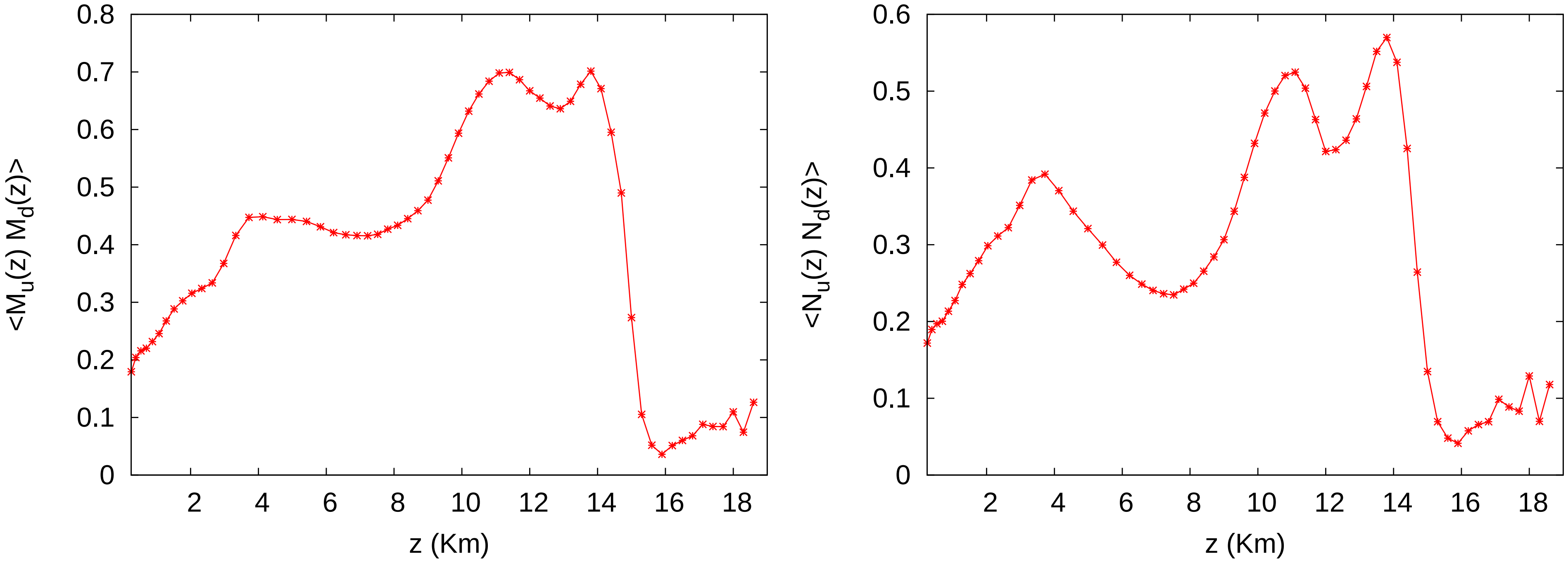
Stochastic variability of down-draught M and N



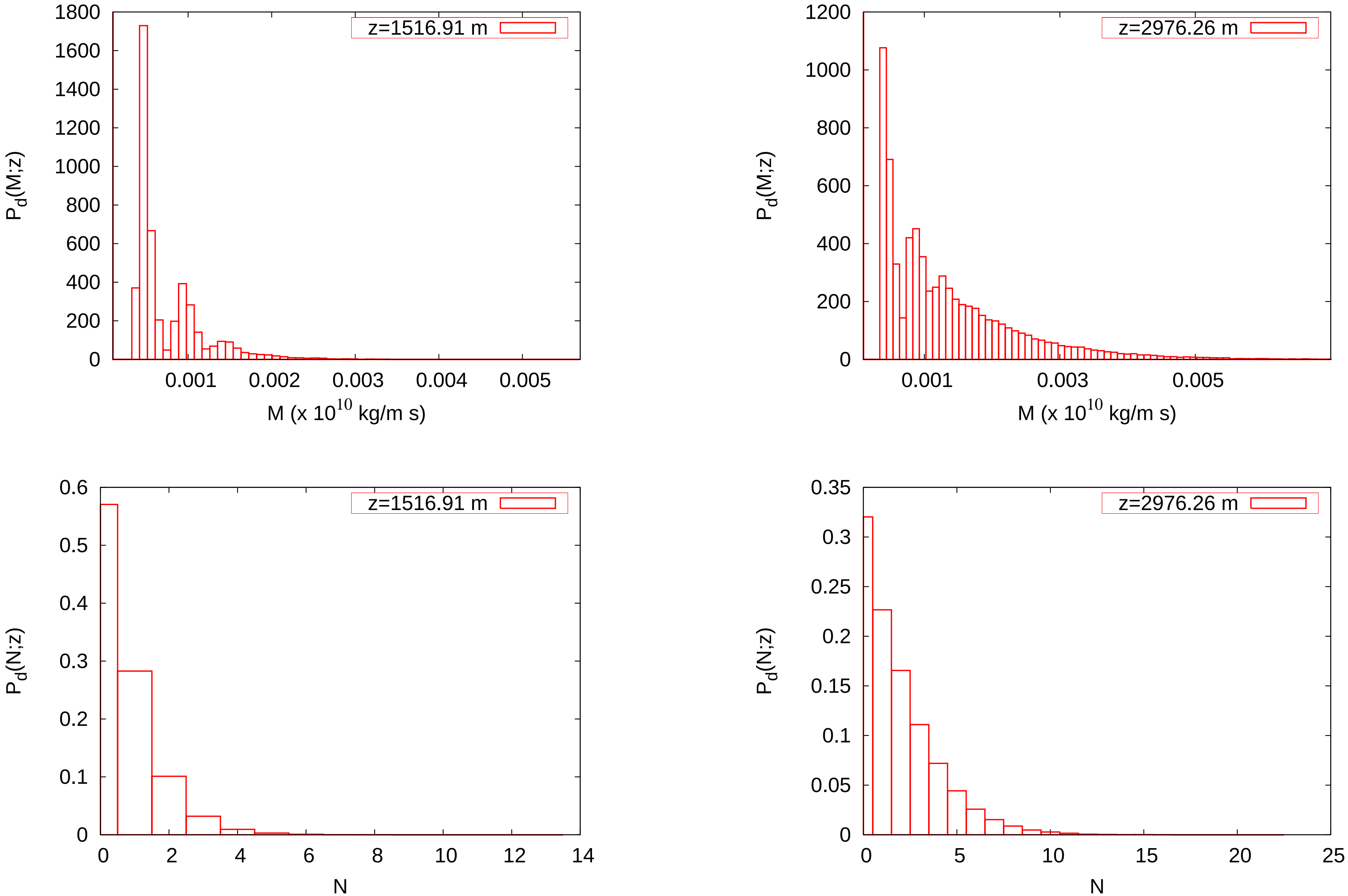
Down draft characteristics



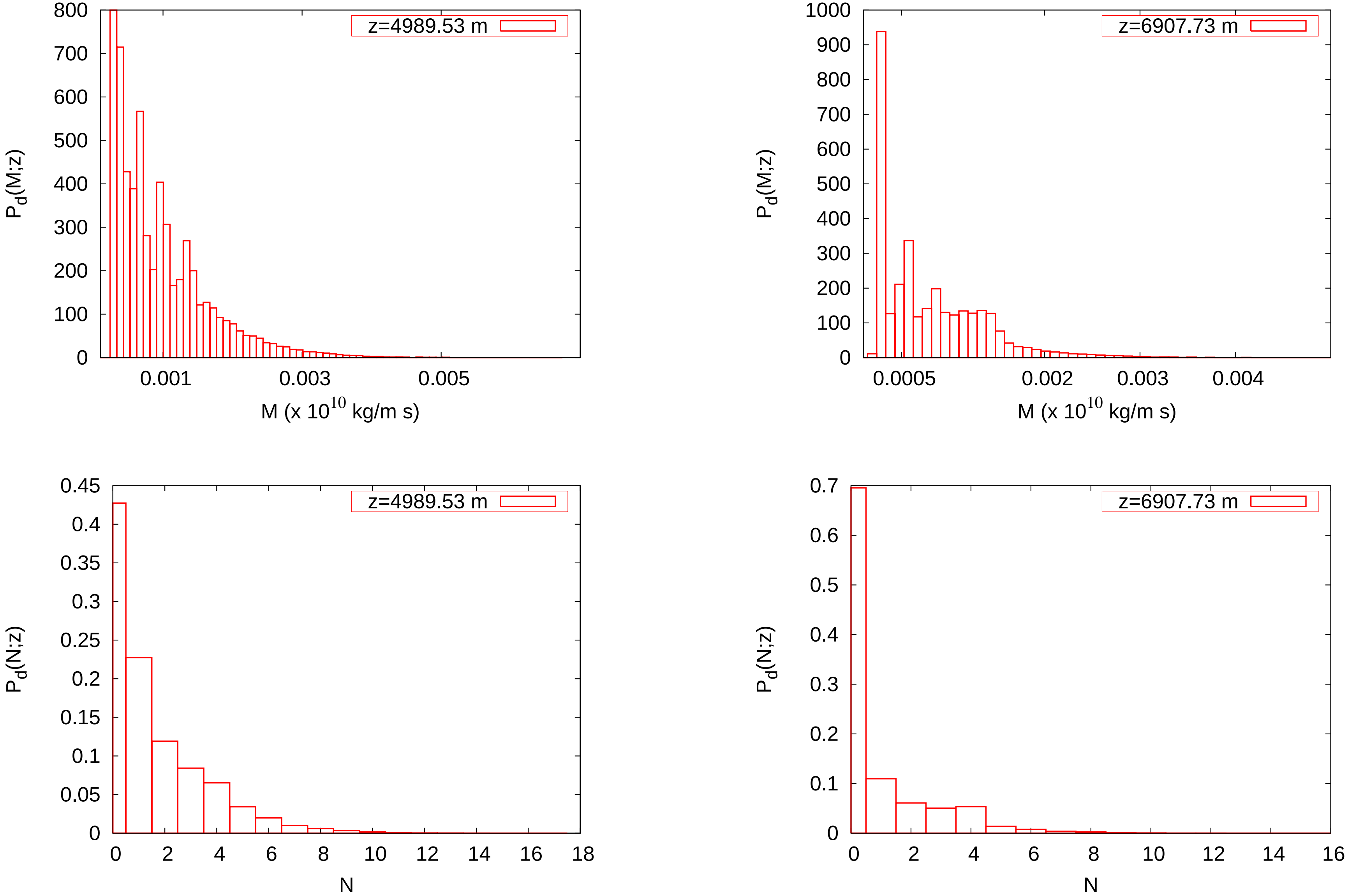
Cross-correlation of down-draught M_d and N_d



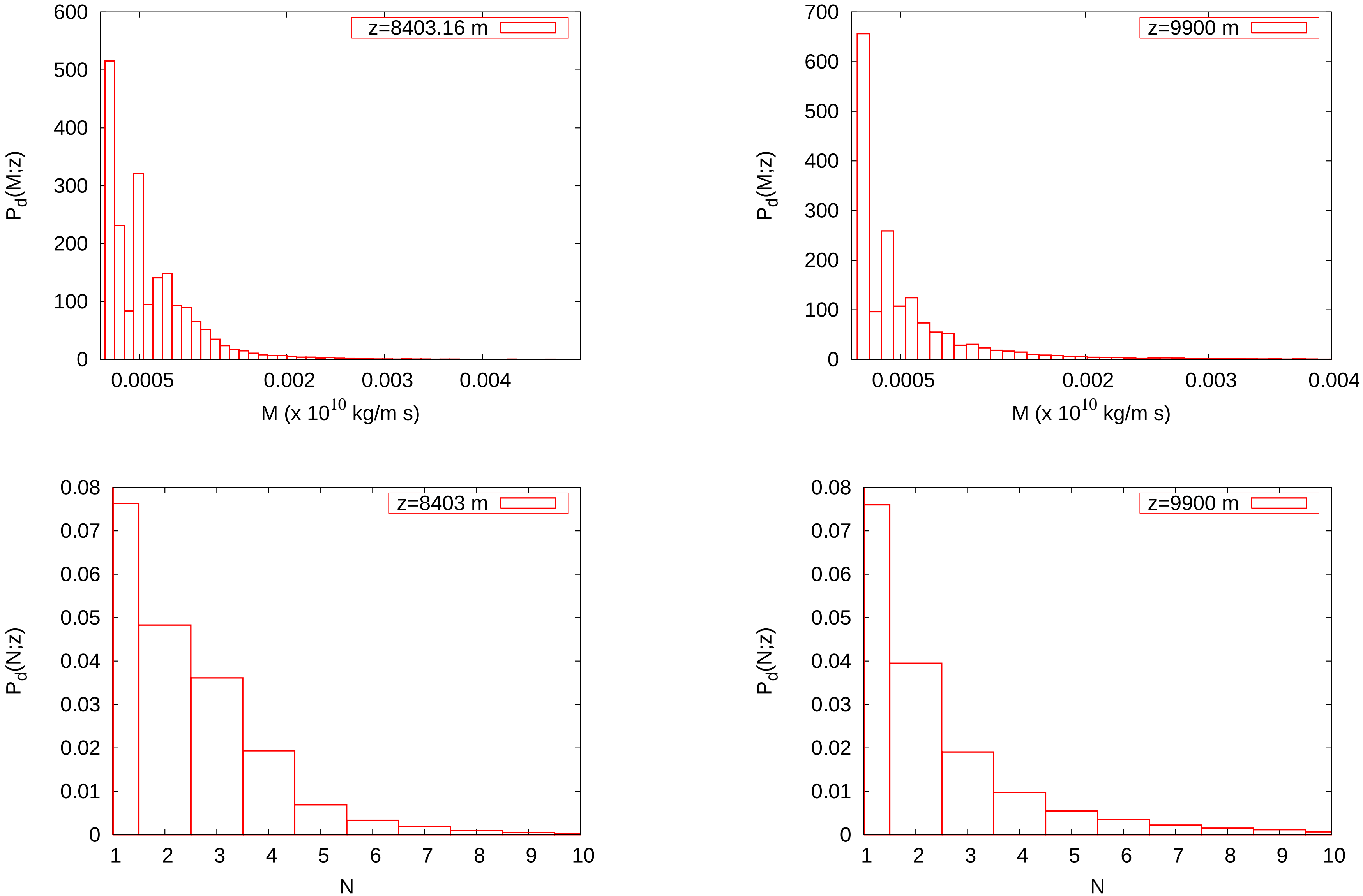
$P_d(M; z)$ and $P_d(N; z)$



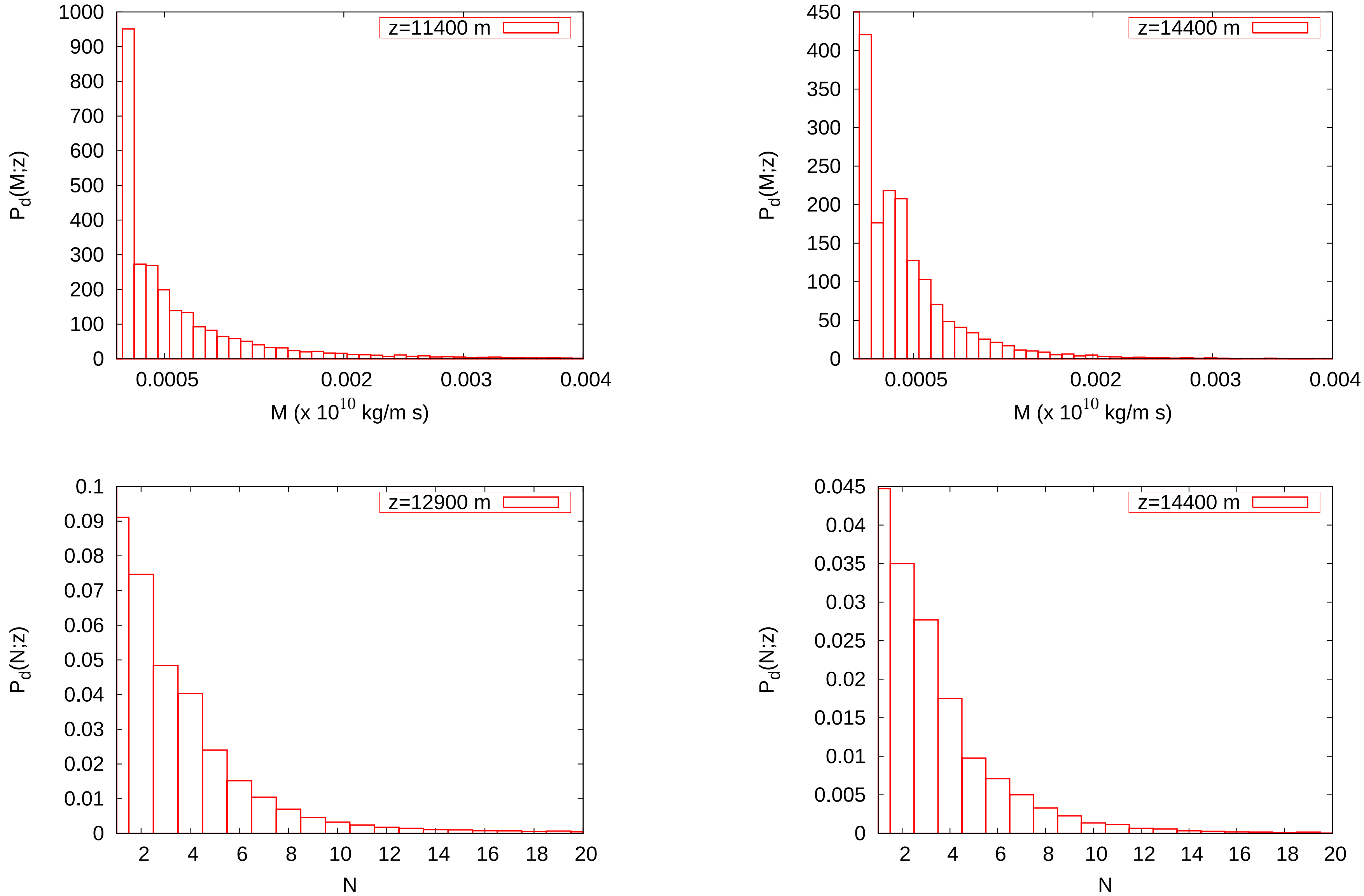
$P_d(M;z)$ and $P_d(N;z)$



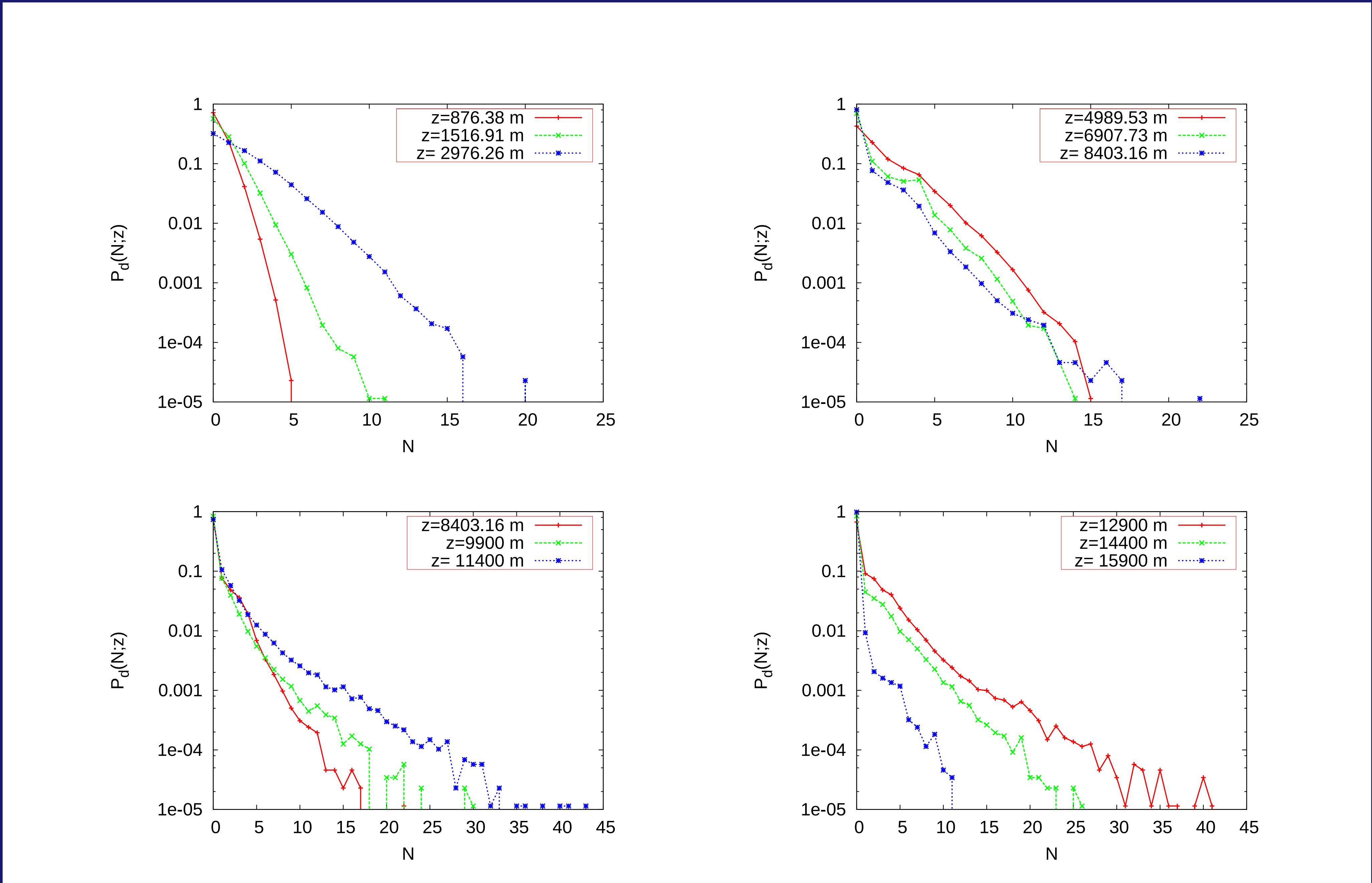
$P_d(M; z)$ and $P_d(N; z)$



$P_d(M;z)$ and $P_d(N;z)$



Tails of $P_d(N; z)$



Summary

- The statistics of the mass flux for the range of heights from cloud base up to $5Km$ matches with the results of the Cohen and Craig (2006).
- At very high altitudes above $10Km$ the statistics of the mass flux is mainly controlled by intermittently penetrating individual plumes which are exponentially distributed.
- Due to the increasing size of a typical cloud the statistics of the number of active grids deviates from the prediction of Poisson theory for the higher altitudes from $6Km$ up to $12Km$.
- Accounting for the typical size of the plumes above $10Km$ shows the consistency of the Poisson statistics with the data.
- Low order conditional and joint statistics of M and N are less compatible with the predictions of the Cohen and Craig (2006).