Games, Puzzles, and Mathematics (Part 1)
Changing the Culture
SFU Harbour Centre
May 19, 2017

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Binary Search Game

An *algorithm* is a finite procedure, governed by precise instructions, moving in discrete steps, whose execution requires no insight or intelligence. However, the process of creating such an algorithm, especially to solve complex real-life problems, requires tremendous intuition and creativity.

Today, we developed an algorithm to determine any English word using only Yes/No questions. Our "binary search" algorithm splits the set of possible words into two smaller sets of roughly equal size, which enables us to quickly find our answer. Binary Search satisfies the three key features of an algorithm: accuracy, simplicity, and efficiency.

We can also do this with numbers rather than words. Have a friend pick any whole number less than 1000 and determine that number by asking at most 10 questions. This is the optimal strategy for winning "The Clock Game" on the Price is Right game show. For more information, check out

`http://priceisright.wikia.com/wiki/Clock_Game`

Here are four questions for further investigation.

I write down a secret 10-letter string consisting of capital letters (e.g. YCBZARIGGQ). Your task is to identify this secret codeword by only asking a series of Yes/No questions.

(a) Design a (simple) algorithm for which you can be guaranteed to correctly guess my codeword within $10 \times 5 = 50$ questions. What is the first question you would ask?

(b) For your algorithm above, if I were to pick a random 10-letter string, what would be the *average* number of guesses you would need to correctly guess my codeword?

(c) Using a calculator, we can show that $2^{47} < 26^{10}$. Use this inequality to explain why there cannot exist an algorithm for which you can be guaranteed to correctly guess my 10-letter codeword within 47 questions.

(d) Does there exist an algorithm for which you can be guaranteed to correctly guess my codeword within 48 questions? If so, what is the first question you would ask?

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Ask your own questions, look for your own examples, discover your own proofs.

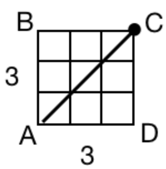
– Paul R. Halmos

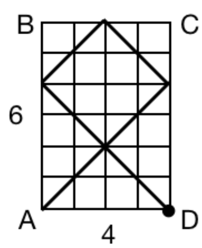
Billiard Ball Mathematics

Topics may include: inductive reasoning, deductive reasoning, planar geometry, integer divisibility, reduced fractions, parity of integers, greatest common divisors, least common multiples, congruence, multiple geometric representations of paths.
Grade level: 3 - 12, post secondary

Hit a ball at a 45° angle from the lower left corner, labeled A, of a rectangular billiards ball table, the ball rebounds off each of the four sides in a new direction but at the same angle. Explore which of the four corners pockets the ball can end up.

Two examples:





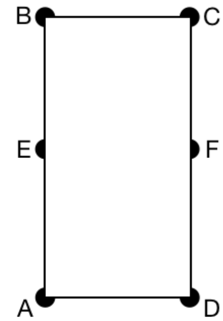
Use graph paper to investigate more examples. Start with tables of small dimensions then work your way up to tables of larger dimensions.

Questions to Consider

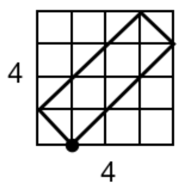
1. Write down a statement about how to determine which of the four corners the ball will end up in terms of the dimensions of the table.
2. Use your statement above to predict which corner the ball will end up for the table with dimensions 108×94?
3. For a general $n \times m$ table how many times does a ball hit an edge before going into a corner hole?
4. For a general $n \times m$ table how many 1×1 squares does the path traverse before going into a corner hole?

Further Investigation

1. Assume there are side pockets at the midpoint of the longer sides of the table (see diagram). Again, a ball is hit at a 45° angle from point A. Explore which of the five pockets the ball will end up. Can you find examples where the ball ends up in each of the five pockets B-F?



2. Starting with the ball at any integer unit along the line from A to D and hitting it at a 45° angle the ball may not end up in a pocket, it may continue in a loop. For example, here is a loop on a 4×4 table.



Is such a loop possible on a table that isn't square. Can you use the dimensions of the table to predict when loops are possible, and how many distinct loops there are?

References

[1] Gardner, Martin. *Martin Garner's 6th book of Mathematics Diversions from Scientific America*. Chapter 2.

[2] Jacobs, Harold R. *Mathematics A Human Endeavor* (3rd ed). Lessons 1 and 2.

Games, Puzzles, and Mathematics (Part 3)
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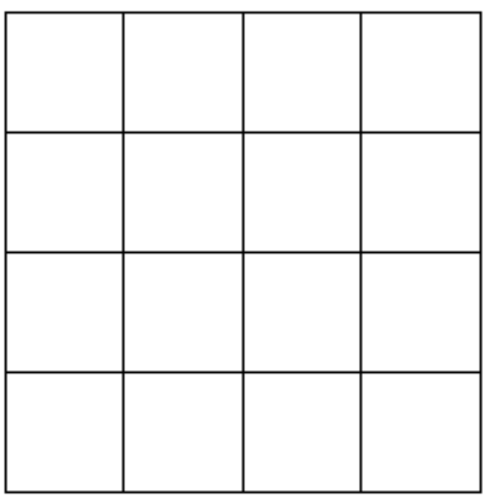
Everyone knows that it is easy to do a puzzle if someone has told you the answer. That is simply a test of memory. You can claim to be a mathematician only if you can solve puzzles that you have never studied before. That is a test of reasoning.

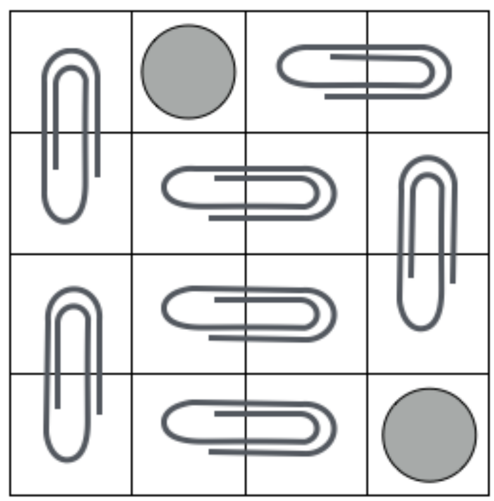
– W.W. Sawyer, Mathematician's Delight

Tiling Problems

A 2-Player Game: Pennies and Paperclips

This is a two player game. Player A begins by placing two pennies on any two squares of the 4×4 board. Player B then places paper clips on the board to cover the remaining squares. Each paperclip covers two adjacent squares, and the paperclips may not overlap each other. See the sample game in the figure on the right. Player B wins if they can cover the remaining squares with paper clips, otherwise Player A wins.





Play a few rounds of the game with your neighbour on the 4×4 board above. Alternate taking turns being Players A and B. Investigate how the placement of the pennies can affect who wins the game.

Extension to 8×8 board: Now play a few rounds on the 8×8 chessboard on page 3. What do the colours of the squares have to do with whether Player A or Player B will win?

Extension to 4 Pennies: Try having Player A place four pennies on the 8×8 board (covering up two squares of each colour). What can you say about this four penny version of the game? Who will win, and under what conditions?

Tiling Questions

1. If you remove two diagonally opposite corners from a 8×8 chessboard can you cover the resulting 62 squares with dominoes?

domino

2. If you remove any two squares of opposite colours from a 8×8 chessboard can you cover the resulting 62 squares with dominoes?
3. If you remove one corner from a 8×8 chessboard can you cover the resulting 63 squares with straight triominoes? If so, how? If not, why not?

triomino

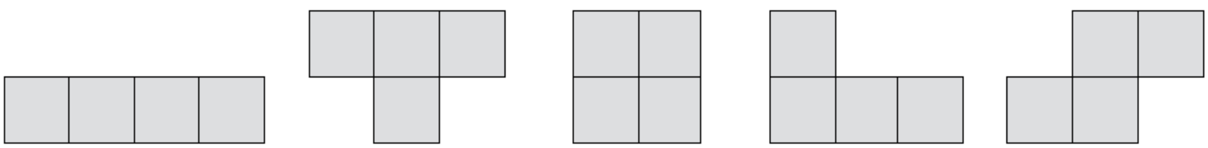
4. For how many of the 64 squares on an 8×8 checkerboard is it true that if we cut out that square, the resulting board, with 63 squares, can be tiled with exactly twenty-one 1×3 triominoes?

Another 2-Player Game: Domineering

A two player game in which each player has a collection of dominoes which they place on the grid in turn, covering up squares. Player A plays tiles vertically, while Player B plays horizontally. The first player who cannot move loses.

Further Investigations with Tetrominoes

1. Can a 4×5 board be covered using the 5 tetromino pieces shown below?



straight tetromino   T-tetromino   square tetromino   L-tetromino   skew tetromino

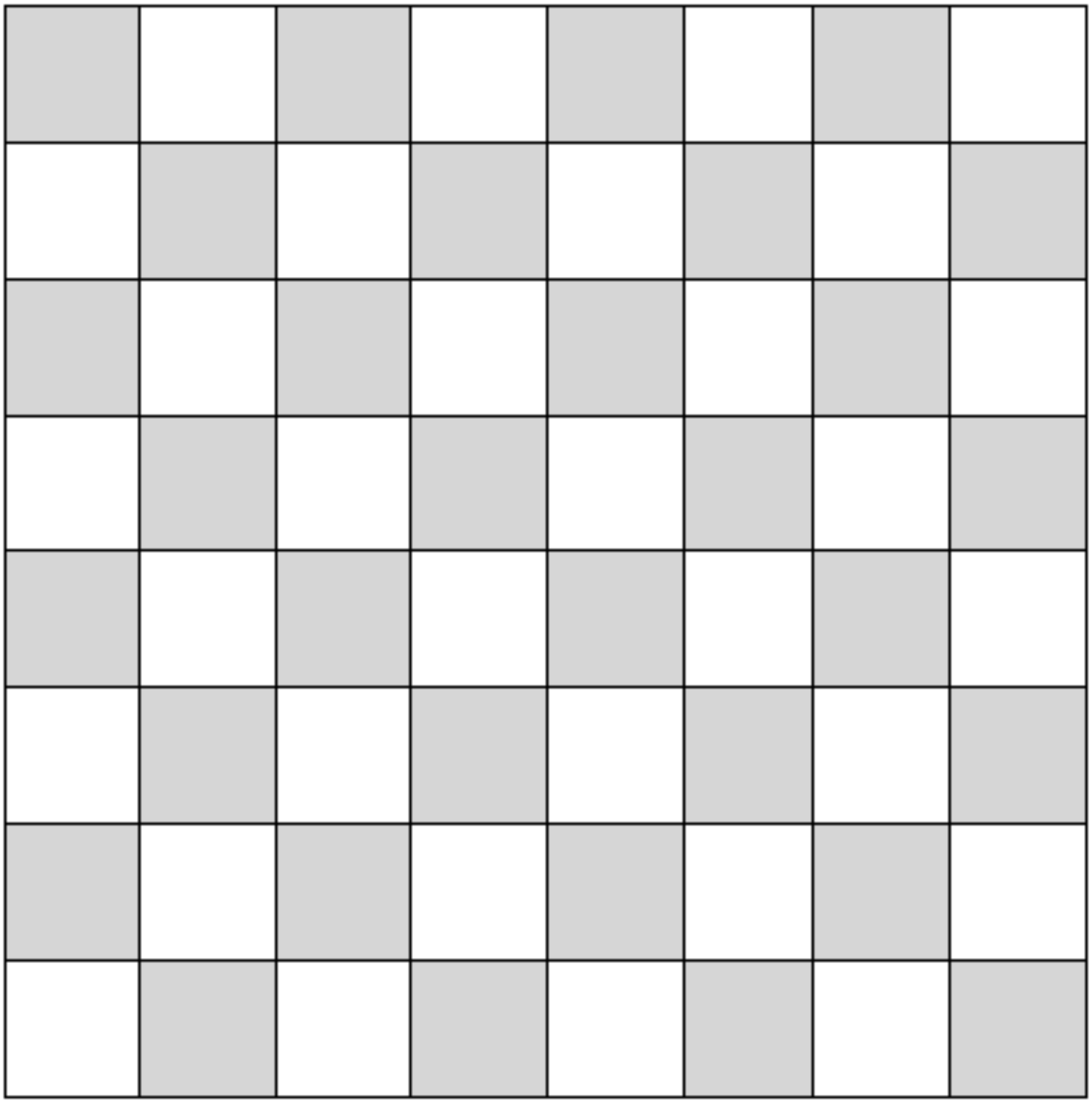
2. Can an 8×8 chessboard be covered by 15 T-tetrominoes and two dominoes?
3. Can an 8×8 chessboard be covered entirely with straight tetrominoes? How about with square tetrominoes, or T tetrominoes, or L tetrominoes?
4. Show that an 8×8 chessboard cannot be covered by 15 L-tetrominoes and one square tetromino. (hint: colour the board differently than a standard checker board.)
5. Show it is impossible to cover the 8×8 board with one square tetromino and any combination of straight and skew tetrominoes.
6. Can a 10×10 chessboard be covered by 25 T-tetrominoes?
7. A 10×10 chessboard cannot be covered by 25 straight tetrominoes.
(hint: colour the board differently than a standard checker board.)

References

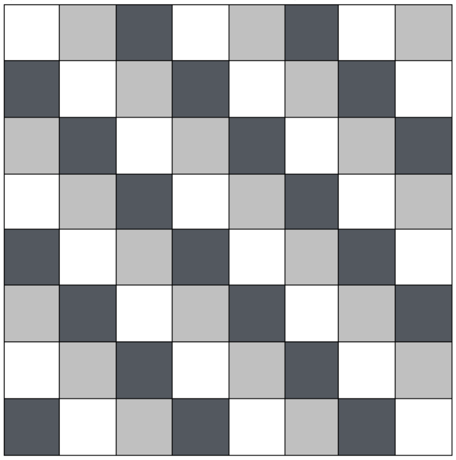
[1] Golomb, Solomon W. *Polyominoes: Puzzles, Patterns, Problems, and Packings*. Princeton Scientific Library.

[2] Jacobs, Harold R. *Mathematics A Human Endeavor* (3rd ed). Freeman & Co. Lesson 5.

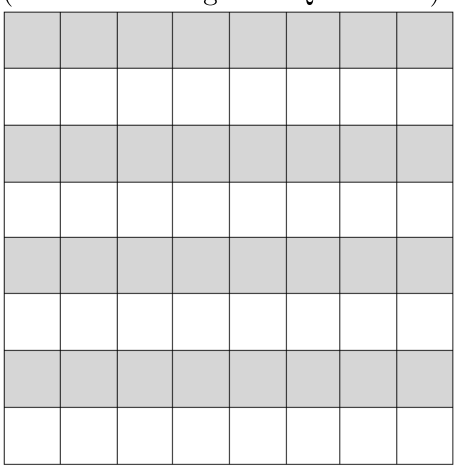
Pennies and Paperclips on an 8×8 board:



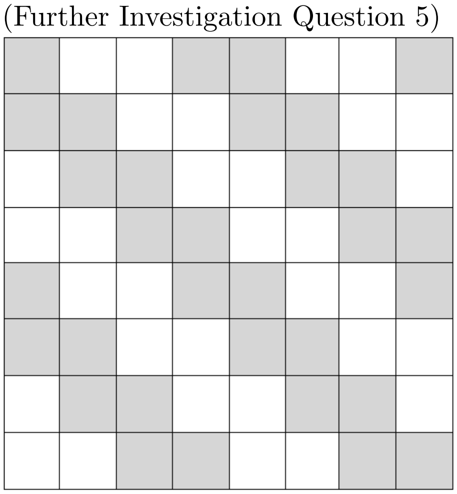
Colouring used to study coverings by straight triominoes.
(Tiling Question 3 & 4)



Colouring used to study coverings by 15 L tetrominoes and 1 square tetromino.
(Further Investigation Question 4)



Colouing used to study coverings by straight and skew tetrominoes and 1 square tetromino.
(Further Investigation Question 5)



Games, Puzzles, and Mathematics (Part 4)
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Dots and Boxes Game

"Dots and Boxes" is a two-player game where players take turns joining a line between two dots that are adjacent, either horizontally or vertically. A player that completes the fourth side of a square (or box) gets a point, and must play again. When all the boxes have been completed, the game ends. Your goal is to score more points than your opponent.

This is an example of a *combinatorial game*, a research topic of tremendous importance, especially in Artificial Intelligence.

Dots and Boxes can be played here:

`https://www.math.ucla.edu/~tom/Games/dots&boxes.html`

Define a "long chain" as a set of three or more adjacent boxes, in which any move in that chain can give all the boxes to the opponent.

Typically at the end of the game, there are two or more long chains. Explain the benefits of the "double-cross" strategy, where we choose not to take all n boxes in a long chain but instead take only $n-2$ of them. Why is this counter-intuitive strategy so powerful?

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Puzzles are made of the things that the mathematician, no less than the child, plays with, and dreams and wonders about, for they are made of the things and circumstances of the world he [or she] lives in.
- Edward Kasner, Mathematics and the Imagination

Thank you so much for coming to our workshop today. Here are some resources that are linked to the activities we shared.

Billiard Table Geometry:
Martin Gardner's 6th book of Mathematical Diversions from Scientific America – Chapter 21
Mathematics A Human Endeavor 3ed, by Harold R. Jacobs – Lessons 1 & 2
http://sigmaa.maa.org/mcst/PosterActivitySessions/documents/BilliardsPoster.pdf

Free Online Domineering Game:
https://www.jasondavies.com/domineering/

Free Online Dots and Boxes Game:
https://www.math.ucla.edu/~tom/Games/dots&boxes.html

Tiling Puzzles (Pennies & Paperclips and beyond):
Mathematics A Human Endeavor 3ed, Harold R. Jacobs – Lesson 5.
Polyominoes: Puzzles, Patterns, Problems, and Packings, by Solomon W. Golomb.
http://www.cut-the-knot.org/do_you_know/chessboard.shtml
http://www.emilangues.education.fr/files/par-rubriques/documents/2008/ressources-pedagogiques/8_Problems_with_dominoes.pdf