DISCOVERY AND EXPERIMENTATION IN NUMBER THEORY

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Abstract. In accordance with the PIMS sponsorship, this is a final scientific report on the workshop *Discovery and Experimentation in Number Theory* associated to the thematic program on the Foundations of Computational Mathematics at Fields.

This past September, as part of the thematic program on the Foundations of Computational Mathematics, our workshop on *Discovery and Experimentation in Number Theory* was held simultaneously, using remote collaboration technology, at the Fields Institute in Toronto, Ontario and at The IRMACS Centre in Burnaby, British Columbia. The event was sponsored by IRMACS, Fields, and PIMS.

Number theory today is characterized by many far-reaching conjectures whose proofs are well out of reach of existing techniques. New techniques have to be developed. An essential place of explicit methods in number theory is to gain numerical insight for such conjectures. In the process of developing methods for even verifying these conjectures in special cases, valuable intuition is gained in the structure of the objects involved. One hopes these insights will help lead to proofs eventually.

The relationship between computational evidence and conjecture must surely be as old as the hills (and thus the hills are older than Archimedes), though in (prime) number theory there is a canonical example, and that is of Gauss' investigations in relation to the prime number theorem. Based on a large amount of computed data (and one must assume a lot of insight), in 1792 Gauss conjectured that, asymptotically, the number of primes less than or equal to n is about $n/\log n$. Gauss' guess turned out to be correct, though it was not until more than one hundred years later that the validity of the prime number theorem was established. Independent proofs were given in 1896 by Hadamard and de la Vallée Poussin. The correct error term for the prime number theorem will have to wait for the proof (or disproof) of Riemann's hypothesis.

While the prime number theorem is still celebrated and the Riemann hypothesis or paramount interest, our workshop focused on the areas of arithmetic of curves over number fields, diophantine equations, the *LLL* algorithm, Mahler measure, irreducibility and related problems. The talks ranged in content from the philosophical, with *Exploratory Experimentation and Computation* given by Jonathan Borwein, to the ultra-technical, with *Addition laws on Elliptic Curves* by Dan Bernstein.

The purpose of this workshop was ultimately the mathematical content, but the experiment went further, or in this case one could say farther, than theorems and formulae. While only one workshop was held, participants had a choice of where they would attend each workshop, Ontario or British Columbia. We found many

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benefits in this scenario. Other than the evident decrease in travel time and cost for most participants, as well as the ecological benefit, we found the number of people that could participate in the workshop increased significantly.

Eleven plenary lectures were given by David H. Bailey (Lawrence Berkeley National Lab), Dan Bernstein (University of Illinois at Chicago), Jonathan M. Borwein (University of Newcastle), Michael Filaseta (University of South Carolina), Jeff Lagarias (University of Michigan), Hendrik Lenstra (Universiteit Leiden), Michael Mossinghoff (Davidson College), Ram Murty (Queen's University), Chris Sinclair (University of Oregon), Cam Stewart (University of Waterloo), and Karen Yeats (Simon Fraser University). Each of the plenary lectures were shared via video link between the two workshop sites. In addition to the plenary lectures, there were also twenty–nine invited lectures which were held at local sites (12 at IRMACS; 17 at Fields), eleven of which were given by students or postdocs.

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