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The Early Evolution of the H-free Process

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The H-free process, for some fixed graph H, is the random graph process defined by starting with an empty graph on n vertices and then adding edges one at a time, chosen uniformly at random subject to the constraint that no H subgraph is formed. Let G be the random maximal H-free graph obtained at the end of the process. When H is strictly 2-balanced, we show that for some $c > 0$, with high probability as $n \to \infty$, the minimum degree in G is at least $cn^{1-(v_H-2)/(e_H-1)}(\log n)^{1/(e_H-1)}$. This gives new lower bounds for the Turan numbers of certain bipartite graphs, such as the complete bipartite graphs $K_{r,r}$ with $r \geq 5$. When H is a complete graph K_s with $s \geq 5$ we show that for some $C > 0$, with high probability the independence number of G is at most $Cn^{2/(s+1)}(\log n)^{1-1/(e_H-1)}$. This gives new lower bounds for Ramsey numbers $R(s,t)$ for fixed $s \geq 5$ and t large. We also obtain new bounds for the independence number of G for other graphs H, including the case when H is a cycle. Our proofs use the differential equations method for random graph processes to analyse the evolution of the process, and also give further interesting information about the structure of the graphs obtained, including asymptotic formulae for a broad class of extension variables.

This is joint work with Tom Bohman.