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Mixing Times of Random Walks Based on Conjugacy Classes of the Symmetric Group

Nathanael Berestycki

DPMMS
University of Cambridge
Wilberforce Rd.
Cambridge, CB4 1JP
UK

N.Berestycki@statslab.cam.ac.uk

Consider a random walk on the permutation group obtained by performing random transpositions at each step. We provide a purely probabilistic proof of a famous result of Diaconis and Shahshahani (1981) that the mixing time of this walk is $(1/2)n\log n$. The proof is based on an explicit coupling argument as well as on a connection to the Erdos–Renyi random graph process. This argument may be generalized to random walks whose step distribution is uniform on a given conjugacy class, that is, on the set of all permutations having a given cycle decomposition. We find that the mixing time of such a random walk is $(1/C)n\log n$, where C is the size of the support of the conjugacy class. This solves a conjecture of Roichman (1996).

This is joint work with Oded Schramm and Ofer Zeitouni.