Cost-Minimizing Regulations for a Wholesale Electricity Market

Alejandro Jofré[2]

CMM-Universidad de Chile

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[2]Joint work with Nicolas Figueroa, CEA-Universidad de Chile

Introduction

The Questions

In an electric network with transmission costs and private information:

- Does the usual (price equal Lagrange multiplier) regulation mechanism minimize costs for a regulator?
- If not, what is the mechanism that achieves this objective?
- How does the performance of both systems compare?

Methodology:

- Algorithmic Game Theory
- Mechanism Design

Related Literature

- Escobar and Jofré (2005), in a symmetric model with complete information, establish that in the presence of transmission costs, equilibrium exists but producers charge a price above marginal cost with the current regulation.
- demand d, s resistance, $c(q) = cq, \bar{c}$ marginal cost, Nash $= \bar{c}/(1 - 2sd)$
- Myerson (1981) for the basic tools of mechanism design. However we require to introduce network constraints.

The Model

Environment

- Two-node network with demand d at each node.
- One producer at each node, with marginal cost of production $c_i \sim F_i[\underline{c}_i, \overline{c}_i]$.
- Transmission costs sh^2, with h the amount sent from one node to another.

Benchmark

The Dispatcher Problem

Given that each generator reveals a cost c_i, the dispatcher solves:

$$\begin{aligned} \min_{q,h} \quad & \sum_{i=1}^{2} c_i q_i \\ s.t. \quad & q_i - h_i + h_{-i} \geq \frac{s}{2}[h_1^2 + h_2^2] + d \text{ for } i = 1, 2 \\ & q_i, h_i \geq 0 \text{ for } i = 1, 2 \end{aligned}$$

The Solution

If we define

$$H(x,y) = d + \frac{1}{2s}\left(\frac{x-y}{x+y}\right)^2 - \frac{1}{s}\left(\frac{x-y}{x+y}\right)$$

and

$$\overline{q} = 2\left[\frac{1-\sqrt{1-2ds}}{s}\right]$$

then the solution to this problem can be written as

$$q_i(c_i, c_{-i}) = \begin{cases} H(c_i, c_{-i}) & \text{if } H(c_i, c_{-i}) \geq 0 \text{ and } H(c_{-i}, c_i) \geq 0 \\ \overline{q} & \text{if } H(c_{-i}, c_i) < 0 \\ 0 & \text{if } H(c_i, c_{-i}) < 0 \end{cases}$$

$$\lambda_i(c_i, c_{-i}) \equiv p_i(c_i, c_{-i}) = c_i \text{ if } H(c_i, c_{-i}) \geq 0$$

The Bayesian Game

The game:

- 2 players. $S_i = C_i = [\underline{c}_i, \overline{c}_i]$.
- Payoff $u_i(s_i, s_j, c_i) = (p_i(s_i, s_j) - c_i)q_i(s_i, s_j)$.

The Equilibrium:

- A strategy $b : [\underline{c}_i, \overline{c}_i] \longrightarrow [\underline{c}_i, \overline{c}_i]$.
- In a Nash equilibrium

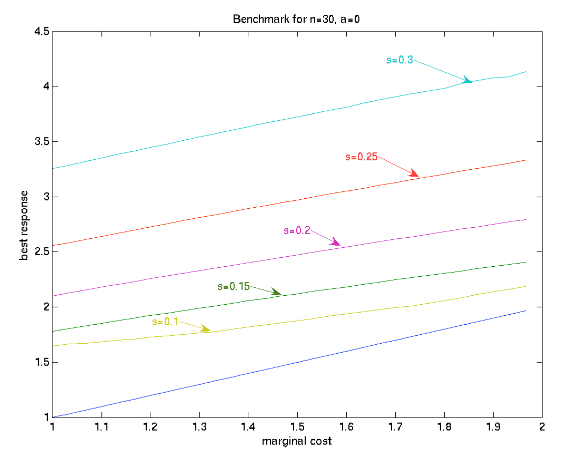
$$\bar{b}(c) \in arg\max_{x} \int_{C_{-i}} [p_i(x, \bar{b}(c_{-i})) - c]q_i(x, \bar{b}(c_{-i}))f_{-i}(c_{-i})dc_{-i} \tag{1}$$

Numerical Approximation

- For simplicity $C_i = [1, 2]$.
- Let $k \in \{0, ..., n-1\}$, and $b(c) = b_k$ for $c \in [\frac{k}{n}, \frac{k+1}{n}]$.
- The weight of each interval is given by $w_k = F(\frac{k+1}{n}) - F(\frac{k}{n})$.
- The approximate equilibrium is characterized by:

$$b_k \in arg \max_x \sum_{l=0}^{n-1} [p_i(x, b_l) - r_k] q_i(x, b_l) w_l \text{ for all } k \in \{0, ..., n-1\} \quad (2)$$

Results


Benchmark for n=30, a=0
best response
marginal cost
s=0.3
s=0.25
s=0.2
s=0.15
s=0.1
4.5
4
3.5
3
2.5
2
1.5
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2

Optimal Mechanism

Mechanisms

- A *direct revelation mechanism* $M = (q, h, x)$ consists of an *assignment rule* $(q_1, q_2, h_1, h_2) : C \longrightarrow R^4$ and a *payment rule* $x : C \longrightarrow R^2$.
- The ex-ante expected utility of a buyer of type c_i when he participates and declares c_i' is

$$U_i(c_i, c_i'; (q, h, x)) = E_{c_{-i}}[x_i(c_i', c_{-i}) - c_i q_i(c_i', c_{-i})]$$

- A mechanism (q, h, x) is feasible iff:

$$\begin{aligned} U_i(c_i, c_i; (q, h, x)) &\geq U_i(c_i, c_i'; (q, h, x)) \text{ for all } c_i, c_i' \in C_i \\ U_i(c_i, c_i; (q, h, x)) &\geq 0 \text{ for all } c_i \in C_i \\ q_i(c) - h_i(c) + h_{-i}(c) &\geq \frac{s}{2}[h_1^2(c) + h_2^2(c)] + d \text{ for all } c \in C \\ q_i(c), h_i(c) &\geq 0 \text{ for all } c \in C \end{aligned}$$

The Regulator's Problem

Using the revelation principle, the regulator's problem can be written as:

$$\min \int_C \sum_{i=1}^{2} x_i(c) f(c) dc \tag{3}$$

subject to (q, h, x) being "feasible"

The Regulator's Problem (II)

It can be rewritten as

$$\begin{aligned} \min \quad & \int_C \sum_{i=1}^{2} q_i(c)[c_i + \tfrac{F_i(c_i)}{f_i(c_i)}]f(c)dc \\ \text{s.t} \quad & \int_{C_{-i}} q_i(c_i, c_{-i})f_{-i}(c_{-i})dc_{-i} \text{ is non-increasing in } c_i \\ & q_i(c) - h_i(c) + h_{-i}(c) \geq \tfrac{s}{2}[h_1^2(c) + h_2^2(c)] + d \text{ for all } c \in C \\ & q_i(c), h_i(c) \geq 0 \text{ for all } c \in C \end{aligned}$$

We denote by $J_i(c_i) = c_i + \frac{F_i(c_i)}{f_i(c_i)}$ the virtual cost of agent i. We assume it is increasing.

Solution

An optimal mechanism is given by

$$\hat{q}_i(c_i, c_{-i}) = \begin{cases} H(J_i(c_i), J_{-i}(c_{-i})) & \text{if } H(J_i(c_i), J_{-i}(c_{-i})) \geq 0 \text{ and } H \\ \overline{q} & \text{if } H(J_{-i}(c_{-i}), J_i(c_i)) < 0 \\ 0 & \text{if } H(J_i(c_i), J_{-i}(c_{-i})) < 0 \end{cases}$$

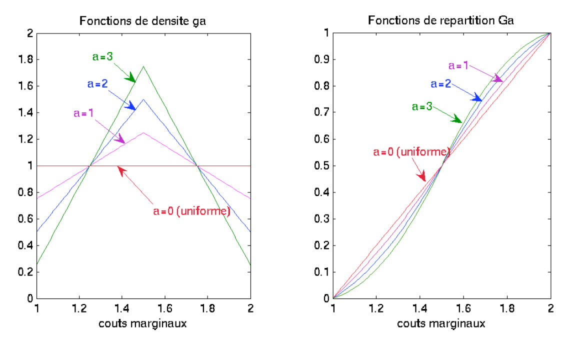
$$\hat{x}_i(c_i, c_{-i}) = c_i \hat{q}_i(c_i, c_{-i}) + \int_{c_i}^{\overline{c}_i} \hat{q}_i(s, c_{-i}) ds$$

Such a mechanism is dominant strategy incentive compatible.

Comparison

We consider the family of distributions with densities

$$f_a(x) = \begin{cases} a(x-1) + (1 - \frac{a}{4}) & \text{if } x \leq 1.5 \\ -a(x-1) + (1 + \frac{3a}{4}) & \text{if } x \geq 1.5 \end{cases}$$


Fonctions de densite ga
a=3
a=2
a=1
a=0 (uniforme)
couts marginaux
Fonctions de repartition Ga
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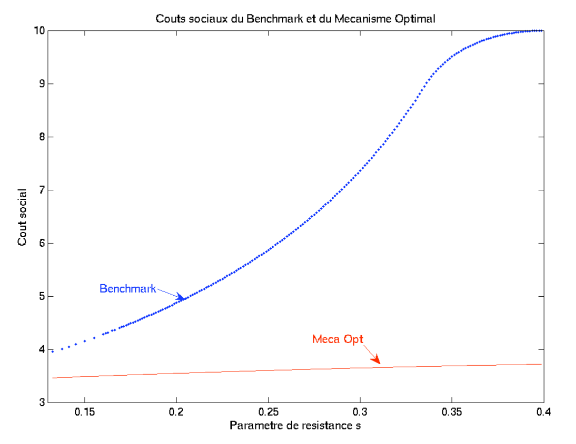
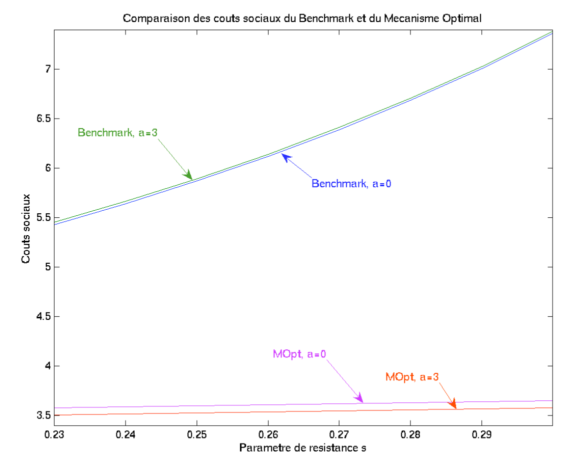
couts marginaux


Couts sociaux du Benchmark et du Mecanisme Optimal
Cout social
Parametre de resistance s
Benchmark
Meca Opt
10
9
8
7
6
5
4
3
0.15
0.2
0.25
0.3
0.35
0.4


Comparaison des couts sociaux du Benchmark et du Mecanisme Optimal
Couts sociaux
Parametre de resistance s
Benchmark, a=3
Benchmark, a=0
MOpt, a=0
MOpt, a=3
7
6.5
6
5.5
5
4.5
4
3.5
0.23
0.24
0.25
0.26
0.27
0.28
0.29

Robustness and Practical Implementation

- The optimal mechanism is detail free. If the designer is wrong about common beliefs, then the mechanism is still not bad:

$$||X_f - X_{\tilde{f}}|| \leq ||x||_1||f - \tilde{f}||_\infty \leq \bar{c}\bar{q}||f - \tilde{f}||_\infty$$

- The assignment rule is computationally simple to implement. It requires solving **once** the dispatcher problem, with modified costs.
- However, the payments are computationally difficult

$$c_i\hat{q}_i(c_i, c_{-i}) + \int_{c_i}^{\bar{c}_i} \hat{q}_i(s, c_{-i})ds$$

- The integral requires solving infinitely many dispatcher problems. But it can be approximated using the risk neutrality of agents.

Future Directions of Research

- Extension to more general networks
- Other interpretations: imperfect substitutes.