

ATELIER « COMBINATOIRE, RANDOMISATION, ALGORITHMES ET PROBABILITÉS »
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Positive Projections

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If A is a set of n positive integers, how small can the set $\{a/(a,b) : a,b \in A\}$ be? Here as usual (a,b) denotes the HCF of a and b . This elegant question was raised by Granville and Roesler, who also reformulated it in the following way : given a set A of n points in \mathbb{Z}^d , how small can $(A - A)^+$, the projection of the difference set of A onto the positive orthant, be?

Freiman and Lev gave an example to show that (in any dimension) the size can be as small as $n^{2/3}$ (up to a constant factor). Granville and Roesler proved that in two dimensions this bound is correct, i.e. that the size is always at least $n^{2/3}$, and asked if this held in any dimension. Holzman, Lev and Pinchasi showed that in three dimensions the size is at least $n^{3/5}$, and that in four dimensions the size is at least $n^{6/11}$ (up to a logarithmic factor), and they also asked if the correct exponent is always $2/3$.

After some background material, the talk will focus on recent developments, including a negative answer to the $n^{2/3}$ question.

This is joint with Béla Bollobás.