Bacterial Microfluidics: 
The physics and engineering of flagellated bacterial motility

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In collaboration with: 
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Stuff going on in my lab

- Electrospray dynamics
- Cilia and flagella in viscous fluids
- Animal Flight
- Bacterial motility
- droplets and contact lines
Motility of flagellated bacteria


In “real life”:
Newtonian and non-Newtonian media
Motility of flagellated bacteria


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Newtonian and non-Newtonian media

H. Berg, Physics Today, Jan 2000;
Interesting questions

- Physics questions:
  - How fast does a “bacterium” swim in a Newtonian fluid?
  - How fast does a “bacterium” swim in a non-Newtonian fluid?
  - How does an elastic flagellum interact with a viscous fluid?
  - What is the flow field associated with rotating flagella?
  - How do adjacent elastic filaments bundle?
  - How do adjacent elastic filaments synchronize using hydrodynamic forces?
  - How important are non-local viscous interactions?

- “Engineering” questions:
  - Can bacteria affect the macroscopic world?
  - Can we harness their motion?
  - Can we control them?
  - etc.

- What next?
 apologies to Picasso, thanks to Christophe Clanet
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Experimental setup (low-Re swimming helix)

fluids with high viscosity:
• silicone oil (Newtonian)
• Polybutene+ Polyisobutylene (viscoelastic)

\[ Re \sim 10^{-3} \]
Fluidic force on a rotating helix (Newtonian)

Fluidic force per unit length saturates with $L > 5 \lambda$

\[ \text{\quad \quad \quad \quad \rightarrow infinitely long helix} \]
Fluidic force on a rotating helix (Newtonian)

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$\rightarrow$ infinitely long helix
Force-free swimming of a rotating helix (Newtonian)

free swimming state \( (F_{HD} = 0) \)

sensitivity of force \(~ 10^{-5}\) N

sensitivity of swimming speed \(~ 0.3\) \(\mu m/s\)
Free swimming speed at different rotation rate
Computation of free swimming speed

1. Resistive force theory
   *(no long-range interactions)*

   \[ f_n = \mu C_n u_n, \quad f_s = \mu C_s u_s, \]

   when swimming freely,

   \[ f_n \sin \theta + f_s \cos \theta = 0 \]

   \[ \frac{V_F}{\Omega R} = \frac{(C_n/C_s - 1) \sin \theta \cos \theta}{(C_n/C_s - 1) \sin^2 \theta + 1} \]

2. Slender body theory
   *(incl. long range, no thickness effects)*

   \[ V_F = \frac{f}{8\pi \mu} \int_{-\infty}^{\infty} \frac{\varphi \sin \varphi \cos \theta \csc^2 \theta}{(\xi(\varphi, \theta))^{3/2}} \, d\varphi, \]

   \[ \Omega R = \frac{f}{8\pi \mu \sin \theta} \int_{-\infty}^{\infty} \left( \frac{\cos \varphi}{(\xi(\varphi, \theta))^{1/2}} + \frac{\sin^2 \varphi}{(\xi(\varphi, \theta))^{3/2}} \right) \, d\varphi \]
Comparison between experiment and theory

\[ \frac{V_F}{\Omega R} \]

\[ \theta = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi \]

- Resistive Force Theory
- Slender Body Theory

\[ d/L = 0.05 \]
\[ d/L = 0.026 \]

\( d \): diameter of the cross-section of helical fiber

\( L \): arc-length within a pitch

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Wednesday, February 23, 2011
Rotating helix in Viscoelastic fluids

Polyisobutylene (PIB) suspended in Polybutene (PB)

![Graph showing Rheology: Boger fluid]

Relaxation time: $\tau = 0.4$ s

![Graph showing Dynamic Moduli $G'$, $G''$]
Rotating helix in Viscoelastic fluids

free swimming speed of rotating helix in viscoelastic fluids

\[ \frac{V_F}{V_0} \]

\[ \Omega \text{ (rad/s)} \]

Next: fluid with stronger elasticity
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- What next?
Elastic filament interacting with a viscous flow

- Motivation
  - Understand simplified behavior of flexible filaments in viscous flows
  - Look at a simpler model for microrobotic propulsion
  - Understand previous numerical experiments
    • (Manghi, Schlagberger & Netz, PRL 2006)

Viscous fluid (Re << 1)

Root set at fixed angle, $\theta$

Filament rotates in two modes:
  - constant torque: $M$
  - constant velocity: $\omega$

Elastic filament
Shape Bifurcation of an Elastic Rotating Rod

Experiment Parameters:
- bending modulus $A \approx 3 \times 10^{-3}$ N/m$^2$
- radius of rod $a = 2.5$ mm
- viscosity $\eta \approx 110$ Ns/m$^2$
- length of rod $L = 200 - 280$ mm
- tilted angle $\dot{\theta} \approx 20^\circ - 30^\circ$
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Reconstructed motion from expt. data
Reconstructed motion from expt. data

Motion around the bifurcation
Theory

- Stokes flow (reversible flow)
  \[ \mu \nabla^2 \mathbf{v} = \nabla p \]
  \[ \nabla \cdot \mathbf{v} = 0 \]
  - Analyze in the rotating frame (steady)
  - No forces due to non-inertial frame
- disregard twist
  - Only bending stiffness
- resistive force theory:
  \[ f_n = \mu C_n u_n, \quad f_s = \mu C_s u_s, \]
- Hydrodynamic forces are local
- balance of forces and moments
- Key non-dimensional parameters:
  \[ \theta, \quad \chi = \mu \omega \frac{L^4}{A} = (L/l_v)^4, \quad M = M_{mot} L/A \]
Comparison between theory and experiment

Intermediate regime: \( M_mL/A \sim \chi^{1/2} \)

High torque: \( M_mL/A \sim \chi^{1/4} \)

Low torque: \( M_mL/A \sim \chi \)

(Qian et al. PRL 2008)
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• What next?
Hydrodynamic interactions

- Cilia and flagella occur in many biological (and engineering) systems.
- Collective motion important for transport and motility
- Interesting questions for flagella and cilia:
  - synchronization through hydrodynamic interactions
  - synchronization vs. chaotic motion
  - role of compliance
  - time-scales for interactions
  - effects of multiple filaments
  - effects of long range hydrodynamic forces
Two-paddle model system

![Diagram of two-paddle model system with symbols and notations]

**TOP VIEW**

- \( \theta_1 \)
- \( \theta_2 \)

**SIDE VIEW**

- \( M_1 \)
- \( M_2 \)
- flexible coupling
- \( 2R \)
- \( 2R \)
- \( D \)
- \( \delta \)
- \( \ell \)
- \( h \)

**Re = \( O(10^{-3}) \)**
Two-paddle interactions

- Rigid shafts – no synchronization
- Rigid shafts – torque mismatch
  - Phase wandering
- Flexible shafts – synchronization!

\[ T_o = 6\pi \mu R^3 / M \]

(Qian et al, PRE 2009)
Numerical simulation - Regularized

\[ \mu \nabla^2 \mathbf{v} - \nabla p + \mathbf{F} = 0 \]
\[ \nabla \cdot \mathbf{v} = 0 \]
\[ \mathbf{F} = \frac{15 \epsilon^4}{8\pi} \frac{1}{(r^2 + \epsilon^2)^{7/2}} \]

(Cortez, 2001)

\[
\begin{pmatrix}
M_1 \\
M_2
\end{pmatrix}
= \begin{pmatrix}
A(\theta_1, \theta_2) & B(\theta_1, \theta_2) \\
B(\theta_1, \theta_2) & A(\theta_1, \theta_2)
\end{pmatrix}
\begin{pmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2
\end{pmatrix}
\]

Reichert & Stark (2005) - flexible coupling leads to synchronization
Simulation and experiment

\[ \frac{\Delta \theta}{\pi} \]

\[ \frac{t}{T_0} \]

symmetric

asymmetric
Analytical approach

velocity on ball $i$, induced by ball $j$

\[ v_i = \frac{F_i}{6\pi \mu a} + \frac{1}{8\pi \mu} \sum_{i \neq j} \left[ \frac{F_j}{|r_{ij}|} + \frac{(F_j \cdot r_{ij})r_{ij}}{|r_{ij}|^3} \right] \]

ignore “far field” term

\[ a \ll R \ll D \]

Sum of forces and moments; for two balls:

\[ -\frac{\Delta M}{M_1} + \Delta \dot{\theta} + \frac{9}{8} \frac{a}{D} \Delta \dot{\theta} \cos \Delta \theta + \frac{3}{8} \frac{a}{D} \cos(2\bar{\theta}) = 0 \]

\[ \frac{\Delta M}{M_1} + 2 - 2\dot{\theta} - \frac{3}{4} \frac{a}{D} \dot{\theta} \left[ -3 \cos \Delta \theta + \cos(2\bar{\theta}) \right] = 0 \]

multiple-scale (slow) evolution equation for phase difference:

\[ \langle \Delta \theta \rangle = -\frac{9}{2} \frac{M_1}{kR^2} \frac{a}{D} \sin \langle \Delta \theta \rangle + \frac{\Delta M}{M_1} \]

I/T time constant for synchronization

Niedermeyer et al. (Chaos, 2008)

Qian et al. (PRE, 2009)
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Motile bacteria as chaotic mixers

- Chaotic advection (Aref 1984)
  - Viscous laminar flow
  - Random switching of “blinking vortices”
- Bacteria have flagella that alternate rotation direction randomly
  - Cell bodies execute random walk
  - Chaotic mixing in zero Re flow?

(Images courtesy of Károly György)
Mixing using freely swimming bacteria

Outlet

Inlet

PDMS Microchannel
Pyrex Glass (0.17 mm)

Length = 28 mm

Width = 200 µm

Depth = 40 µm

Fluorescence
No Fluorescence

24 mm  20 mm  16 mm  12 mm  8 mm  4 mm  0.5 mm

y

x
Theory for standard diffusion.

Diffusion Equation: \[ U \frac{\partial I}{\partial x} = D \frac{\partial^2 I}{\partial y^2} \]

Intensity: \[ I(y, x) = \frac{1}{2} I_0 \text{erfc} \left( \frac{y}{2\sqrt{D\tau}} \right) \]

Intensity Gradient: \[ -\frac{dI}{dy} = \frac{1}{2} \frac{I_0}{\sqrt{\pi D\tau}} \exp\left( -\left( \frac{y}{2\sqrt{D\tau}} \right)^2 \right) \]

Similarity Variable: \[ \tau = \frac{x}{U} \]
Enhanced diffusion due to bacteria

- Diffusion of small molecule rises from 20 – 80 $\text{um}^2/\text{s}$
  - Large molecules and 1 um beads show 50x enhancement
- Diffusion faster than standard “Fickian” diffusion
  - Introduction of new time scale

\[ \frac{I}{I_o} \propto \left( \frac{\tau_o}{\tau} \right)^\alpha \]

(Kim & Breuer, Phys Fluids 2004)
Bacterial carpets and enhanced mixing

Brownian motion

Motion (μm) of 0.5μm fluorescent particles during 1/30 sec

(Darnton, Turner, Breuer & Berg Biophys J. 2003)
Bacterial pumps

- Pumping arises spontaneously
- Direction determined by “last flush”
- “Crystallization” process

(Kim & Breuer, Small 2008)
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(Kim & Breuer, Small 2008)
Bacterial barges

- Rotation: 33 deg/s
- Surface area: 1400 um²
- Long axis: 65 um
- Bacteria: 300

- Translates: 5 um/s
- Rotation: 4.6 deg/s
- Surface area: 4350 um²
- Long axis: 92 um
- Bacteria: 1100
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PDMS barge

Glass substrate
Bacterial barges

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Summary:

- Force-free swimming speed measured:
  - newtonian fluids
  - viscoelastic fluids underway

- Fluids-filament interactions:
  - coiling of an elastic filament due to viscous stresses

- Synchronization of adjacent filaments
  - hydrodynamic interactions
  - elastic compliance necessary

- multiple paddles illustrate significantly more complex (& chaotic) behavior
• Acknowledgements: Tom Powers, Bin Liu, Min Jun Kim, Qian Bian, Dave Gagnon
• Support: NSF