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The Unscaled Limit of Invasion Percolation

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Let G be a finite or infinite connected rooted graph and assign independently to each edge of G a random weight w_e with distribution $\mathrm{Unif}[0,1]$. The invasion percolation cluster is the tree T grown inside G as follows : start with the root, add the lowest weight edge leaving the root, then at each step thereafter add the lowest weight edge leaving the current tree. When applied to finite weighted graphs this procedure is the well known Prim's algorithm for growing a minimum weight spanning tree. Less is known about invasion percolation on infinite graphs ; it is a subject of current research. We study invasion percolation on Aldous' Poisson-weighted infinite tree, which may be viewed as a limit of invasion percolation on either the complete graph K_n (as $n \to \infty$) or the d-ary infinite tree (as $d \to \infty$). Furthermore, this limit picture, which may be obtained via a Poisson point process on the upper right quadrant $[0,\infty) \times [0,\infty)$, provides a suitable setting for short proofs of many interesting properties. We also define a related stationary process which can be seen as capturing the behaviour of invasion percolation far from its starting point. This behaviour is closely linked with that of critical bond percolation, and so serves as an example of self-organised criticality.

This is joint with Louigi Addario-Berry and Ross Kang.