Submittee: Alejandro Morales
Date Submitted: 2016-04-13 15:00
Title: Combinatorial Structures in Perturbative Quantum Field Theory
Event Type: Conference-Workshop

Location:
Simon Fraser University, Burnaby, British Columbia

Dates:
March 21, 2016

Topic:
- Combinatorial interpretations of the perturbative expansion
- Renormalization group equation
- Combinatorial Fourier and Legendre transforms
- Dyson-Schwinger equations
- Combinatorial Hopf algebras

Methodology:
The one day conference consisted of one 50 minute plenary talk, seven 25 minute talks, and a 25 minute open problem session (four problems were presented). There was videoconferencing during all the talks. People at the University of Saskatchewan used this service. The participants came from different fields in mathematics (discrete math, algebraic geometry) and physics (quantum field theory) with overlapping interests. This allowed having interaction despite the different background. The problem session also helped realize that (1) we were seeing similar positivity questions in finite field counts in our respective fields and (2) we could use tools from other fields represented in the audience to attack our problems. The conference was on a Monday and four participants from Europe and the US stayed the rest of the week to continue discussing the questions raised on the conference.

Objectives Achieved:
Bringing together participants from different fields (discrete math, algebraic geometry, physics) that are working on related problems: combinatorics of Feynman graphs and Feynman integrals and point counting over finite fields in graph hypersurfaces.

Scientific Highlights:
- Introduce a toric orbit used in studying a certain q-analogue of rook placements (arxiv:1011.4539) to make progress on positivity conjectures of point counts in graph hypersurface point counts at t=q-1. Erik Panzer used this method to understand the constant and linear term of the point count coming from wheel graphs.
- From the participants working in physics and algebraic geometry (Matilde Marcoli, Paolo Aluffi) we

learned that q-positivity and (q-1)-positivity in point counts might come from decompositions into cells and tori respectively.
- In the open problem session, Erik Panzer (Oxford) asked about an algorithm for computing the coproduct of a certain new Hopf algebra of Feynman graphs by Francis Brown, on "divergent" graphs. A graduate student Iain Crump (SFU) made progress on this question.
- One of the speakers, Michael Borinsky (Humboldt U.), was a physics graduate student from Germany studying asymptotics of certain generating functions that often appear in quantum field theory. He interacted with local experts in analytic combinatorics, Marni Mishna and Julien Courtiel (SFU, PIMS) to compare the tools and the state of the art in the fields.
- One of the speakers commented that the miniconference "was unique in that he could take something home from all of the talks." He found that it connected people with overlapping interests.

Organizers:
Yeats, Karen, Mathematics, Simon Fraser University
Morales, Alejandro, Mathematics, University of California Los Angeles

---

Speakers:
1. Michael Borinsky, Physics, Humboldt-Universitaet Berlin,
Generating functions' asymptotics' generating functions,

Sequences which admit some factorial growth are common in combinatorics and omnipresent in physics. I will talk about the algebraic properties of power series of this type. These power series form a subring of the ring of power series which is closed under composition. This subring can be equipped with a derivation which maps a power series to its asymptotic expansion. Leibniz and chain rules for this derivation can be deduced. In many cases full asymptotic expansions can be obtained easily in this formalism.

2. Julien Courtiel, PIMS UBC,
Terminal chords in connected diagrams chords,

The topic of this work, resulting from a collaboration with Karen Yeats, is the enumerative study of connected chord diagrams, within the specific framework of quantum field. In fact, the solutions to certain Dyson-Schwinger equations can be defined in terms of connected chord diagrams with a particular parameter: the terminal chords.

We study some statistics about these terminal chords: their asymptotic number, the position of the first terminal chord, their distribution with respect to the leading-log expansions, etc. We establish the means, the variances and the limit laws of some of these variables, and show the physics applications.

3. Matilde Marcoli, Division of Physics, Mathematics, and Astronomy, Caltech,
Renormalizations, Motives, Determinant Hypersurfaces, and Kausz Compactifications

I will discuss a renormalization procedure (different from the physical one) for Feynman integrals, based on mapping the integral to the determinant hypersurface complement, and to a Kausz compactification. The talk is based on joint work with Paolo Aluffi (arXiv:0901.2107) and with Xiang Ni (arXiv:1408.3754)

4. Alejandro Morales, Mathematics, UCLA,
Combinatorics of the Legendre and Fourier transforms in PQFT,

The partition function, Z, the Fourier and Legendre transforms, are fundamental to quantum field theory. When Z is regarded as a function analytic obstacles are encountered, and heuristic arguments are introduced to go around them. Since Z may be formulated in terms of Feynman diagrams, it is reasonable to ask whether combinatorial methods may be used to determine which of its structures are independent of analytic assumptions. We demonstrate that this is so to an extent by constructing both algebraic and combinatorial analogues of the Legendre and Fourier transforms. Joint work with Achim Kempf and David M. Jackson.

5. Emad Nasrollahpoursamami, Mathematics, Caltech,
Differential Equations for Feynman Amplitudes

A Feynman diagrams in scalar quantum field theory in dimension D, is a graph and a collection of external data which are vectors in R^D corresponding to the vertices. The amplitude of the diagram is a function of the external data which is defined by an integral which does not necessary converges. One can construct a maximal system of algebraic differential equations satisfied by the integral. It can be shown that in the case of divergent diagrams, the analytic continuation of the integral with respect to D is still a solution to these differential equation.

The structure of these equations can be described by combinatorics of polytope which is constructed from the graph. The set of differential equations is "regular holonomic", which in particular implies that the space of solutions is finite dimensional. One can construct a basis for the space of solutions as both integrals and gamma series.

In this talk I will talk about the construction of the polytope and differential equations, and show that certain gamma series are solutions to it.

6. Erik Panzer, All Souls College, Oxford,
Hopf algebras of infrared divergences,

Feynman graphs and their ultraviolet subdivergences form a Hopf algebra which was defined by Connes and Kreimer. Recently, Brown generalized this Hopf algebra by taking into account also the infrared divergences. I will define this Hopf algebra and demonstrate the corresponding new factorization of the second Symanzik polynomial.

7. Julian Purkart, physics, Humboldt-UniversitÃ¤t Berlin,
The signed permutation group on Feynman graphs,

The Feynman rules assign to every graph an integral which can be written as a function of a scaling parameter L. Assuming L for the process under consideration is very small, so that contributions to the renormalizaton group are small, we can expand the integral and only consider the lowest orders in the scaling. The aim is to determine specific combinations of graphs in a scalar quantum field theory that lead to a remarkable simplification of the first non-trivial term in the perturbation series. It will be seen that the result is independent of the renormalization scheme and the scattering angles. To achieve that goal we will utilize the parametric representation of scalar Feynman integrals as well as the Hopf algebraic structure of the Feynman graphs under consideration. Moreover, we will present a formula which reduces the effort of determining the first-order term in the perturbation series for the specific combination of graphs to a minimum.

8. Karen Yeats, Mathematics, SFU,
Field diffeomorphisms and Bell polynomials,

Kreimer and Velenich observed that if a diffeomorphism is applied to a free field theory then the terms in the perturbative expansion must cancel. I'll prove this at tree level using Bell polynomials.

Links:
https://sites.google.com/site/combstructpqft16/
