Mathematical modeling of complex systems
Part 1. Overview

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Summary

1. What is a complex system ?

Complex system

- System with locally interacting agents
  - emergence of spatio-temporal coordination
  - patterns, structures, correlations, synchronization
  - No leader / only local interactions





2. Examples

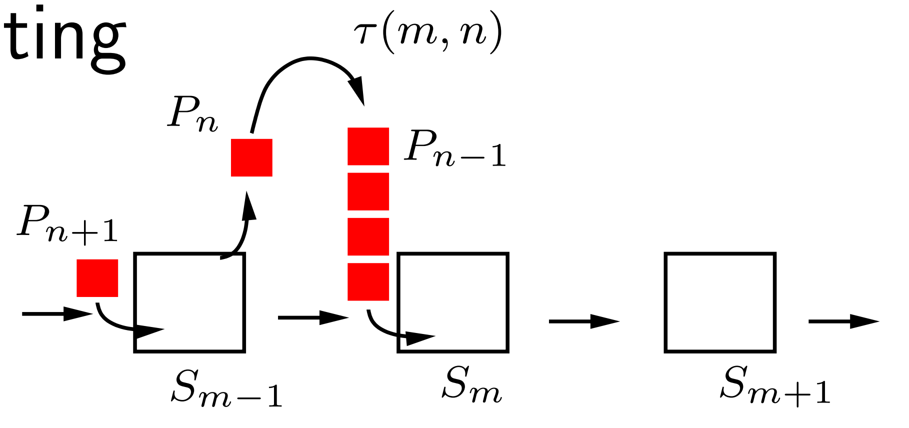
Example 1: Vehicular traffic

- Observations:
  - Free/Congested phase transitions $\longrightarrow$ large scale struct.
  - Spatio-temporal oscillations: stop-and-go waves
  - Coordination and synchronization (e.g. between lanes)
  - Stochasticity, high sensitivity to perturbations

- References:
  - [Daganzo], [Helbing], [Klar et al], Traffic forecast, . . .
  - [D., Rascle et al]: non-overlapping constraint

Example 2: Supply chains

- Network of stations along which parts are circulating
  - factory
  - economic circuit

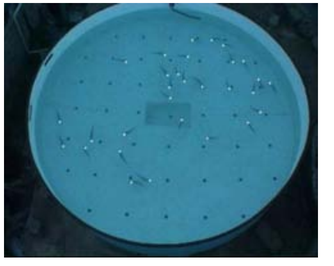


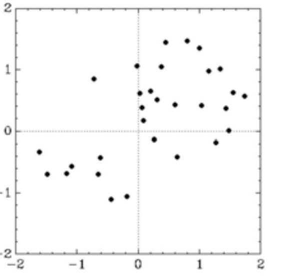
- Observations:
  - Spatial patterns (stocks, depletions)
  - Temporal patterns (economic cycles)
  - Instabilities (bullwhip effect)

- References:
  - [Armbruster, D., Ringhofer], [Klar, Herty et al]: Fluid models

Ex 3-1: collective motion

- Onset of collective motions in gregarious species
  - Example: Fish (experiments by Theraulaz group)





- Observations:
  - trajectory fitting requires new stochastic models
  - cohesion depends on group size
  - alternating leadership
- References:
  - [Gautrais et al], [D. & Motsch] Persistent Turning Walker

Ex 3-2: Herding

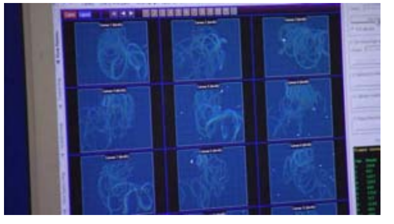
- Herding in mammalian species
  - exp. in Theraulaz group, ANR Project 'Panurge'


- Observations
  - stimulation by neighbours / time lag before response
  - large scale structures: herd size $\ll$ animal size
  - non-overlapping constraint (Navoret et al, in progress)
  - positive feedback reinforcement (e.g. trail formation)

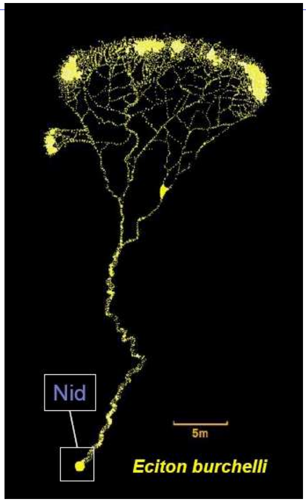
Ex 3-3: Pedestrian traffic

- Pedestrians (ANR project 'Pedigree')
  - density-depend. phase transitions (free/lane/clogging)
  - synchronization, oscillations (e.g. gate crossings)
  - path optimization, collision avoidance
  - cognitive processes
  - control by environmental variables



Ex 3-4: collective decision making



- Ant trail formation (coll. w. C. Jost)
  - positive feedback / pheromone deposition
  - (E. Boissard et al, in progress)
- Constructions by social insects (coll. with G. Theraulaz & C. Jost)
  - complex structures (sponge-like, layers, . . .)
  - organization & functionalities
  - temporal dynamics
  - (C. Sbai et al, in progress)

Striking features

- Self-organization, pattern formation, time synchronization,
  - large-scale complex and evolutive structures
  - organization emerges from local interactions only
  - no leader : Stigmergy [Grassé, 1959]
- Aggregation, clogging, spatial constraint
  - transition from compressible to incompressible regime
- Stochasticity in space & time
  - structures build up from local fluctuations
  - role of feedback (activator-inhibitor, cf Turing)

3. Models

3 classes of models

- Microscopic (particle like)
  - Individual-Based Models (IBM's)
  - state of each individual followed in time
  - coupled Ordinary or Stochastic Differential Equations
- Mesoscopic (kinetic)
  - probability distribution of individuals in state-space
  - e.g. (position, velocity, activity)
- Macroscopic (continuum-like)
  - density, mean velocity, mean activity, . . .
  - not conservation eqs in general (except mass . . .)

Example: ant displacement

- Particle level:

$$\dot{X} = u(\theta)$$
$$d\theta = \sqrt{2b}\, B_t$$

- Kinetic level:

$$\partial_t f + u(\theta) \cdot \nabla_x f = b\, \partial_\theta^2 f$$

- Continuum level:

$$\partial_t \rho = D \Delta \rho$$

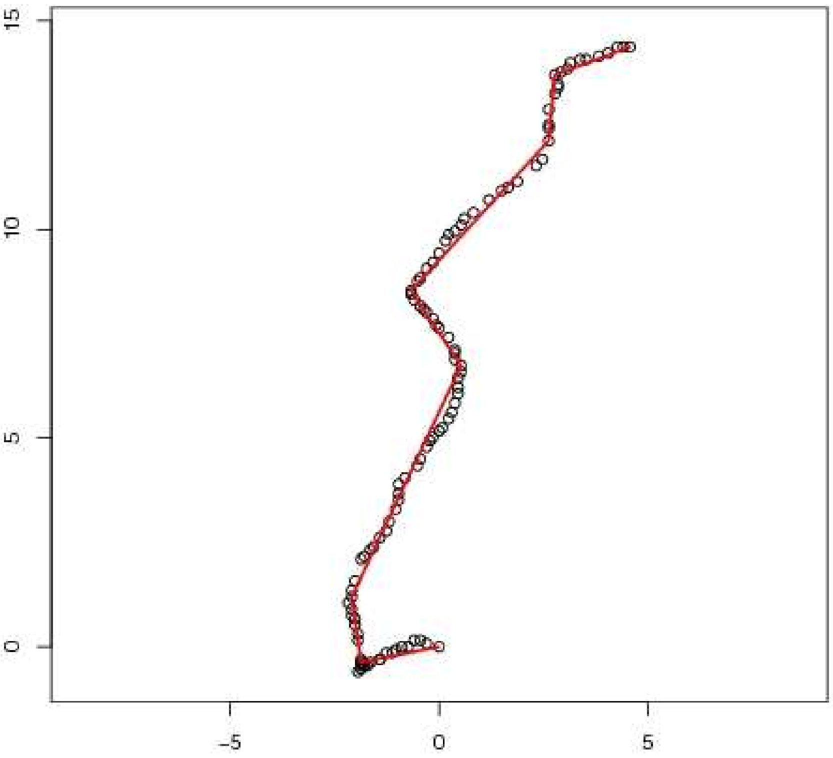
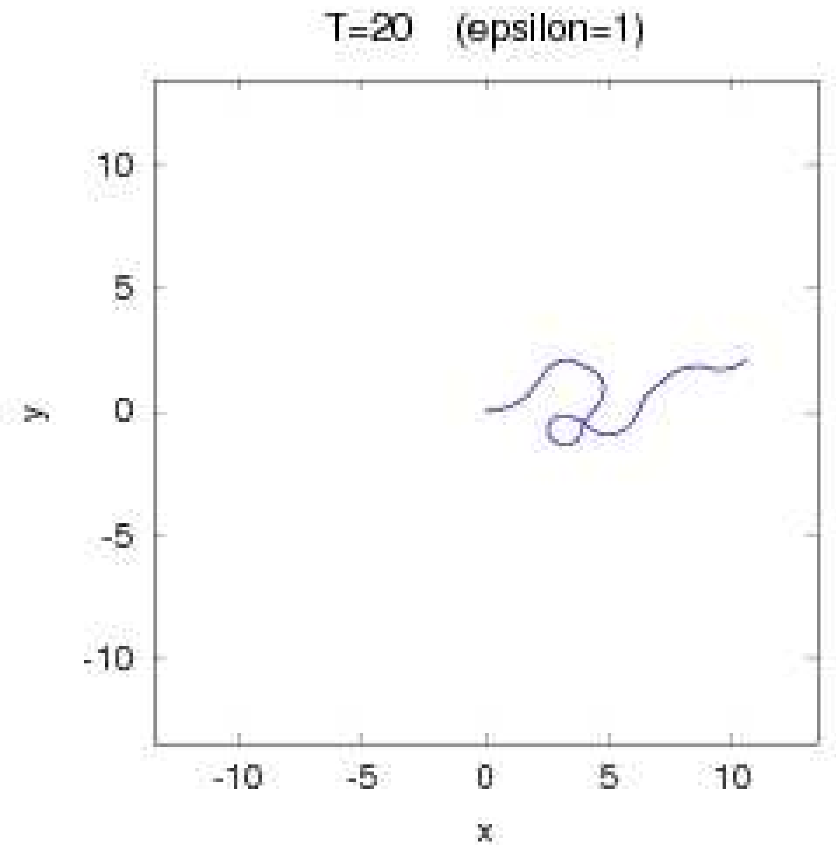


Illustration (Persistent Turning Walker)



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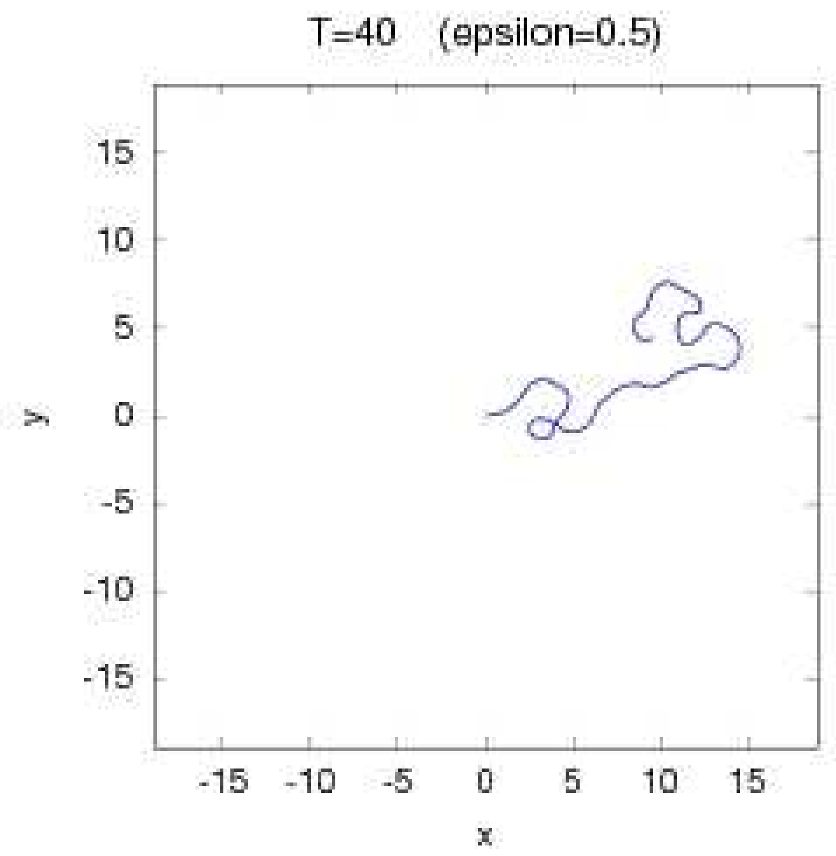

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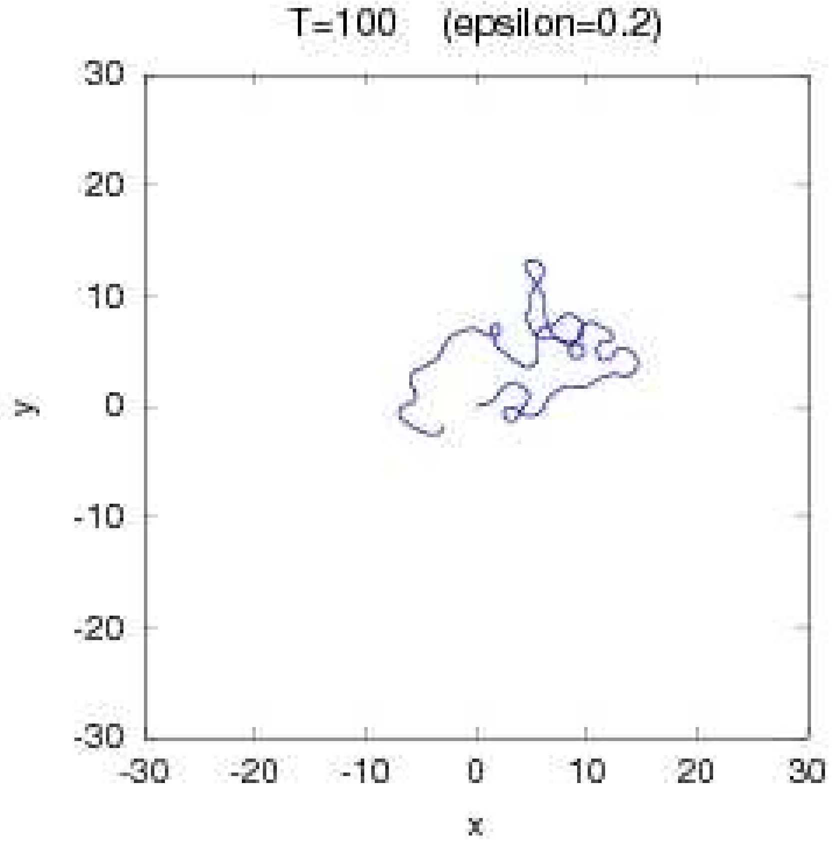

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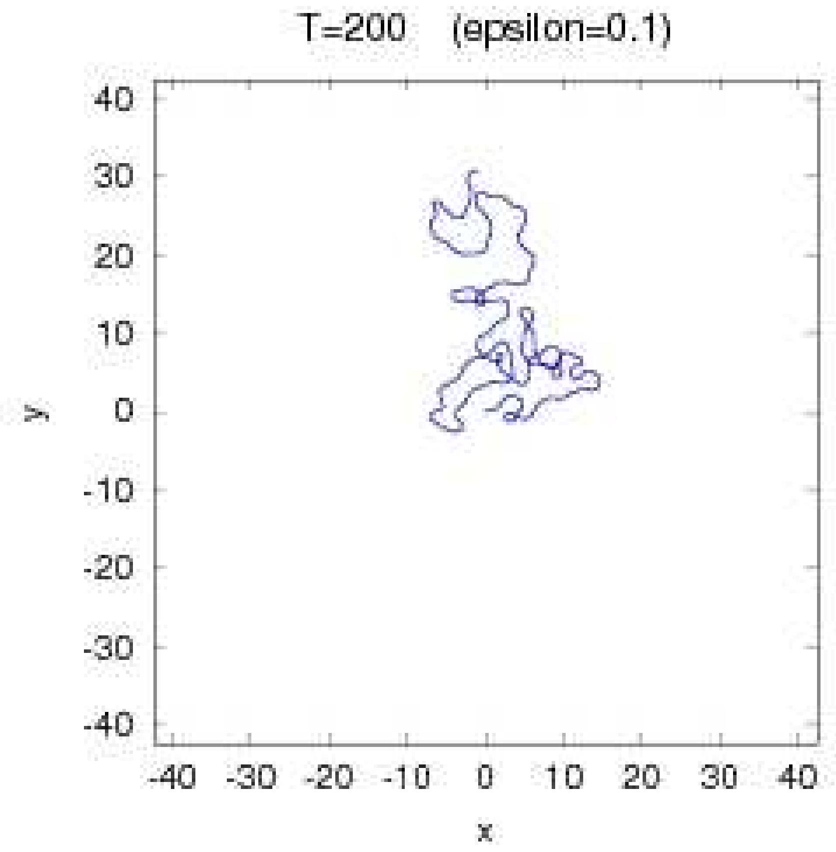

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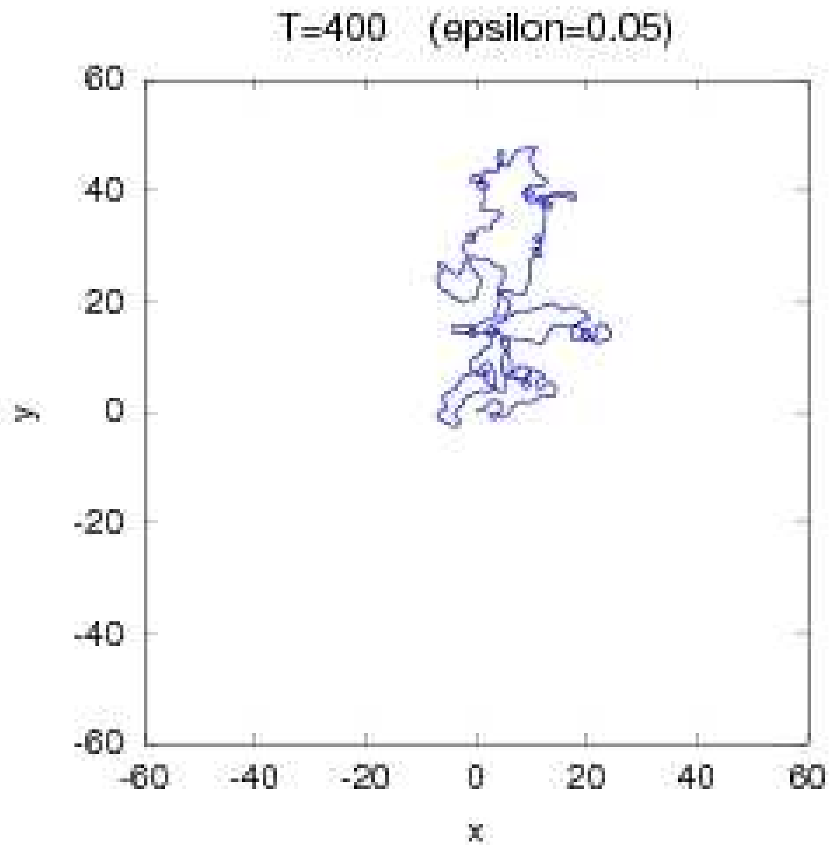

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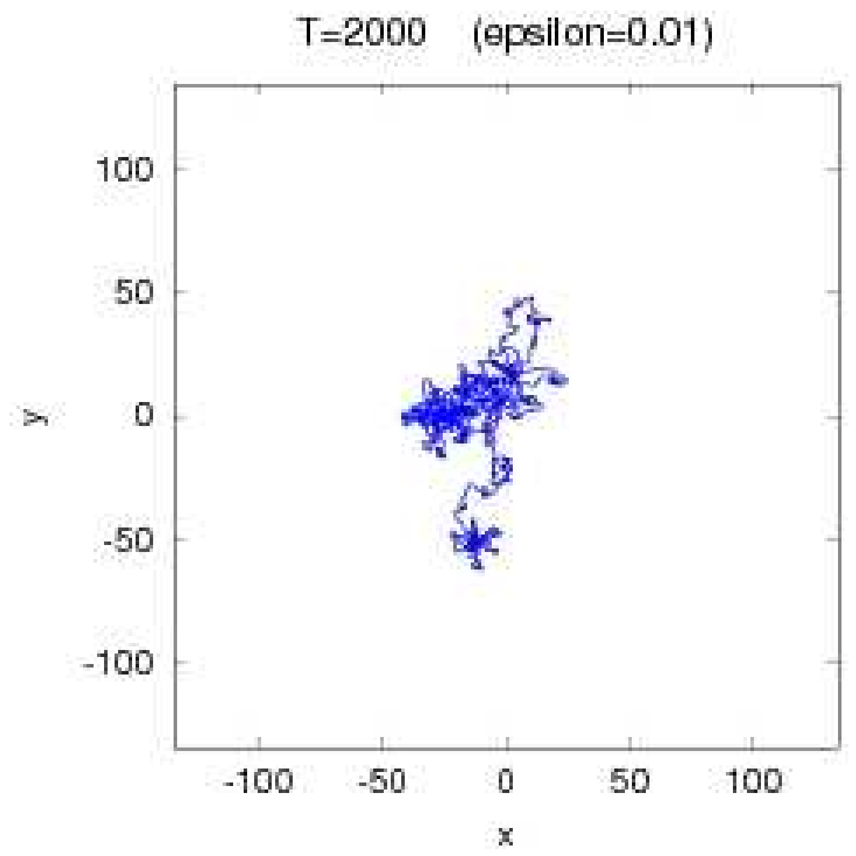

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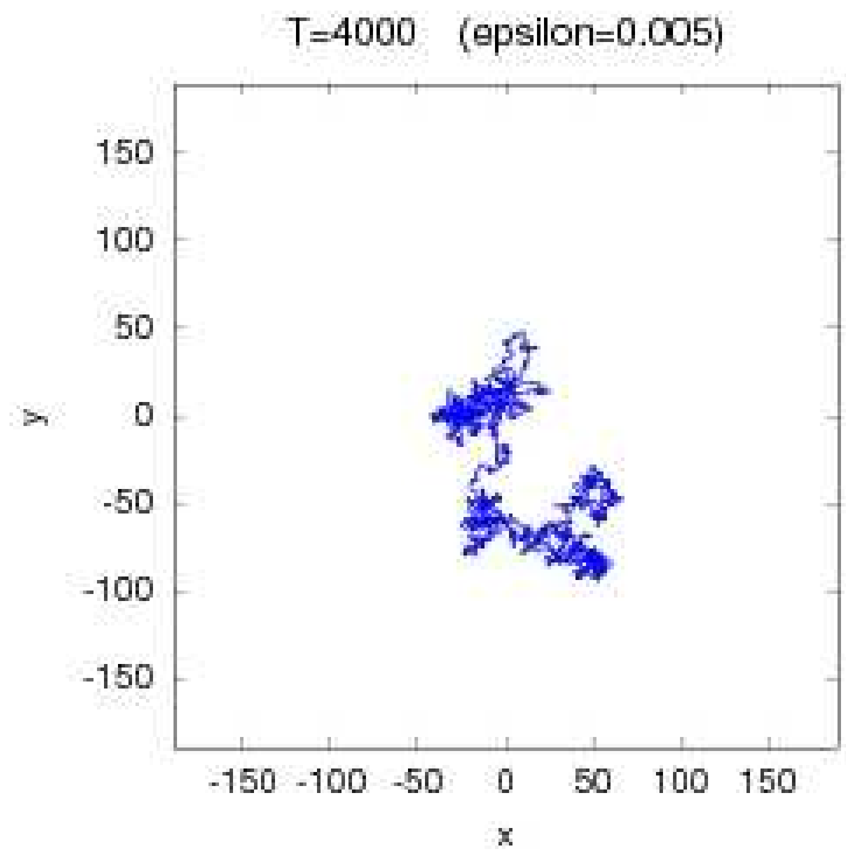
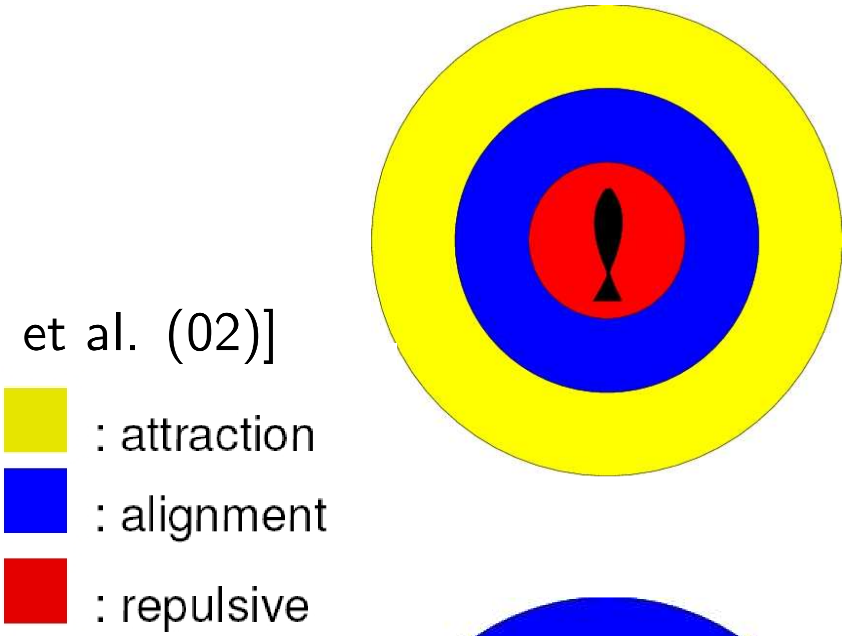
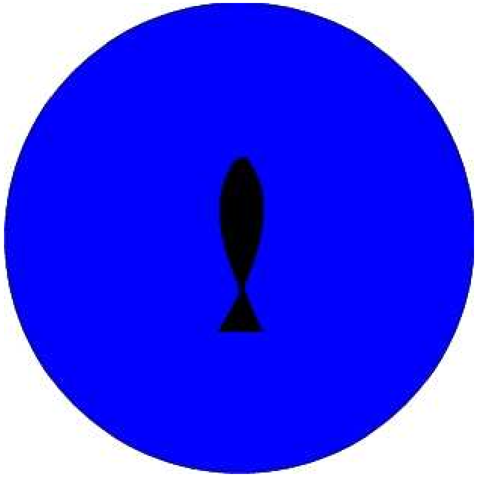


Model of cohesive displacement

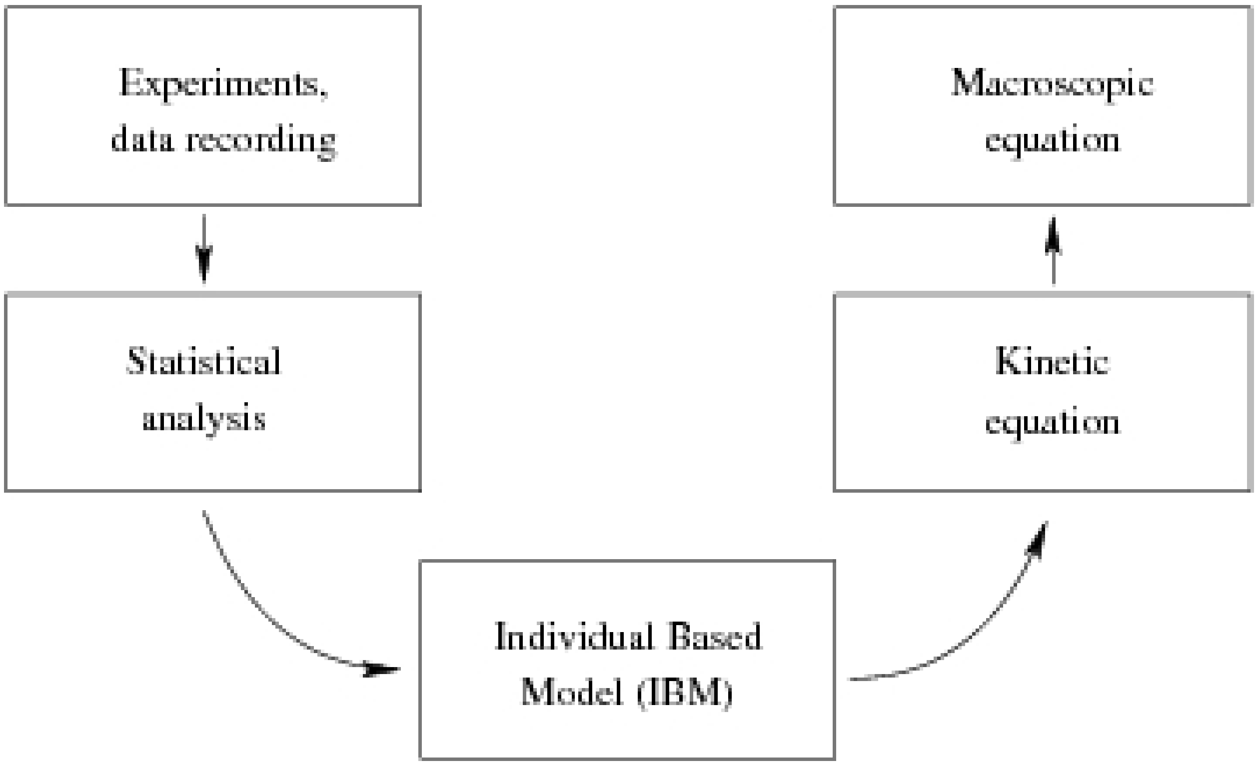
- 3-zones model
  - [Aoki (82)] [Reynolds (86)]
    [Huth & Wissel (92)] [Couzin et al. (02)]



- Alignement only
  - [Vicsek et al (95)]



Data analysis - modeling loop



Pros & Cons

- Pros for IBM's
  - easy to implement
  - behavioral rules can be directly incorporated
- Pros of continuum models
  - computational efficiency for large systems
  - parameter identification, control & optimization
  - morpho-genesis & morpho-analysis easier

Validity of macroscopic approach

- Large statistics
  - validity for small groups ?
  - statistical answer
- Independent particle assumptions
  - chaos assumption
  - when $N \longrightarrow \infty$, particle are nearly independent
  - description by 1-particle distribution valid
  - validity for self-organization processes ?

4. What are we looking for ?

Predict is not explain (R. Thom)

- Probe the systems by means of the models
  - use models with minimal set of parameter
  - which exhibit 'universality' features
  - to provide clues for the observed structures
- Can microscopic diversity be encoded in macroscopic universality ?
  - large variety of microscopic behaviours
  - result in the same kind of macroscopic behaviour
  - e.g. diffusion (fractional), transport, nonlinearity, . . .

What we would like to understand

- What parameters at the micro-level determine
  - the class of macroscopic models
  - the qualitative behavior (e.g. line formation, . . .)
  - the quantitative features (e.g. scaling laws, . . .)
- Are macroscopic descriptions still valid ?
  - is propagation of chaos still true ?
  - what if chaos assumptions fails ?
- Animal models ?
  - labs for stochastic collective motion without leaders
  - observations and models useful for other fields (neuro-science, engineering, social sciences, . . .)