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Title: Curvature of the central path in linear programming
Event Type: Conference-Workshop

Location:
University of Calgary, Calgary AB

Dates:
Start Date: 05/26/2015
End Date: 05/29/2015

Topic:
The field of optimization is ubiquitous to countless applications in engineering and sciences. Within the past 3 decades, the field of convex optimization was revolutionized with the introduction of the so-called Interior Point Methods (IPM). These methods allow solving many large-scale optimization problems very efficiently and already found ways into a number of leading commercial numerical software packages like CPLEX, Gurobi, Mosek, etc. Despite reach and beautiful theory of these methods, some essential gaps in our understanding of convex optimization still exist. For example, a question of whether a linear optimization problem can be solved in strongly polynomial time is cited by the Fields medalist Stephen Smale as one of the top mathematical problems for the XXI century.

Central path - a real analytic curve underlying the complexity analysis of majority of IPM - has first drawn the attention of researchers since the very inception of IPM some 20+ years ago. It is believed that the complexity and thus the numerical efficiency of IPM are intimately connected to the geometric properties of the path. Despite that, the geometric-algebraic properties of the path are still poorly understood, even for linear optimization. Specifically, we still do not have a full understanding of the worst-case total curvature of the central path, which in turn holds one the greater promises for further algorithmic improvements. Some connections to IPM algorithm complexity were established over the course of 3 decades of IPM theory development. However, insofar all of the previous attempts - Sonnevend's curvature surrogate, Vavasis-Ye crossover events, Tsuchiya-Monteiro analysis of the two - fall short of connecting with the true geometric curvature, in large part due to the fact that the curvature itself remained elusive. Interestingly, despite a well-grounded interest in understanding the true curvature of the path, no major advancements were made until 2000, when a novel set of methods largely borrowing on algebraic geometry - tools mostly uncommon in large optimization community - was used for the first time.

In early 2000 Dedieu, Malajovich and Shub established first pivotal result towards understanding the total curvature of the path, essentially bounding the average curvature with the number of embedding dimensions. In 2008 in the work of Deza, Terlaky and Zinchenko, a few pivotal observations were made regarding the worst-case curvature and a relationship to the combinatorial structure of the underlying polytope was hypothesized. Using more algebraic techniques, some refinements of these results soon followed in work of De Loera, Sturmfels and Vinzant in 2010. In 2014, using the techniques of tropical geometry, in a unexpected break-through work of Allamigeon, Benchimol, Gaubert and Joswig it was shown that in fact the total curvature of the path may grow exponential in the dimension. The latter result not only disproves a number of pre-existing

conjectures, but possibly even sheds light on possibility of the existence of strongly-polynomial IPM-based method for linear programming.

Methodology:
Lectures and working sessions

Objectives Achieved:
(1) a well-substantiated working conjecture was made on the worst-case lower bound for the total curvature of the central path in the number of inequalities in generic dimension, further extending the work of Allamigeon, Benchimol, Gaubert and Joswig; preliminary numerical tests substantiate the working hypothesis very well.

(2) a detailed examination of possible set of attack angles onto the problem of establishing a tight upper bound on the total curvature was made; one new (and possibly simpler) re-parametrization for the curvature estimation was proposed.

(3) a substantiated working hypothesis that the knowledge of total curvature alone would not result in improved IPM-type method with better complexity estimates was formulated and discussed in length.

Scientific Highlights:
Besides the objectives listed above, the event strengthened and further broadened collaborative international -- Canada, France, USA -- network of researchers working on the investigation of the curvature of the central path. The workshop and working group discussions were extremely helpful to deepen the understanding of the Allamigeon-Benchimol-Gaubert-Joswig construction that uses fairly exotic (to optimizers) tools of tropical algebraic geometry and model theory, with its relationship to infinite games.

Organizers:
Deza, Antoine, Computing and Software, McMaster University
Zinchenko, Yuriy, Mathematics, University of Calgary

Speakers:
Allamigeon, Xavier, INRIA Saclay / Ecole Polytechnique - Control and Optimization
Benchimol, Pascal, EDF R&D - Control and Optimization
Deza, Antoine, McMaster University - Computing and Software
Gaubert, Stephane, Saclay / Ecole Polytechnique - Control and Optimization
Lee, Yin Tat, MIT - Mathematics
Skomra, Mateusz, INRIA Saclay / Ecole Polytechnique - Control and Optimization
Sidford, Aaron, MIT - Mathematics
Terlaky, Tamas, Lehigh University - Industrial and Systems Engineering
Zinchenko, Yuriy, University of Calgary - Mathematics

Links:
http://www.pims.math.ca/scientific-event/150526-ccplp

Comments / Miscellaneous:
Thank you very much for supporting this event & PIMS CRG on Optimization! Please accept my apologies for submitting the report so late.
