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A New Probability Inequality and Concentration Results

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We prove a new probability inequality. Using it, we prove the best possible concentration for stochastic bin-packing with discrete item sizes settling a question of Talagrand's and others. We deal with several problems on sparse random graphs—for the chromatic number, we prove the first exponential tail bounds and for the number of triangles, we prove the first exponential tail bounds for small deviations (starting with $O(1)$ standard deviations). We also deal with number of s cliques for s almost upto $O(\log n)$. For the Longest Increasing Sequence problem, the traveling Salesperson problem and some others we give simple proofs matching Talagrand's results. We also prove the Johnson–Lindenstruss theorem on random projections. Our probability inequality can be viewed as a substantial strengthening of Azuma's. It has 3 salient features : Instead of an absolute bound on each variable as in Azuma's inequality, we will assume only upper bounds on conditional moments of each variable (conditioned on the previous ones). Second point : We use information on "typical" as well as "worst-case" conditional moments. Point 3 : we assume a weaker condition than Martingales, namely only a negative correlation which makes it applicable to all negatively associated random variables among others. Some comparison with traditional inequalities like Burkholder's and Efron-Stein will be given.