The theory of storage and the convenience yield

The theory of storage

<u>and</u> the normal backwardation theory

explain the relationship

between the spot and futures prices in

commodity markets

- The **theory of normal backwardation** focuses on:
  - the balance between traders' positions
  - the risk management function of the derivative market

- The **theory of storage** is centered on:
  - storage costs
  - the motives of stock holding on the physical market
  - the price discovery function of the futures markets

- There are still a lot of researches on these theories

- The storage theory has the stronger influence

A few definitions

- **Backwardation**

  Spot price > Futures price

  $S(t) > F(t,T)$

- **Contango**

  Spot price < Futures price

  $S(t) < F(t,T)$

- **Basis** (temporal basis)

  Futures price – spot price

  $F(t,T) - S(t)$

  Backwardation = discount

  Contango = premium

Section 1. The role of inventory in commodity markets

Section 2. The analysis of contango and backwardation

Section 3. The convenience yield

Section 4. Empirical tests of the storage theory

Section 5. Critiques of the theory

Section1. The role of inventory in commodity markets

1.1. Why are they so important?

1.2. Storage costs

1.3. Different kind of stocks

1.1. Why are stocks important?

- Rigidity
- Uncertainty

Rigidity

- Consumption is an inelastic function of price
  - Equipment
  - Consumer habits
  - Prices of commodities represent a low part of the prices of final products

- Supply is an inelastic function of price
  - High fixed costs of production (mineral resources)
  - High fixed costs of transportation (gas facilities)
  - Seasonality (agricultural products)
  - Joint production processes (petroleum products)

Uncertainty

- **Supply may abruptly change:**
  - Weather conditions (agricultural products)
  - Failure in production / transportation / transformation capacities
  - New discovery (mineral resources)
  - New plants
  - Technological changes (energy products)

- **Demand**
  - Weather conditions (energy)
  - GDP growth

The role of inventory

- Buffering effect :

  The stocks absorb prices fluctuations

  Conditions: - overcapacity

  - large storage facilities

- Avoid disruptions in the flow of goods and services

- Link between the present and the future

  Imperfect link : non negativity constraint on inventory

1.2. Storage costs

- Fixed costs

    (as long as storage capacities are not saturated)

    - Insurance costs

    - Warehouse costs

- Deterioration and obsolescence
- Handling costs
- Maintenance costs
- Financial costs

1.3. Different nature of stocks

- There may be several kinds of stocks :

        - hedged / unhedged stocks

        - speculative / industrial stocks

        - physical / paper stocks

        - certified stocks

     - strategic stocks

        - stocks underground (mineral reserves)

        - stocks in processing facilities, in transportation facilities…

Section 2. Storage theory and the analysis of contango and backwardation

2.1. Arbitrage operations

2.2. The analysis of contango

$$F(t,T) > S(t)$$

2.3. Convenience yield and backwardation

$$F(t,T) < S(t)$$

2.1. Arbitrage operations

2.1.1. Surplus stocks

2.1.2. Scarce inventory

2.1.1. Surplus stocks

1) The level of contango can not stay higher than the storage costs C

- Reason: Cash and carry operations

If $F - S > C$

then: buy the spot   - S

sell the futures   + F

finance the storage costs: - C

Result: > 0

2) Backwardation is impossible:

- Reason: Reverse cash and carry operations

2.1.2. Scarce inventory

- Backwardation (S > F)

- Reverse cash and carry are unlikely to happen

- Non negativity constraints on stocks

- **In contango (F>S),** the basis:

  - is stable (as long as storage capacities are available)
  - is limited to storage costs

- **In backwardation (S>F), the basis:**

  - is not stable

  - is determined by the spot price that operators are willing to pay: there is no objective limit to the basis

- **Asymmetrical behavior of the basis**

Crude oil stocks at Cushing vs. WTI 1ˢᵗ time spread
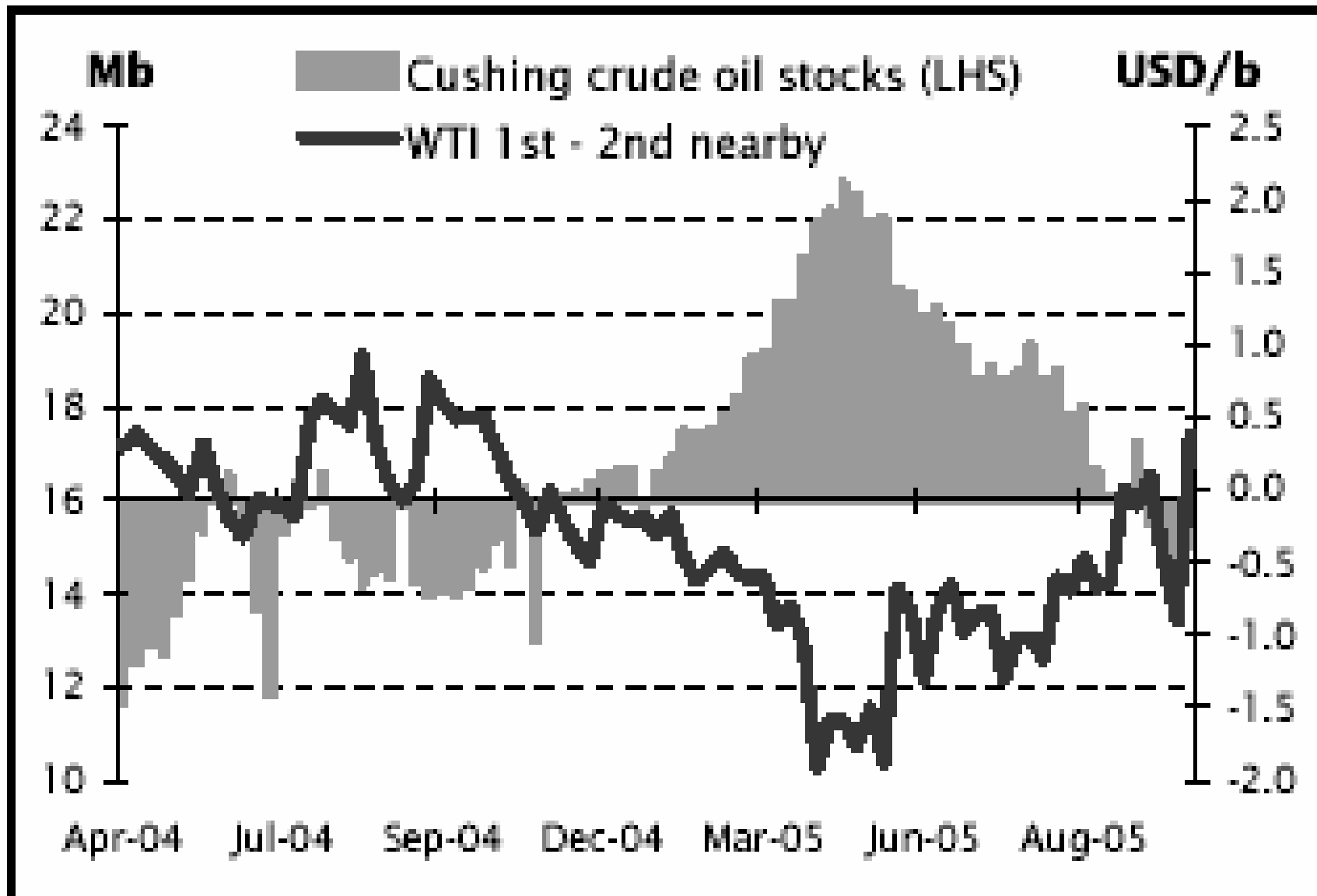
Source: US DOE Weekly, NYMEX & SG Commodities Research

2.2. The analysis of contango

The spread between futures and spot prices is related to the cost of holding commodities over time (carrying charges):

$$F(t,T) - S(t) = C_S(t,T)$$

- F(t,T) : Futures price at t for delivery at T
- S(t) : Spot price at t
- $C_s(t,T)$: Storage costs between t and T

2.3. Backwardation and convenience yield

- Why are spreads prices less than full carrying charges?

- Because "stocks of all goods possess a yield": the convenience yield (Kaldor, 1939).

- The convenience yield is low when stocks are abundant; it is positive when stocks are rare

The storage theory

$$F(t,T) = S(t) + C_S(t,T) - C_y(t,T)$$

- $C_S(t,T)$: "pure" storage costs
- $C_y(t,T)$: convenience yield
- $C_S(t,T) - C_y(t,T)$: net storage costs

Section 3. The convenience yield

- The convenience yield is an *implied* return on inventories
- Holding inventories allows for:

  - reducing the costs and delay of furniture

  - being able to answer to unexpected demand rises

  - insure the continuity of exploitation

- There are a lot of debates on convenience yield

  - Does it really exist or is it an *ad-hoc* theoretical construction?

  - What does it stand for?

  - How can we measure it?

3.1. Convenience yield and risk premium

3.2. The price of storage

3.3. Stock-out and coverage yields

3.4. Inventory and the demand for money

3.5. Convenience yield, forward and futures contracts

3.1. Convenience yield and risk premium

- Brennan, 1958 : supply and demand **of inventory**

$$E_t\left[S(t+1)\right] - S(t) = C_{St}(L_t) + \pi_t(L_t) - C_{Yt}(L_t)$$

    - S(t): spot price,

    - C_{St}: marginal storage cost,

    - L_t: inventory' level,

    - π_t: marginal risk premium on inventory,

    - C_{Yt}: marginal convenience yield

The convenience yield is an advantage, in terms of less
   delay and lower costs

Inventory allows for :

   - keeping regular customers satisfied

   - taking advantage of a rise in demand and price

3.2. The price of storage (Working, 1934-1949)

- Empirical observation:

   - Certified stocks in registered warehouses

   - For all commodities, stocks never fall to zero

- There is always some connection between the present and the future prices
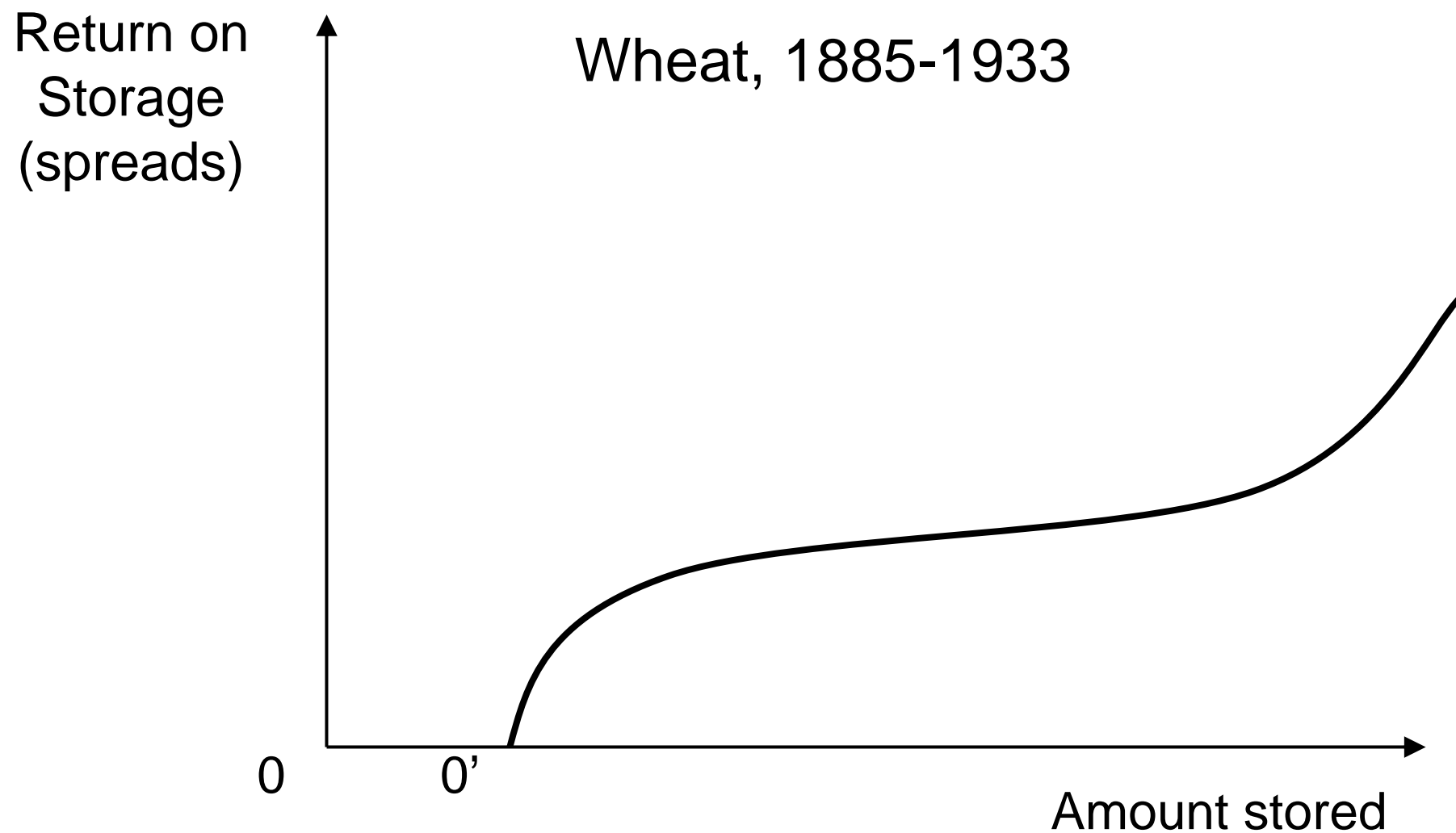
- Working (1939):

  A known return for storage is a price of storage

  The price of storage is not quoted directly
  It must be derived by taking the difference between
   quoted prices for two different dates of delivery

- Supply and demand **on storage capacities**

The price of storage (Working, 1934)

Return on
Storage
(spreads)

Wheat, 1885-1933

0   0'

Amount stored

- Similar results on other markets :

  - Telser (1958), Gray & Peck (1981) : wheat

  - Howel (1956), Telser (1958) cotton

  - Weymar (1974) : cocoa

  - Brennan (1958) : shell eggs, cheese, butter, oats

- Working (1949) : Generalization

The supply of storage curve (Working,1949)
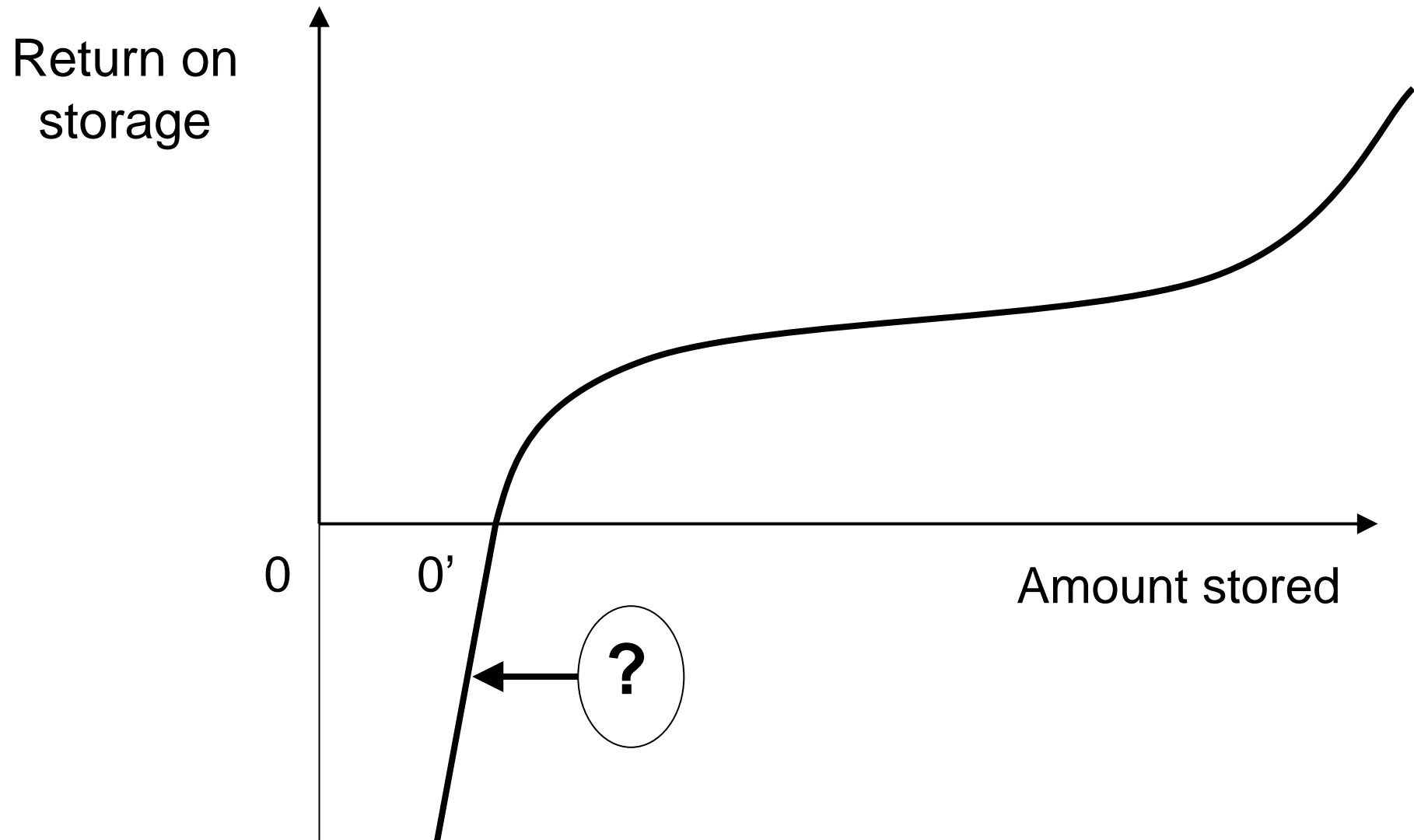
- **Empirical observation :**

  Nonlinear relationship between stocks and spreads

- What makes spreads between futures and spot prices fall below full carrying charges ?

- **Why do firms store commodities at a loss ?**

- Return on storage is the result of the equilibrium between storage demand and supply

  This return may be negative for two reasons
  - high fixed costs in storage activity
  - convenience yield

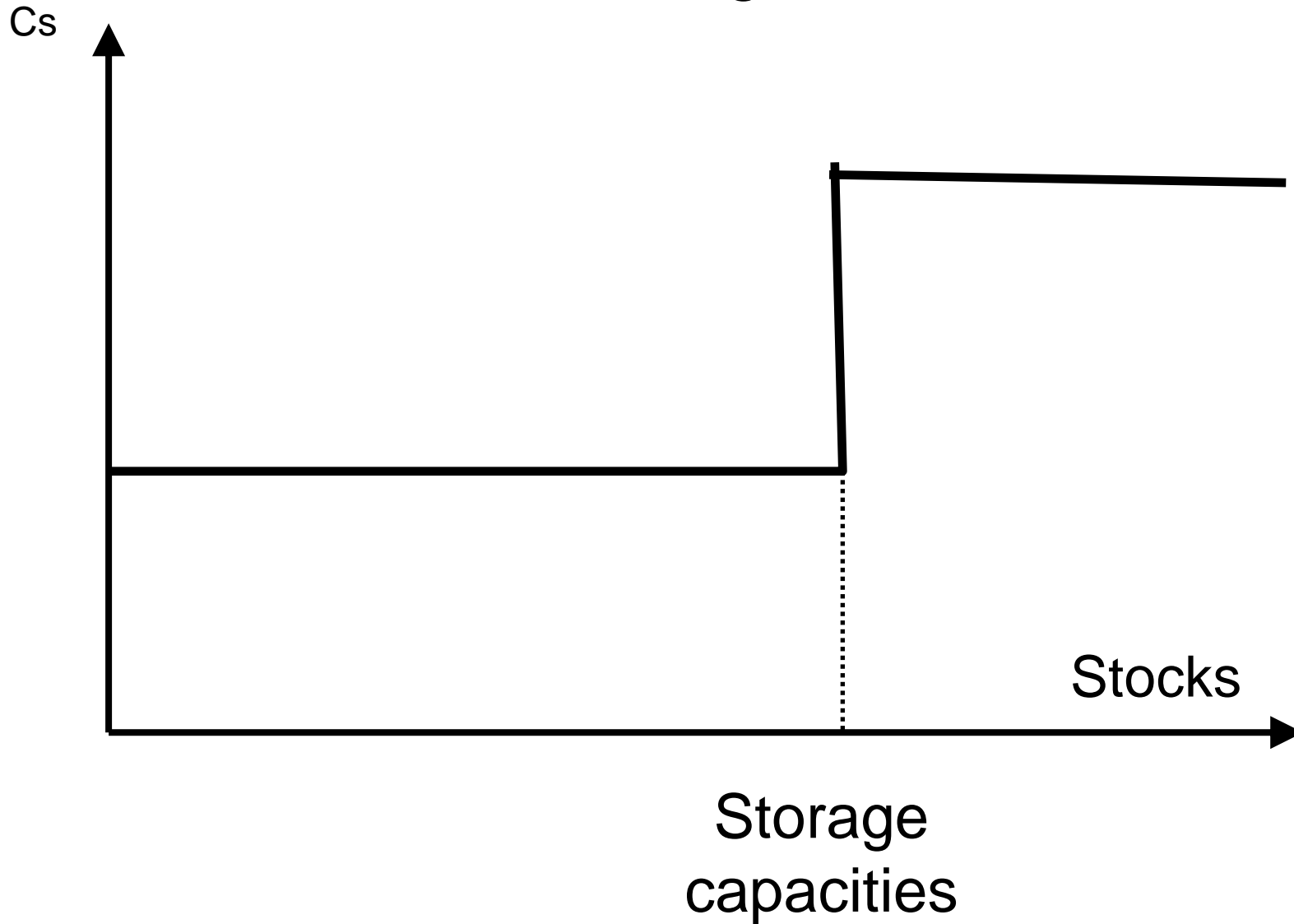# 3.3. Stock-out yield and coverage yield
Weymar (1968)

Marginal inventory holding costs depend on:
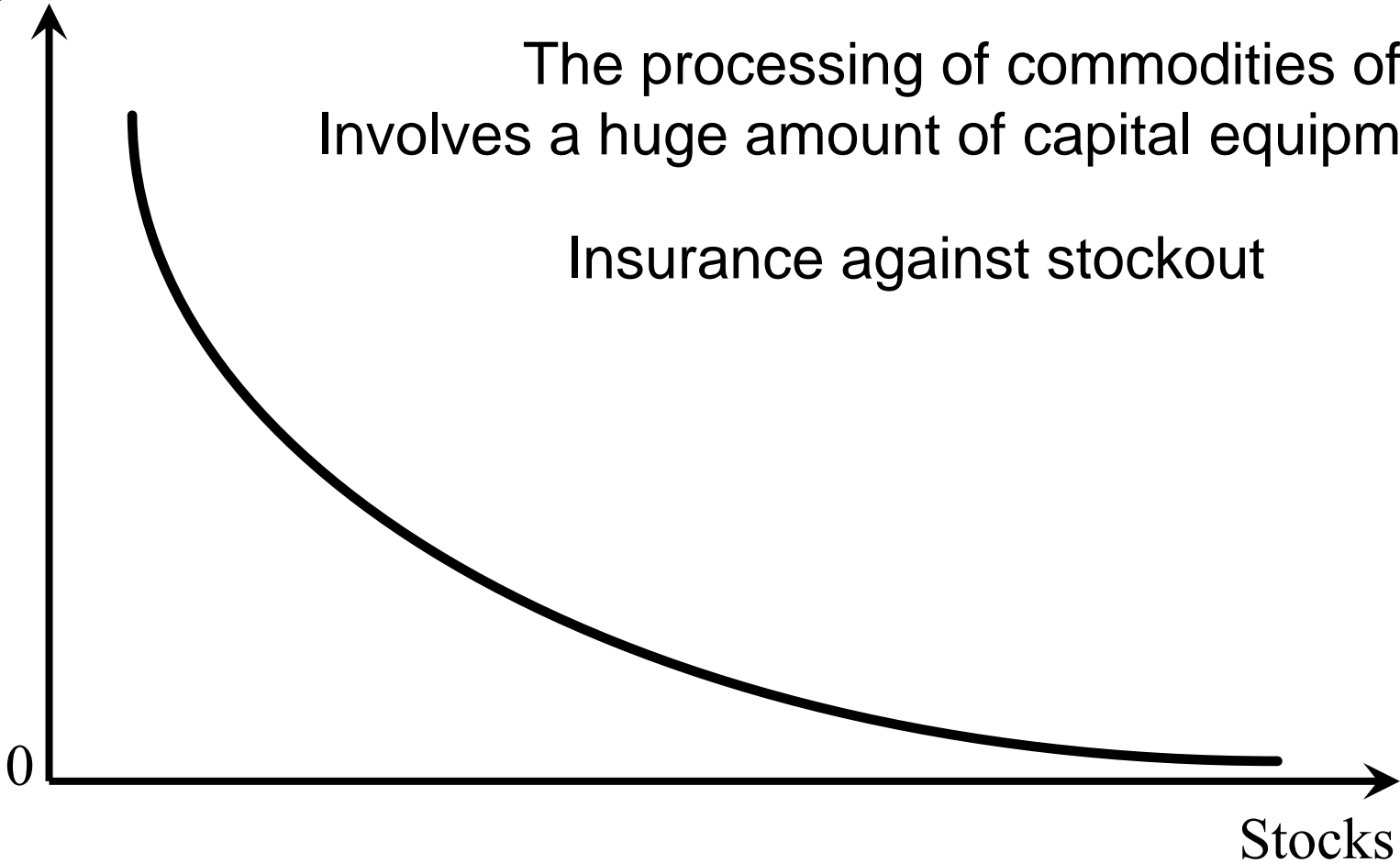
Pure storage costs

Stock-out yield
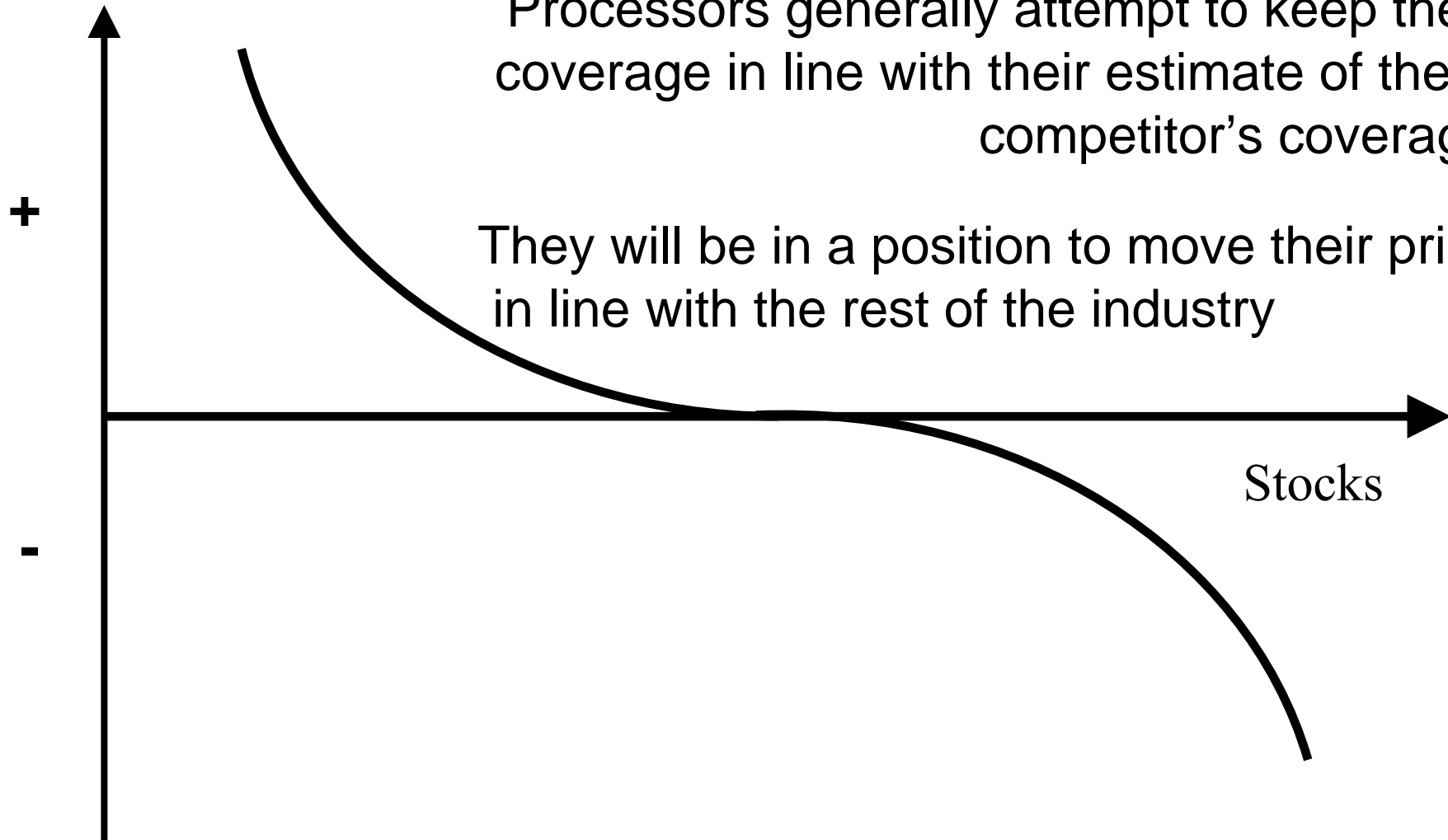
Coverage yield

Pure storage costs

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Stockout yield

Stockout
yield

The processing of commodities often
Involves a huge amount of capital equipment

Insurance against stockout

0

Stocks

Coverage yield

Coverage yield

Processors generally attempt to keep their coverage in line with their estimate of their competitor's coverage

They will be in a position to move their prices in line with the rest of the industry
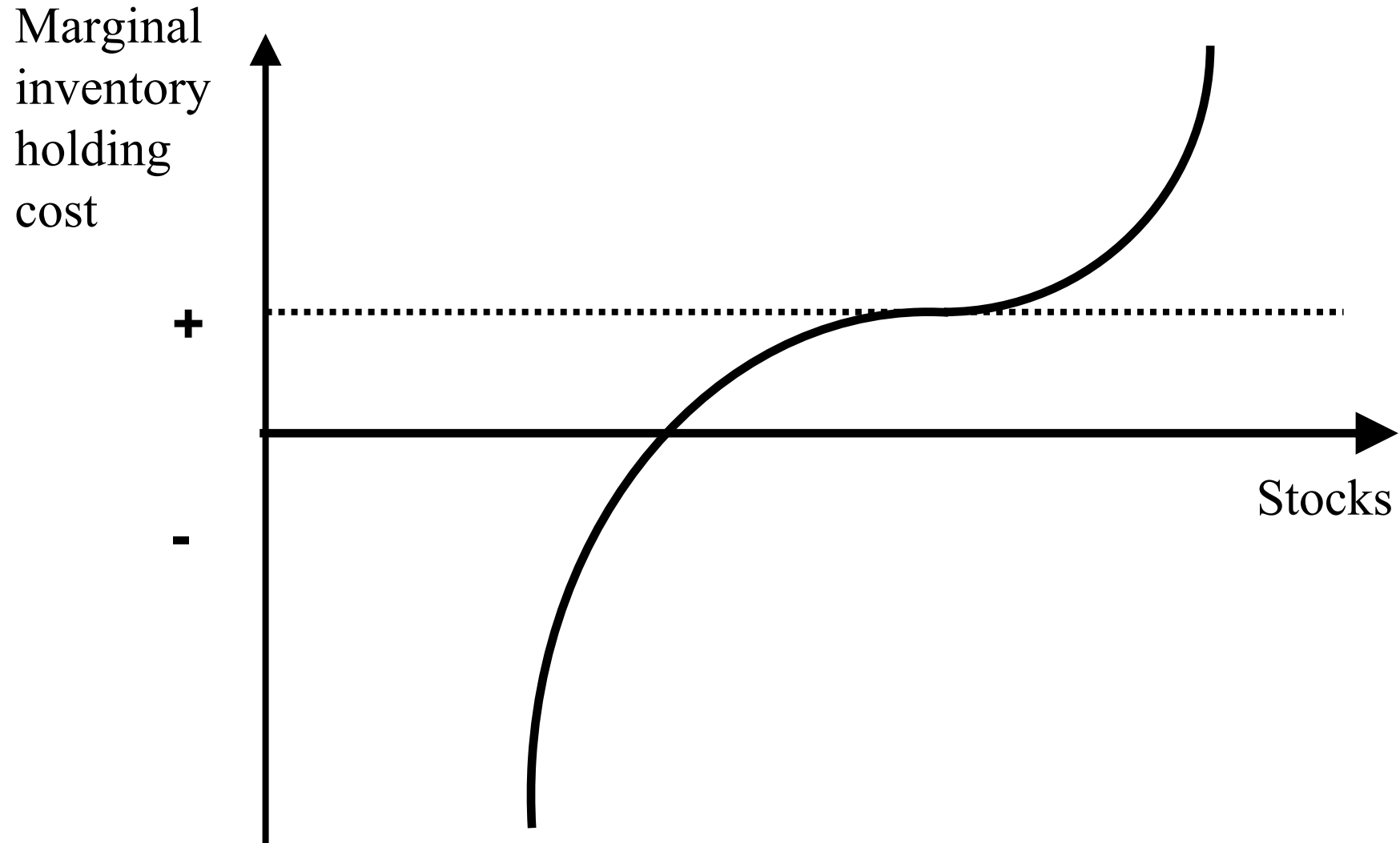
+

-

Stocks

Marginal inventory holding costs

=

Pure storage costs

−

Stock-out yield

−

Coverage yield

Marginal inventory holding cost

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3.4. Inventory and the demand for money (Williams)

- Firms hold inventories for the same reasons they hold money

- The negative component to spreads is what firms pay for holding stocks. This is equivalent to the expense of holding cash.

- Futures markets may be viewed as implicit loan markets :

  "A short hedging operation (the spot purchase of a commodity and its simultaneous sale for future delivery), amounts to borrowing the commodity over an interval of time while lending money".

- Conventional models of the demand for money demonstrate that even risk-neutral firms desire to hold cash

- Four reasons for holding inventory:

    - pure storage

        to smooth out consumption

    - speculative storage

        rare, except for precious metals, because futures contracts are a superior vehicle for speculation

    - transactions demand

    - precautionary demand

- Transactions and precautionary demands are important for the spreads analysis

Transaction demand for inventories

- Transformation costs give rise to the transaction demand for money and for inventory

- Transformation costs are much higher for commodities than for money

- Transformation costs for commodities are:

  - The costs of buying and selling the commodity

  - Processing and transportation costs

- Stocks give the possibility to undertake transactions immediately; They insure the access to the merchandise

- There is a transaction demand even if there is no uncertainty

Transaction demand for inventories

- Example :
- Two periods t_1 t_2
- A farmer holds W bushels of wheat

$$W = w_1 + w_2$$

- w_1 : amount marketed at t_1
- Prices are known : p_1 p_2
- H1. $p_1 > \delta$ p_2

  where δ is the discounting factor

1) No marketing costs : sell W at t_1 (no stock)
2) Marketing costs : cw^2 with c a positive constant

   Problem : Optimal storage policy

   Find w_2 that will be stored in the first period

Transaction demand for inventories

$$Max_{w_2}\left(p_1 w_1 - c w_1^2 + \delta\left[p_2 w_2 - c w_2^2\right]\right)$$

$$w_2^* = W + \frac{\delta p_2 - p_1}{2c} \cdot \frac{1+r}{2+r}$$

Positive inventories as long as :

$$p_1 - \delta p_2 > 2cW$$

The larger the spread, the lower the inventories

Precautionary demand for inventories

- Even when there is no uncertainty, there is still a transaction demand for inventories

- Precautionary demand

  - comes from uncertainty

  - is directly linked to transformation costs

- Uncertainty in the supply / demand

- A firm that is risk neutral still holds inventories as a precaution against irregularities in its receipts, ordered materials, or sales

Precautionary demand for inventories

- Example / rigidity and uncertainties in production
- Rigidity :

  - A miller has a fixed production capacity K

  - Variable cost : raw material (wheat or corn)

  - The firm loses money as soon as it does not operate at full capacity

  Problem: Minimize the shortage costs (expected)

Precautionary demand for inventories

- Uncertainty on supply:

  - The firm is unable to control:

    - the amount of wheat being forwarded to it,

    - its time to arrival

  - It holds a precautionary stock I

  - Let f(z) being the probability that a particular amount z arrives

  - If z is too low to operate at full capacity, then the firm will suffer a shortage cost :

$$(K - I - z) \; c$$

  where c is a constant loss from the shortage

Precautionary demand for inventories

- Expected shortage cost :

$$\int_0^{K-I} \left[(K - I - z)c \right] f(z)\, dz$$

- How much inventory to keep in order to avoid these costs?

- Balance between shortage and storage costs

- How much is the miller willing to pay to ensure his access

  to raw material ?

Precautionary demand for inventories

- Minimizing expected total costs :

$$MIN_{I \geq 0} (EC) = IP_A + \int_0^{K-I} [(K - I - x)c] f(x) dx$$

- I stands for a line of credit : it gives immediate access to the raw material

- P_A is : - the price for this services per unit of I

  - the cost of holding the commodity

Precautionary demand for inventories

- Optimum value of I : I* such as

$$\frac{\delta EC}{\delta I} = 0 = P_A - \int_0^{K-I^*} cf(x)\,dx$$

- The firm willingness to pay up to P_A for access to inventory

3.5.Convenience yield, forward and futures contracts

- Futures and forward contracts give the possibility to obtain a physical delivery at expiration
- They insure the future availability of the merchandise
- There is a convenience yield associated to these contracts

$$C_y(t,T)$$

$$CY_{Contracts} < CY_{Inventories}$$

1. Quality, volume, localization (Blau, 1944-1945)
2. Only inventories give the possibility to benefit from an unexpected prices rise (Brennan, 1958)
3. There is no stockout yield associated with the holding of a contract (Weymar, 1968)

3.6. Dynamic behavior of the convenience yield

1) The convenience yield is deterministic. It is positively correlated to the spot price.

$$Cy(t) = c. S(t), \text{ where c is a constant}$$

2) The convenience yield has a mean reverting behavior

Stocks have the capacity to reconstitute themselves

There is a level of stocks which satisfies the needs of the industry in normal conditions. The behaviour of operators in the physical market guarantees that this level is maintained.

Mean reverting convenience yield
Schwartz 1997

Dynamic of states variables

$$\begin{cases} dS = (\mu - C)Sdt + \sigma_S Sdz_S \\ dC = [k(\alpha - C)]dt + \sigma_C dz_C \end{cases}$$

- μ drift of the spot price S,

- σ_i volatility of variable i,

- α : long-run mean of the convenience yield C,

- κ : speed of adjustment of the convenience yield,

- *dzi* : Brownian motion .

$$E[dz_S \times dz_C] = \rho dt$$

3) Asymmetrical convenience yield

Direct consequences of :

- the non negativity constraint on inventory

- the imperfections in arbitrage operations

- the asymmetry in the basis

Convenience yield as a real option

Section 4. Empirical tests on the theory of storage

4.1. Empirical implications of the theory

4.2. Empirical tests : a few results

4.1. Empirical implications of the theory

Direct implication
- Positive correlation between the basis and the inventory level
- For seasonal products, the convenience yield must rise when the harvest comes near

Indirect implication
- Basis is more volatile in backwardation
- In backwardation, spot prices are more volatile than futures prices
- For seasonal products, futures prices with an expiration date situated before or after the harvest have a different behavior

4.2. Empirical tests : a few results

- There is a convenience yield in almost all commodity markets, except for precious metals (gold, silver)

- The convenience yield :

  - is high when prices are high, and is otherwise low

  - changes with the level of pure storage costs

  - has as seasonal behavior

  - is affected by changes in economic cycles (during economic recovery, inventories are low, and convenience yield is high)

Conclusion on empirical tests

- The theory of storage is generally validated

- The basis behavior changes with the particularities of the commodity considered

- **Empirical examination of the relationship between prices and stocks, which is at the center of the theory, remains relatively scarce**

Section 5. Critiques of the theory

5.1. Convenience yield and the nature of stocks

5.2. Transformation costs

5.3. Marketing costs

5.1. Convenience yield and the nature of stocks

- The results of empirical tests change with :
    - Quality of stocks : Certified / non certified
    - Availability : Strategic / non strategic stocks
    - Localization : distance to the futures exchange

5.2. Transformation costs

- The convenience yield is often over-estimated because there is an aggregation phenomenon on stock data (However, it is very difficult to know where the aggregation phenomenon starts and where it stops)

5.3. Marketing costs

- Stocks are not held because they are profitable to keep, but because they are expensive to sell

Conclusion

- Recurrent question :

  Why do firms hold inventory in backwardated markets ?

- 3 variables explaining the behavior of the futures price :

  - Spot price S

  - Convenience yield CY

  - Storage costs (interest rate)

- S & CY positively correlated

- Asymmetrical behavior of the basis

- What about non storable commodities?

Light Sweet Crude Oil, 1989-2008