Bacterial Microfluidics: The physics and engineering of flagellated bacterial motility

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In collaboration with:

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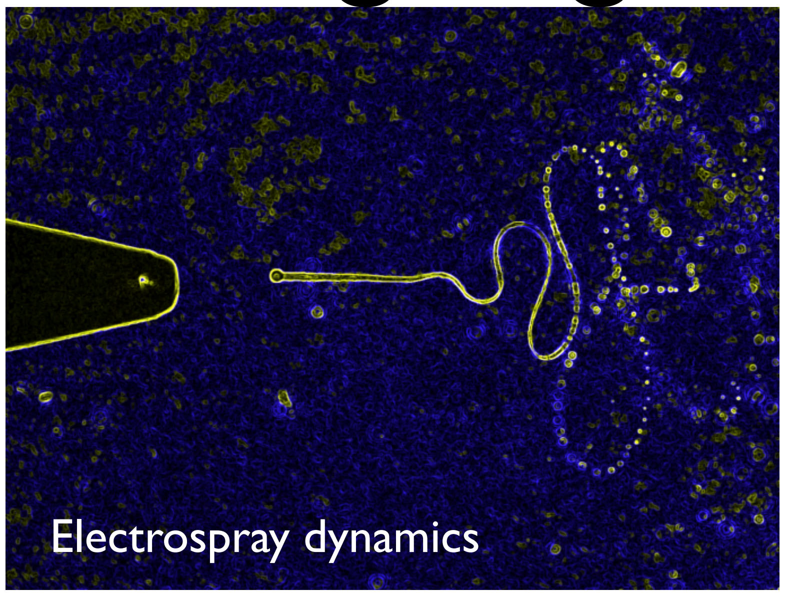
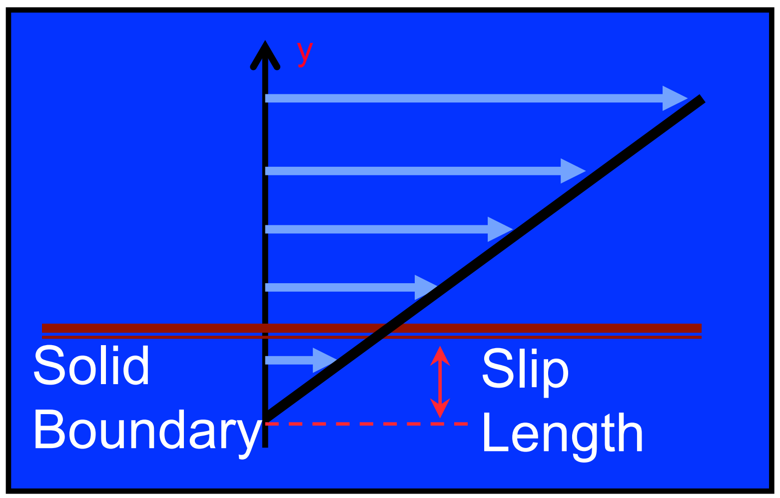
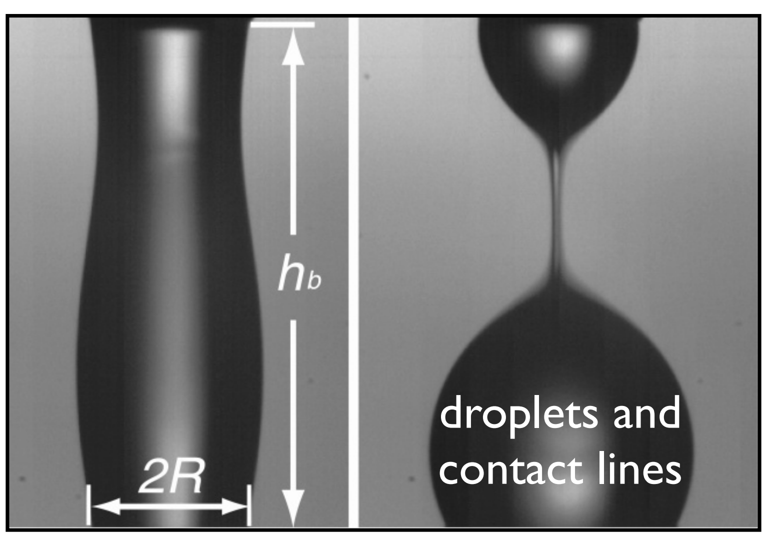
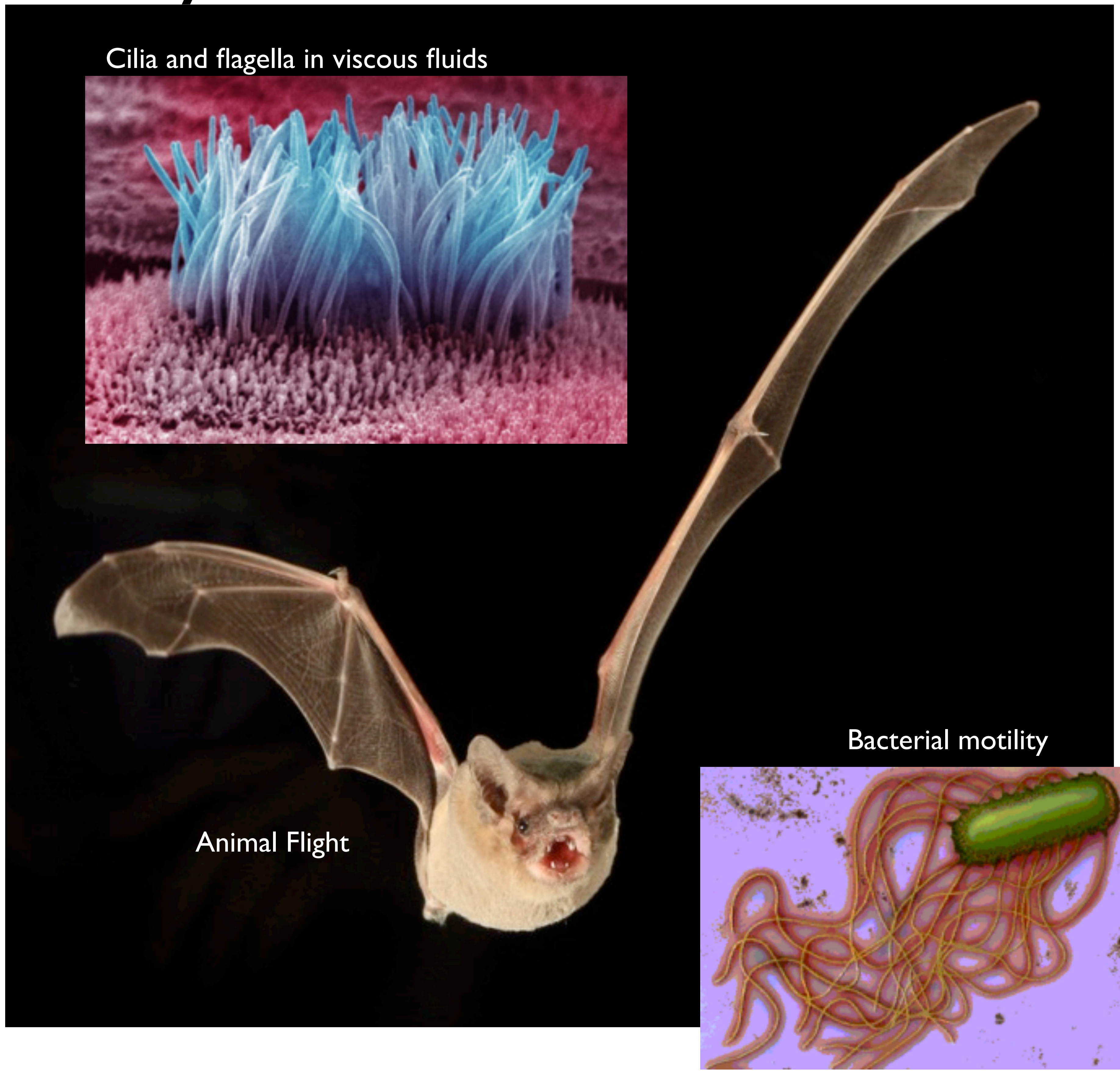
Stuff going on in my lab



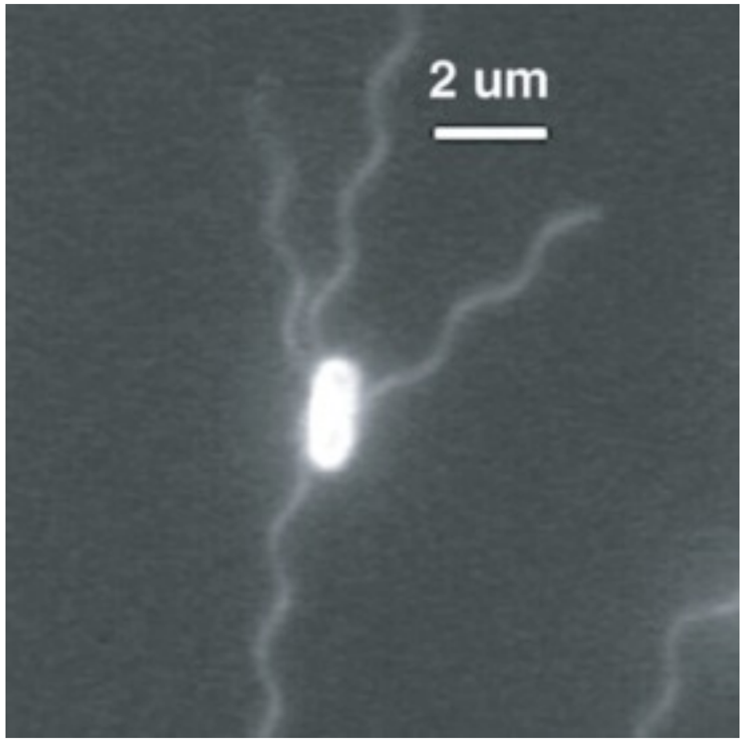


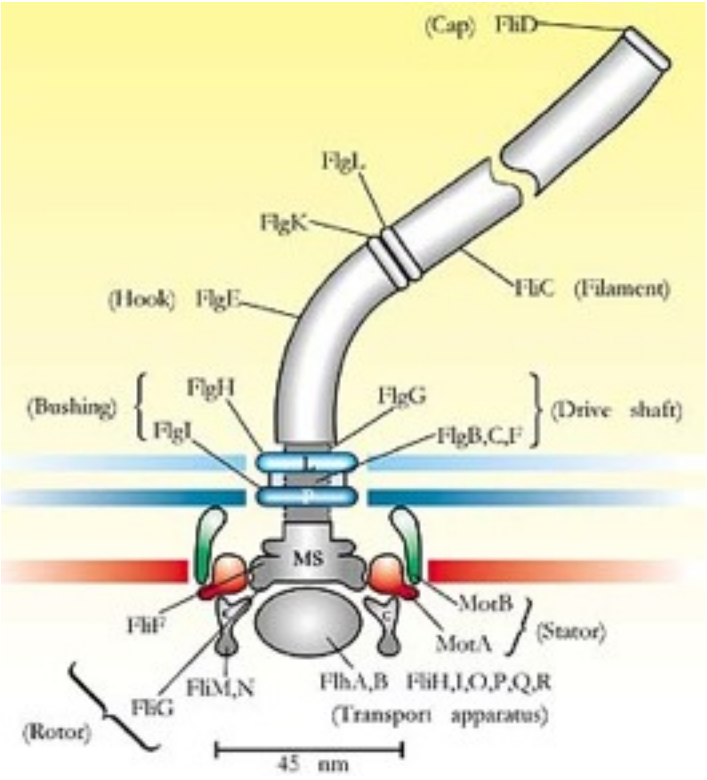


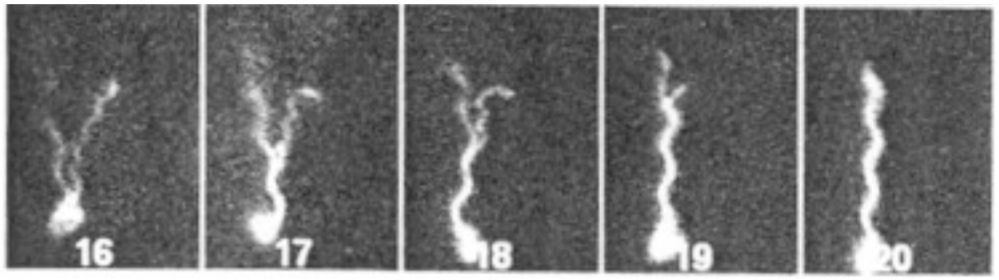


Motility of flagellated bacteria

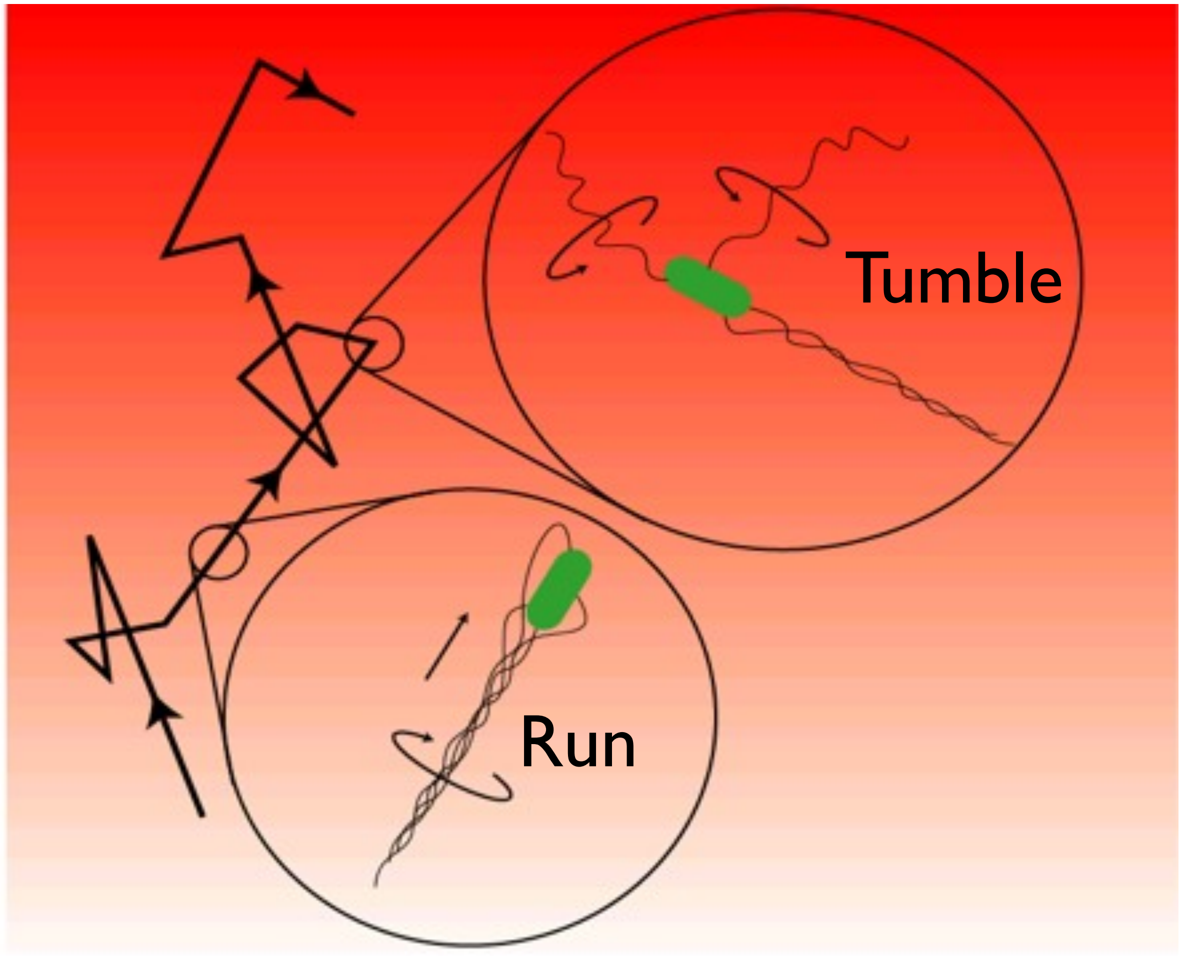
H. Berg, Physics Today, Jan 2000;







L. Turner, W. S. Ryu, and H. C. Berg (2000)



In "real life":
Newtonian and non-Newtonian media

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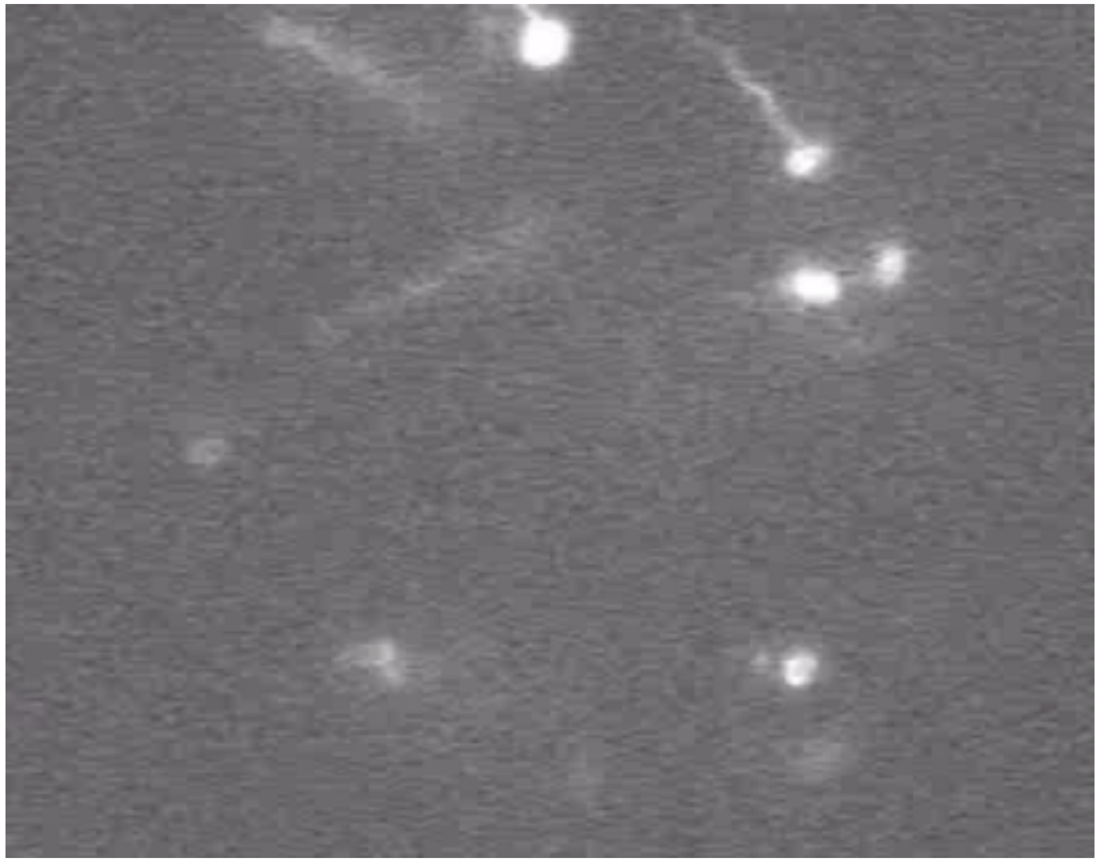

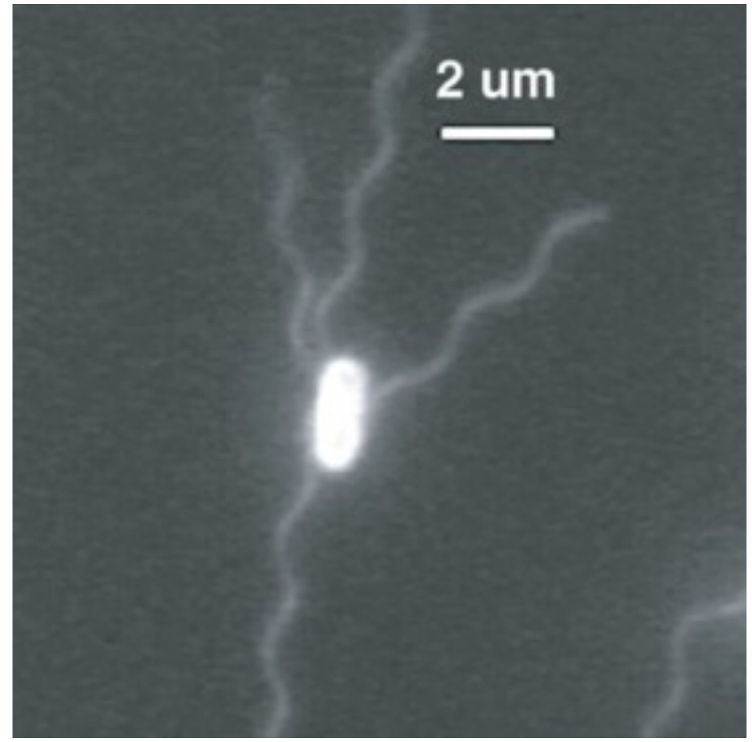

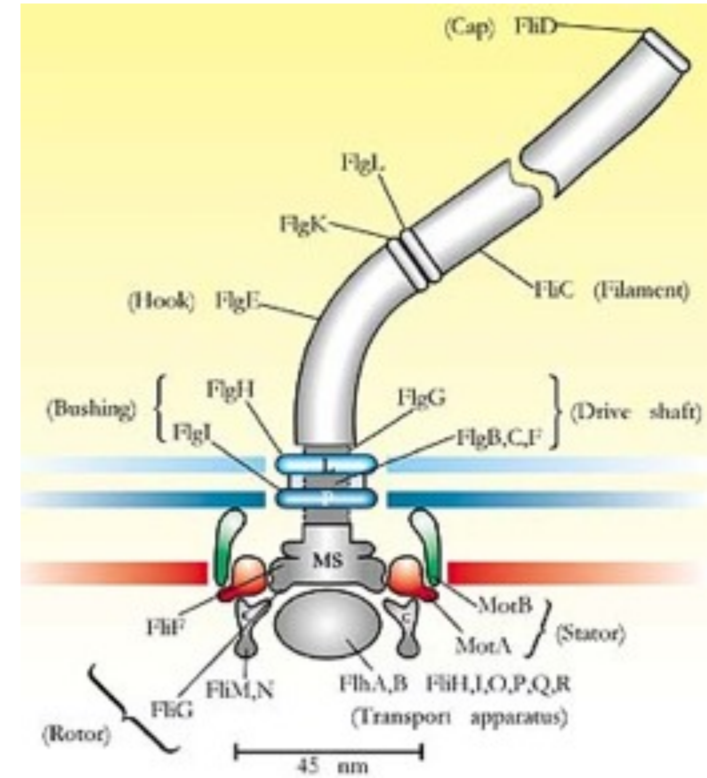

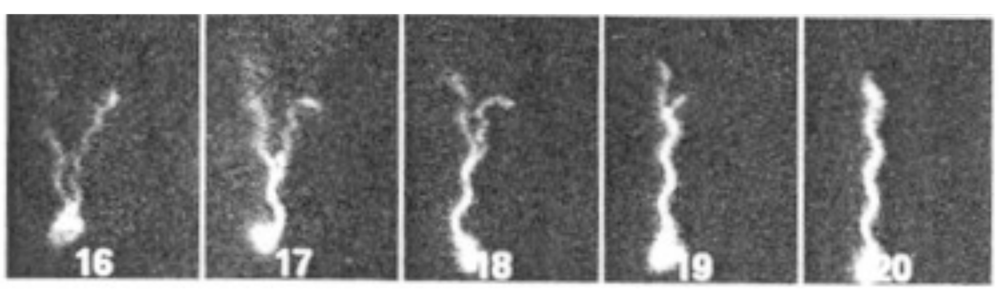

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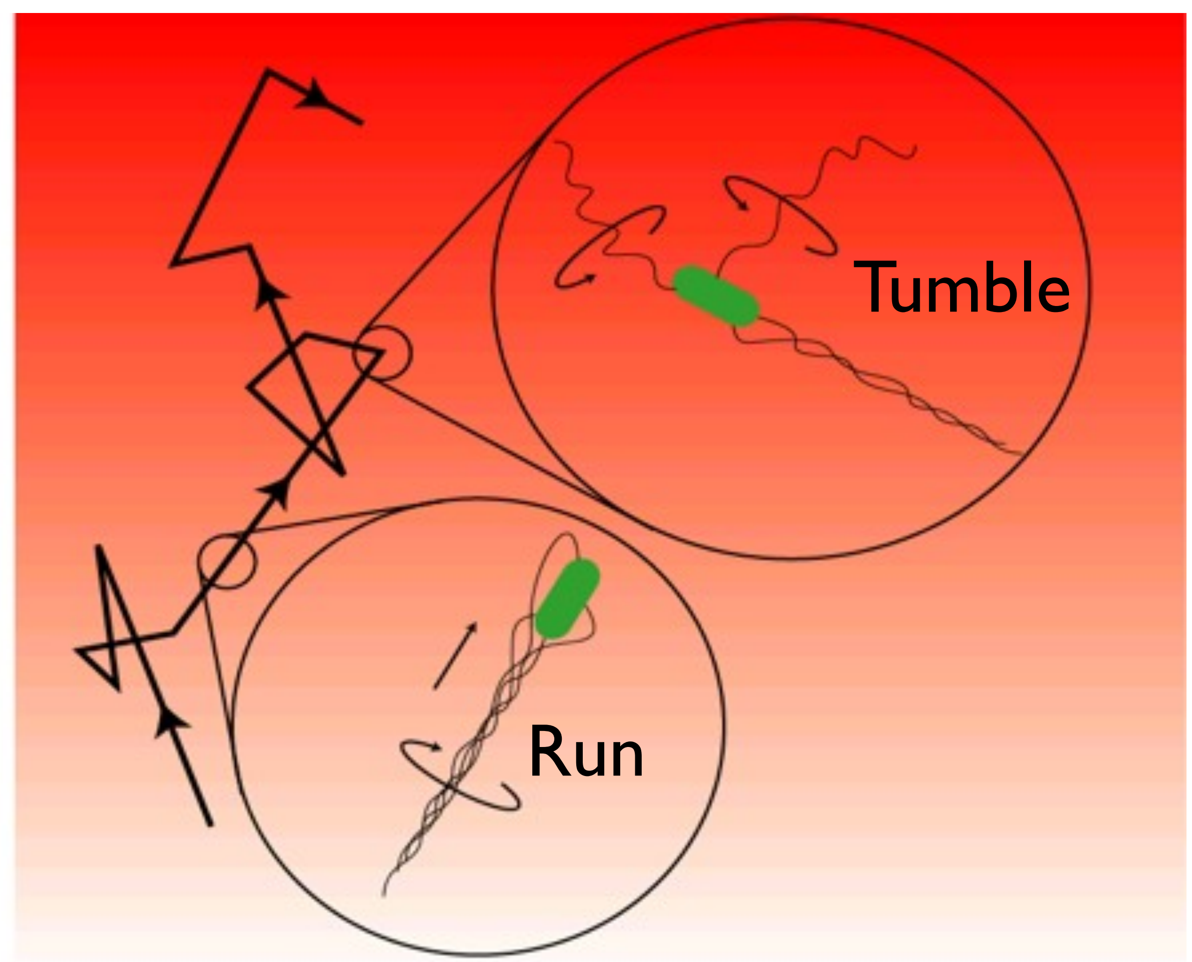

Interesting questions

- Physics questions:
    - How fast does a "bacterium" swim in a Newtonian fluid?
    - How fast does a "bacterium" swim in a non-Newtonian fluid?
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    - Can bacteria affect the macroscopic world?
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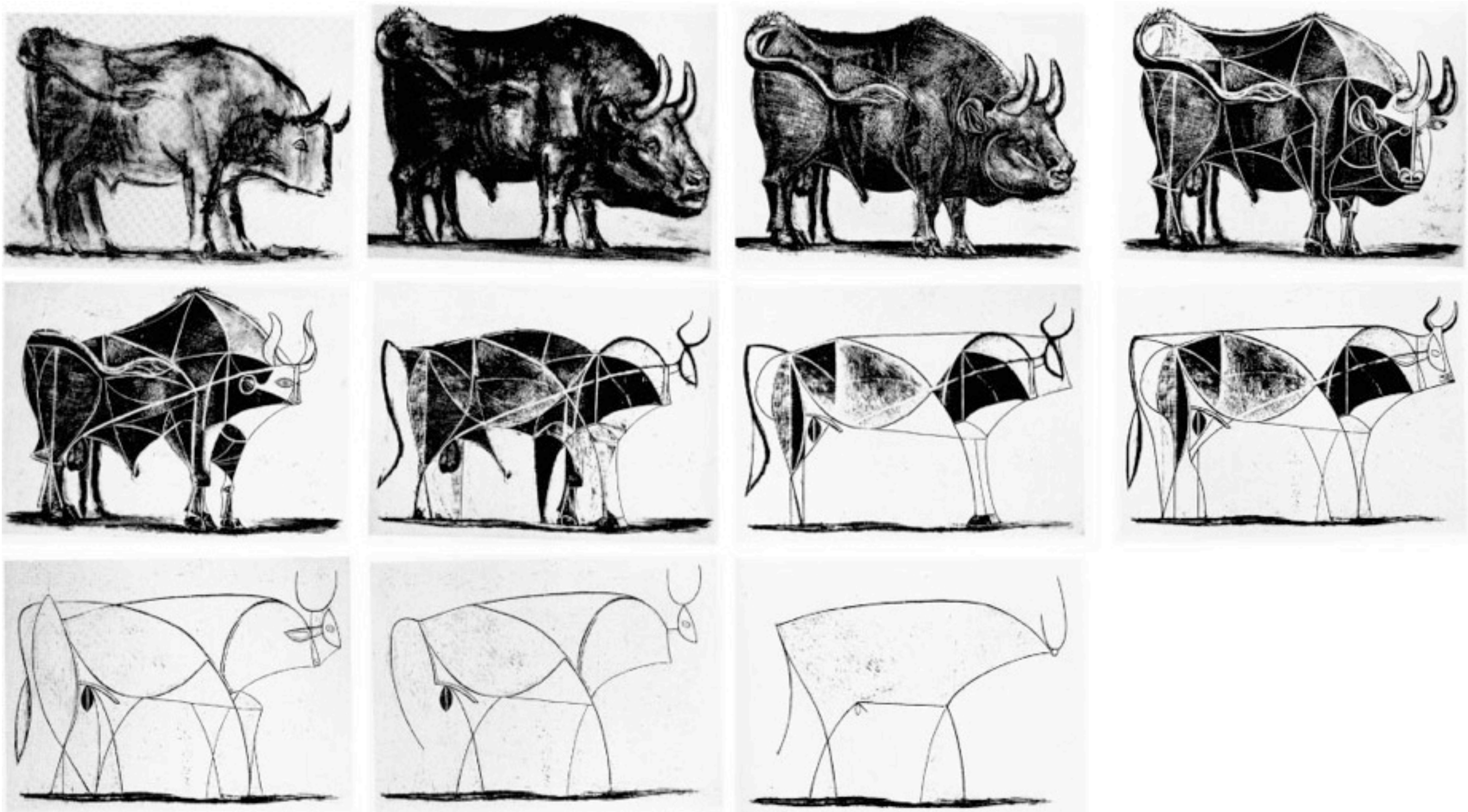
    - etc.
- What next?

Bull #11



apologies to Picasso, thanks to Christophe Clanet

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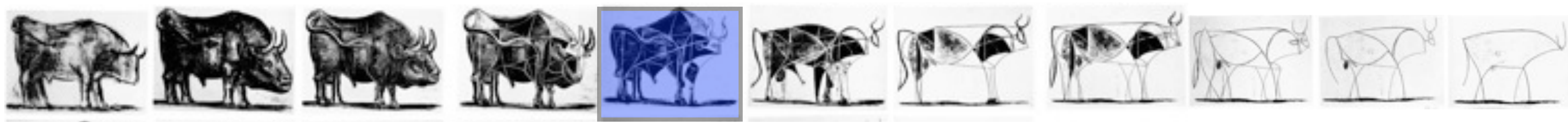
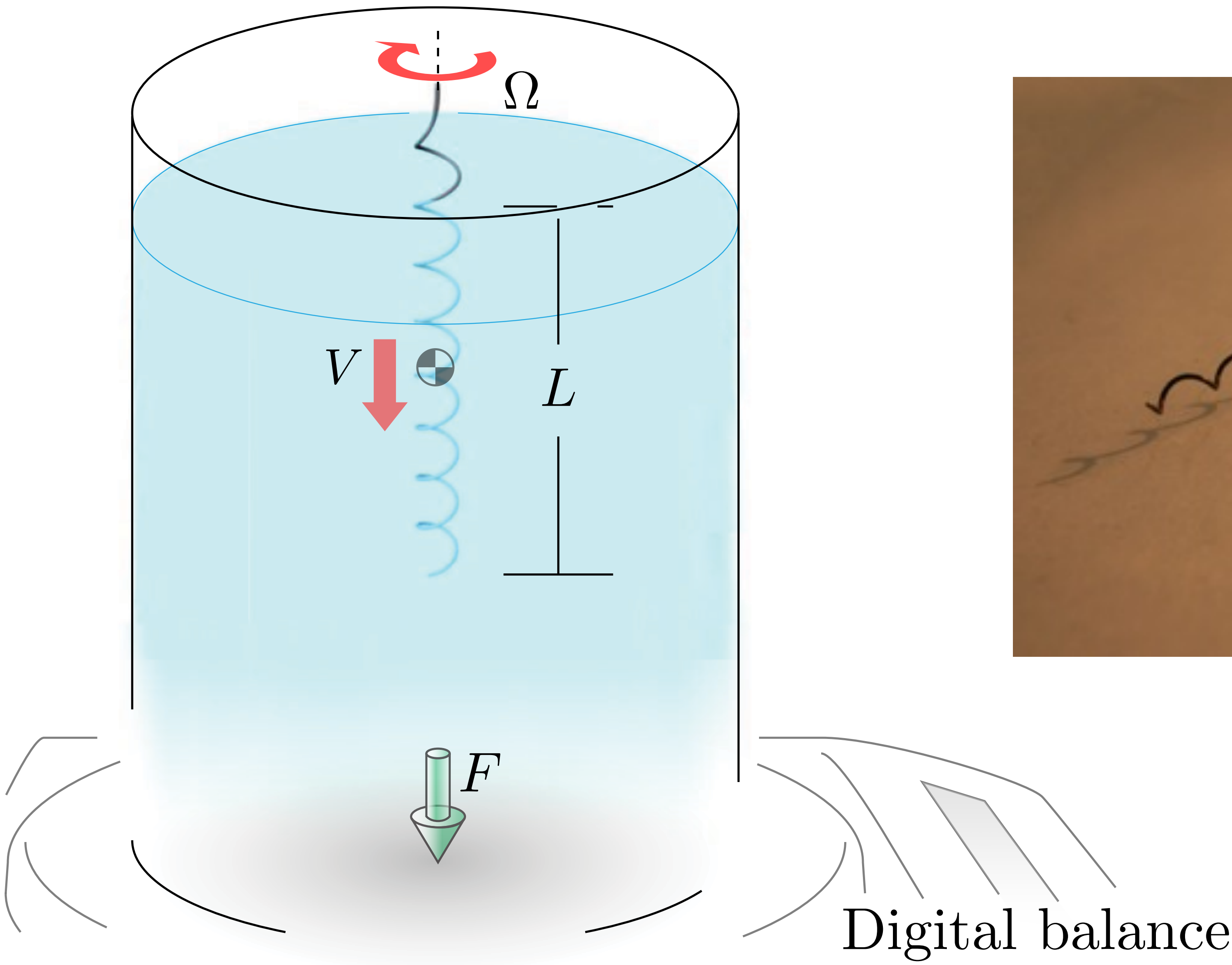
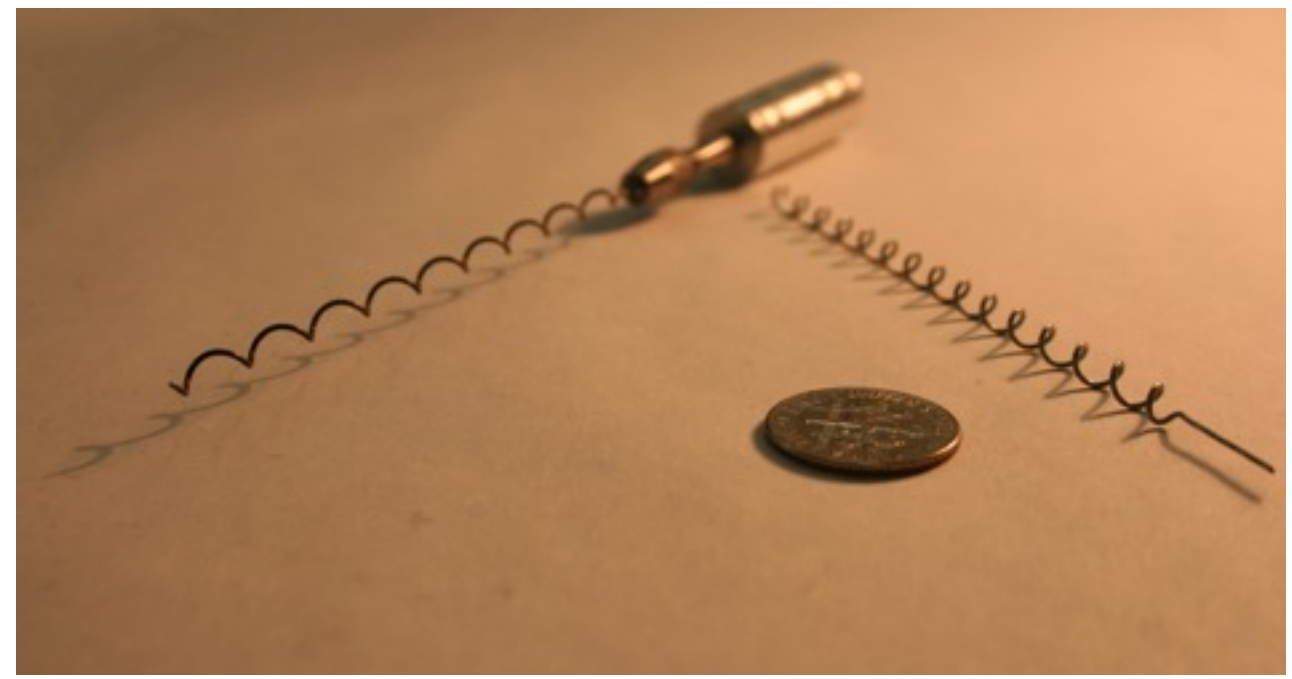
Experimental setup (low-Re swimming helix)







fluids with high viscosity:

- silicone oil (Newtonian)
- Polybutene+ Polyisobutylene (viscoelastic)

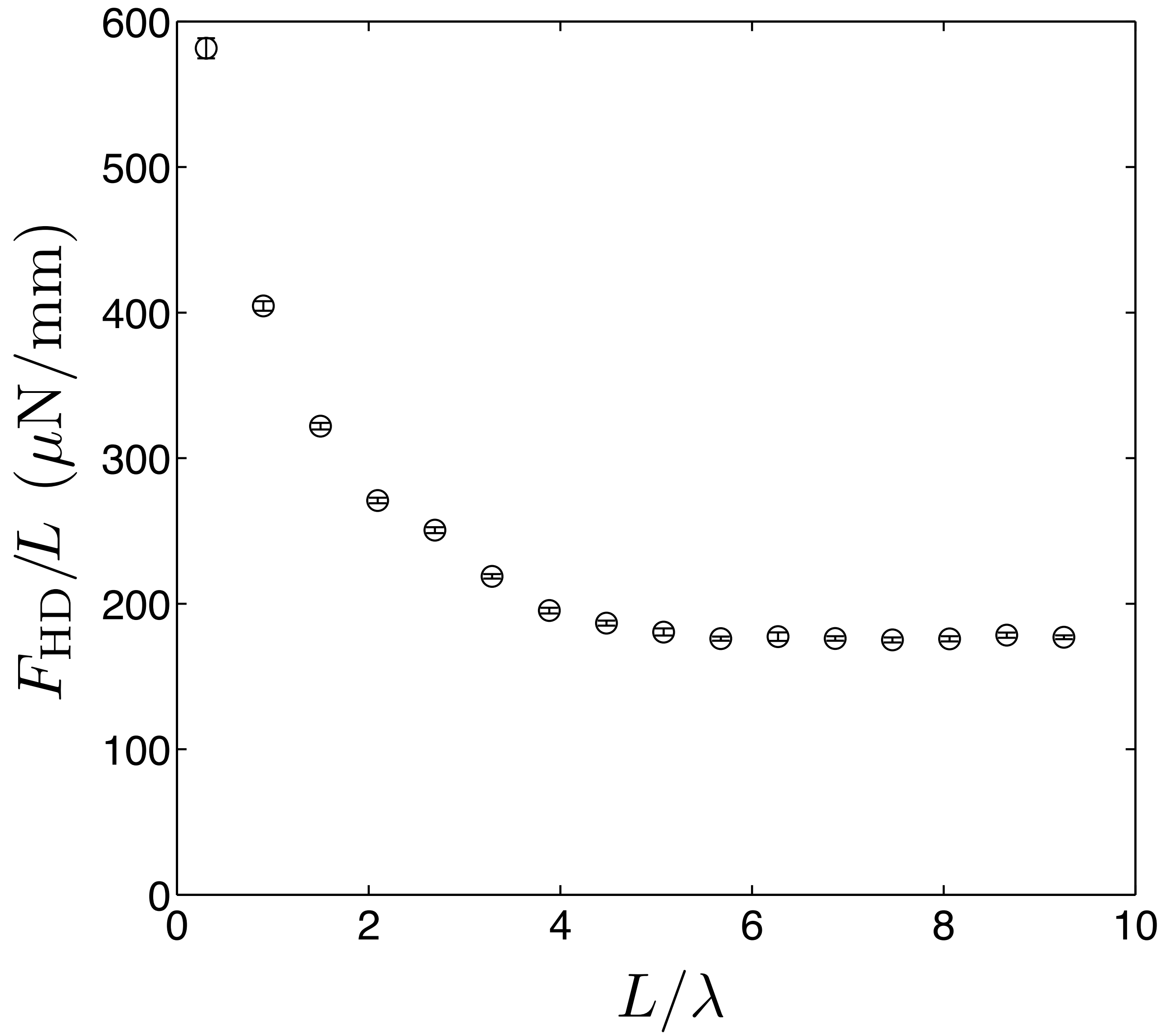
$$\mathrm{Re} \sim 10^{-3}$$

Fluidic force on a rotating helix (Newtonian)

Fluidic force per unit length saturates with L>5 λ

$\longrightarrow$ infinitely long helix

Fluidic force on a rotating helix (Newtonian)



Fluidic force per unit length saturates with L>5 λ

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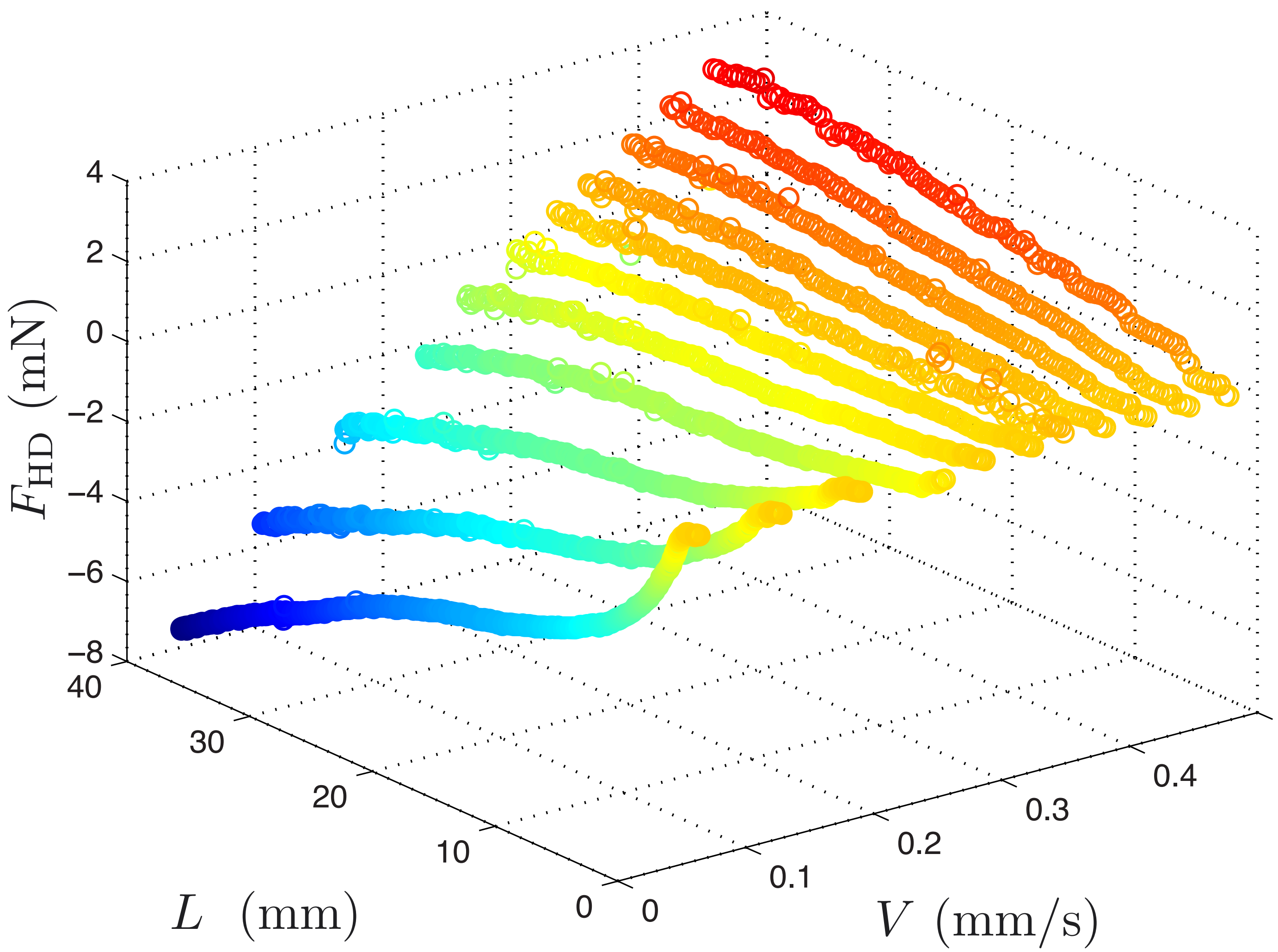
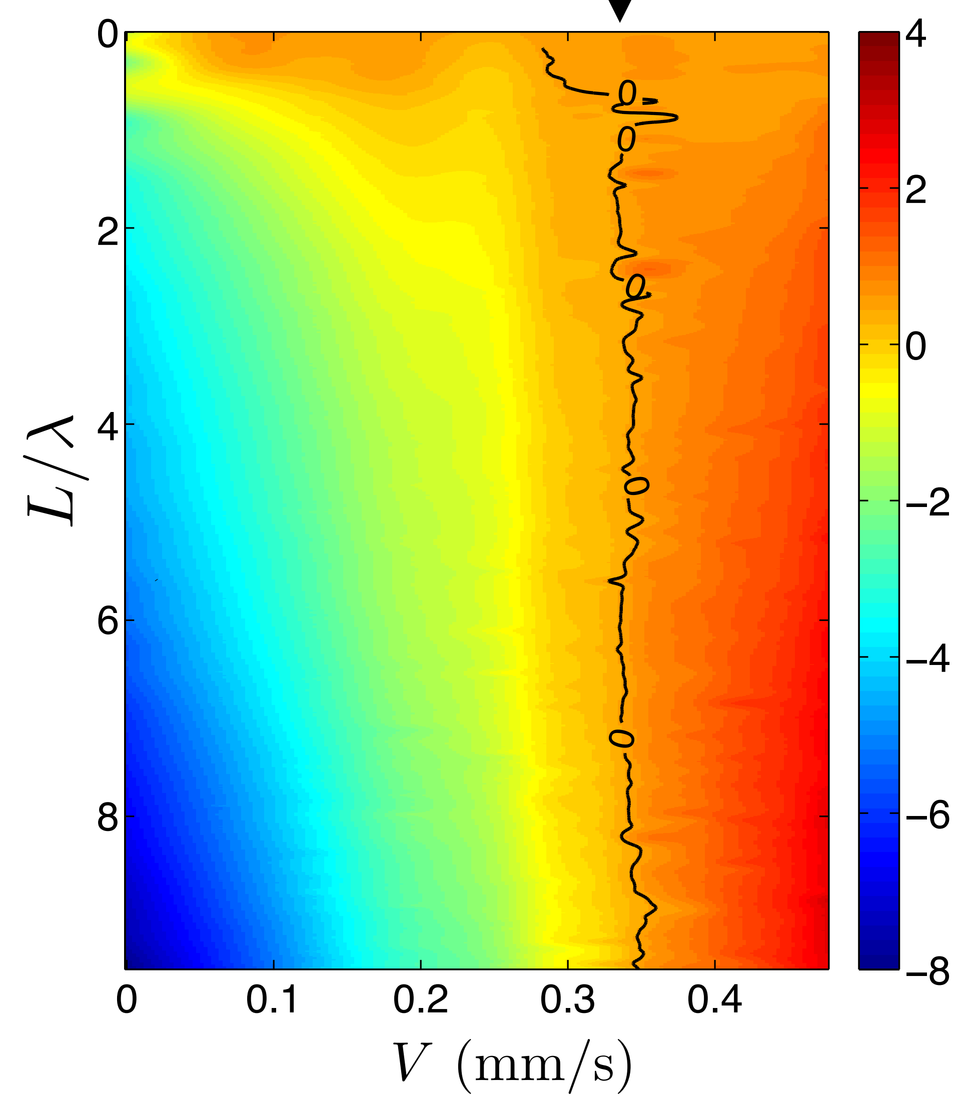
Force-free swimming of a rotating helix (Newtonian)



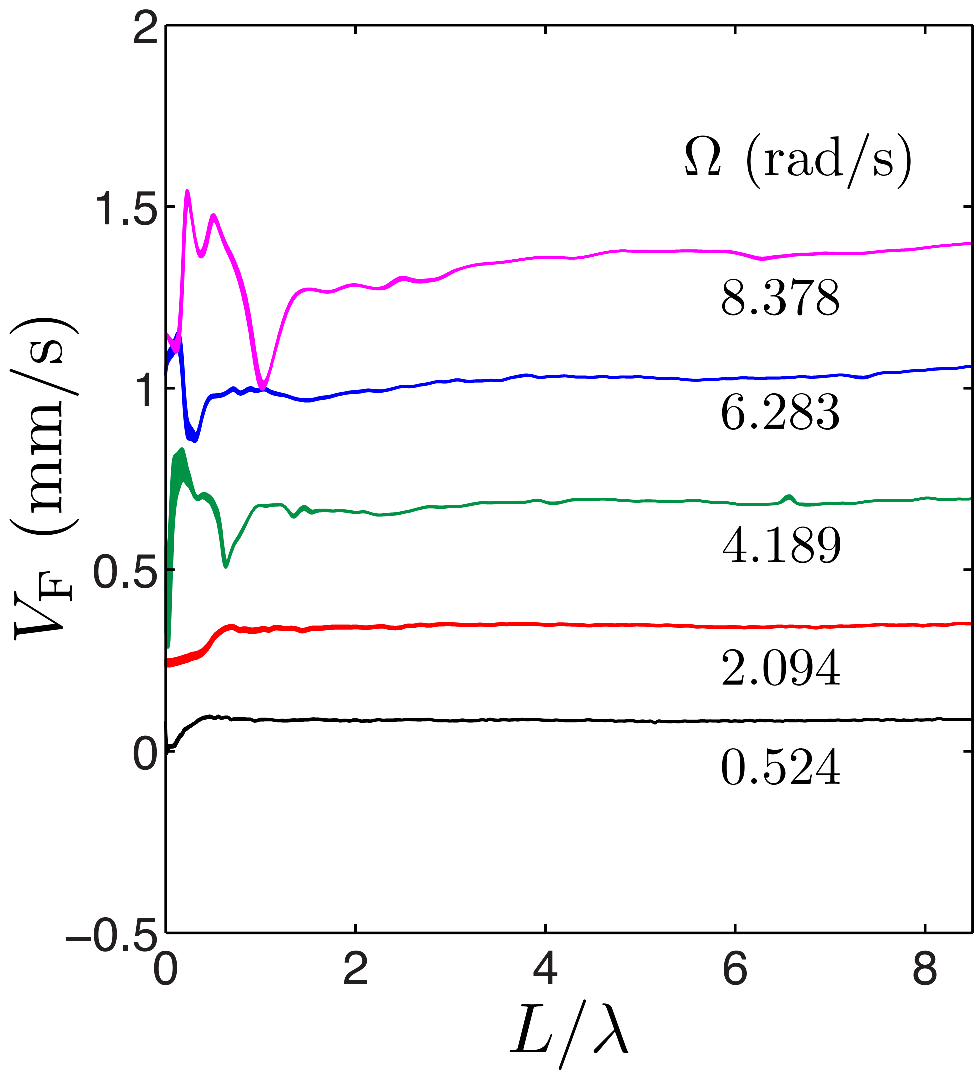
free swimming state ($F_{\mathrm{HD}} = 0$)

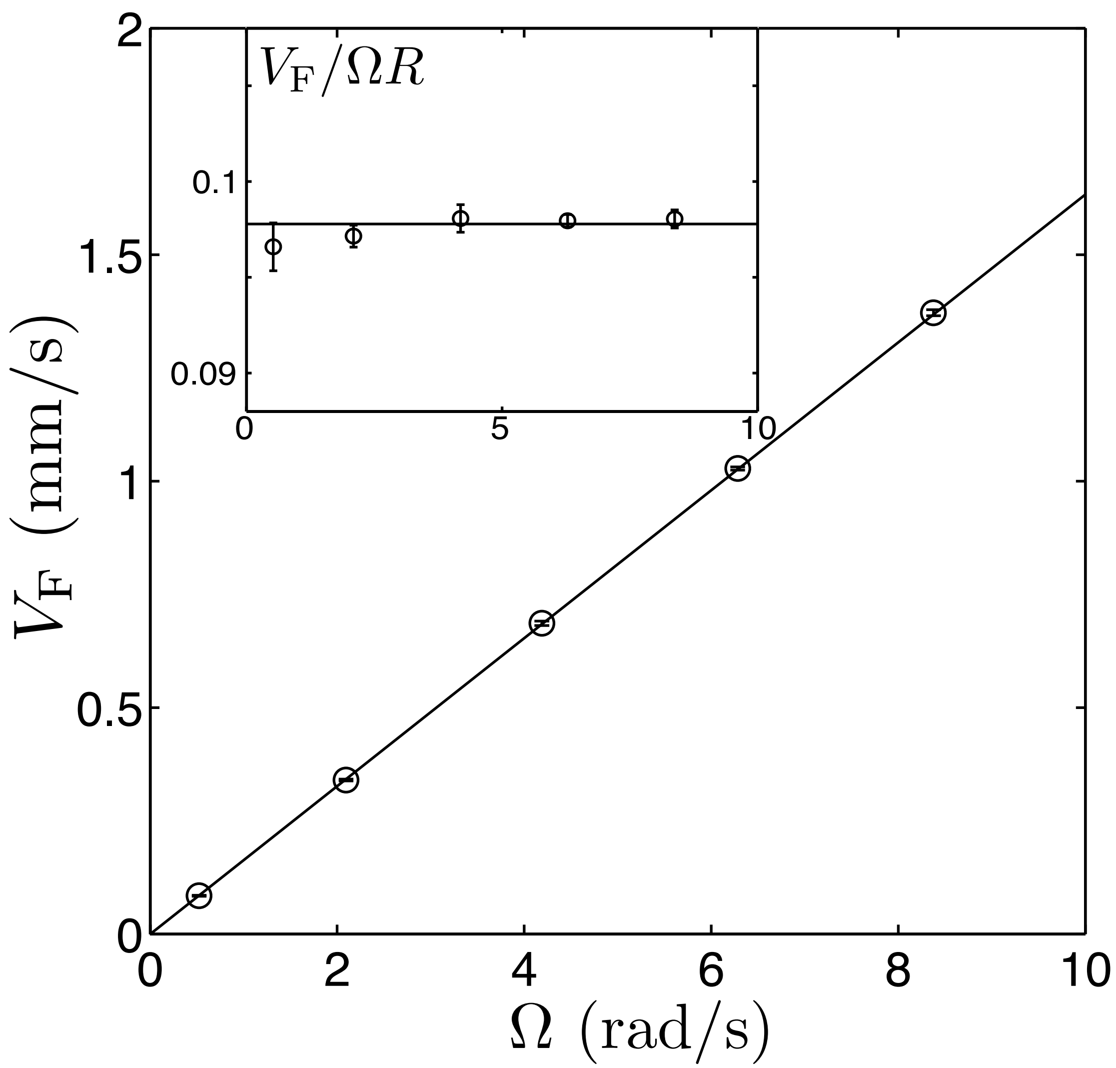


sensitivity of force ~ $10^{-5}\mathrm{N}$

sensitivity of swimming speed ~ 0.3 $\mu\mathrm{m/s}$

Free swimming speed at different rotation rate





Computation of free swimming speed

1. Resistive force theory
(no long-range interactions)

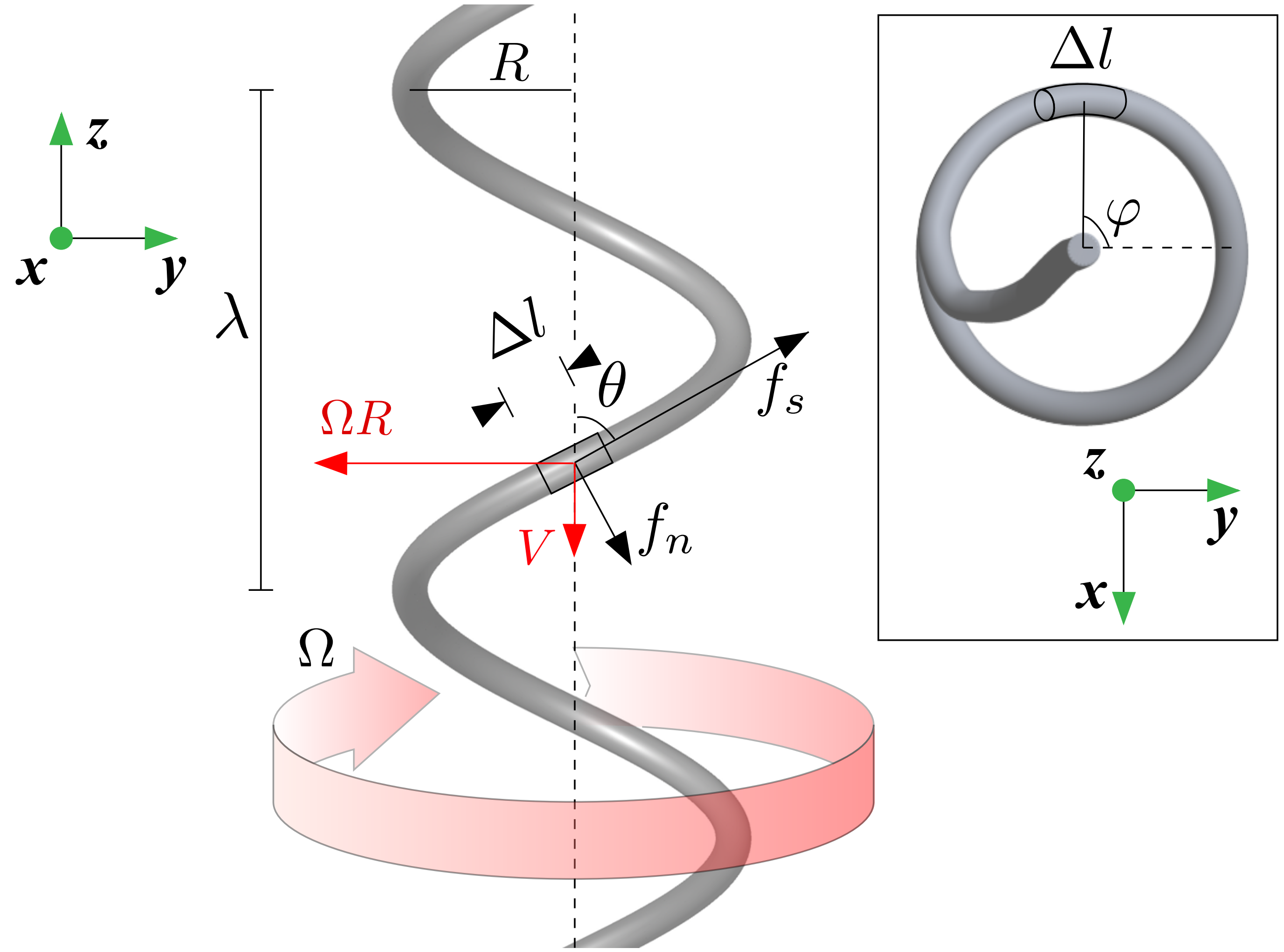
$$f_n = \mu C_n u_n, \quad f_s = \mu C_s u_s,$$

when swimming freely,

$$f_n \sin\theta + f_s \cos\theta = 0$$

$$\downarrow$$

$$\frac{V_\mathrm{F}}{\Omega R} = \frac{(C_n/C_s - 1)\sin\theta\cos\theta}{(C_n/C_s - 1)\sin^2\theta + 1}$$

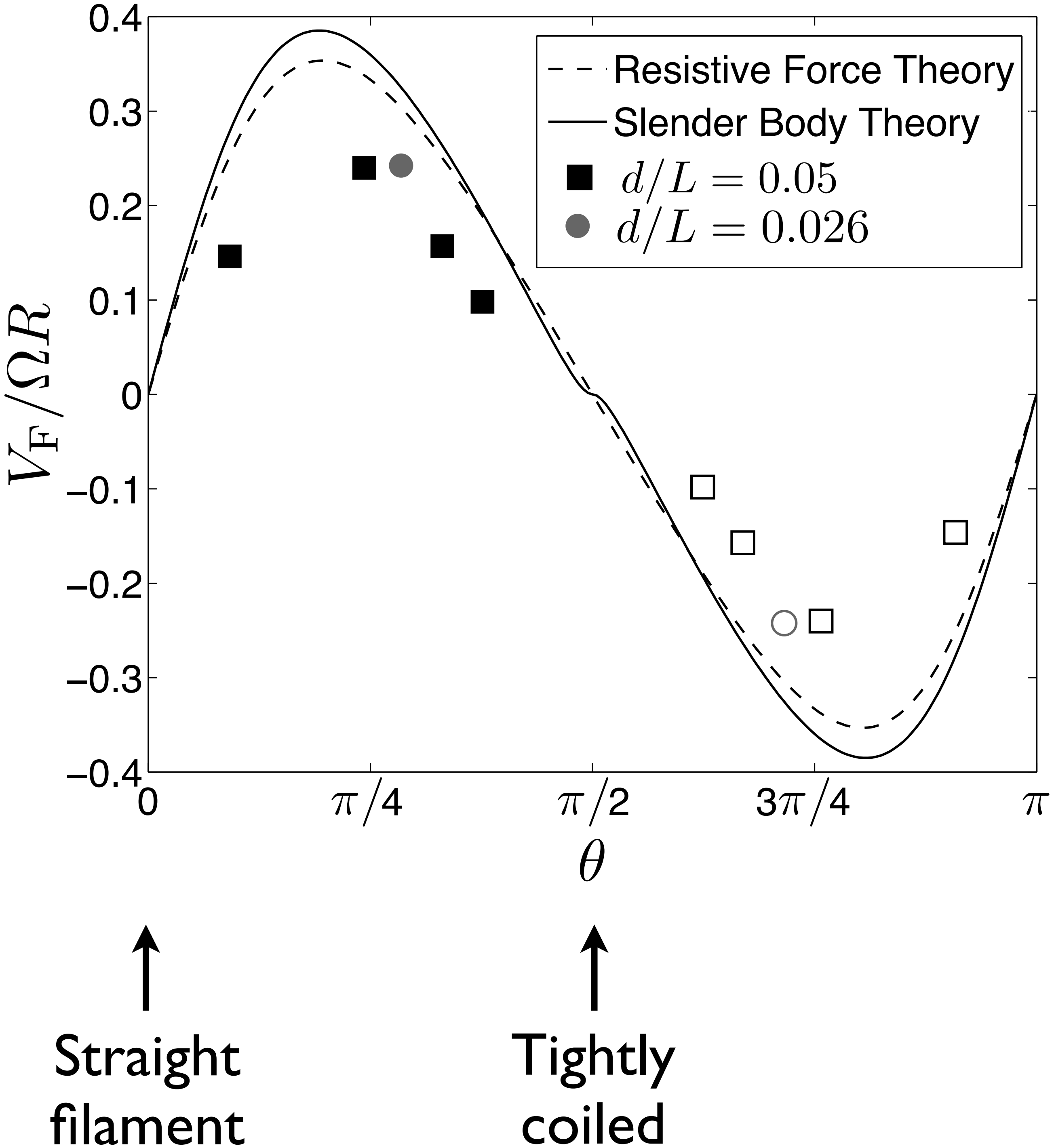


2. Slender body theory
(incl. long range, no thickness effects)

$$V_\mathrm{F} = \frac{f}{8\pi\mu}\int_{-\infty}^{\infty} \frac{\varphi\sin\varphi\cos\theta\csc^2\theta}{(\xi(\varphi,\theta))^{3/2}} d\varphi,$$

$$\Omega R = \frac{f}{8\pi\mu\sin\theta}\int_{-\infty}^{\infty}\left(\frac{\cos\varphi}{(\xi(\varphi,\theta))^{1/2}} + \frac{\sin^2\varphi}{(\xi(\varphi,\theta))^{3/2}}\right) d\varphi$$

Comparison between experiment and theory

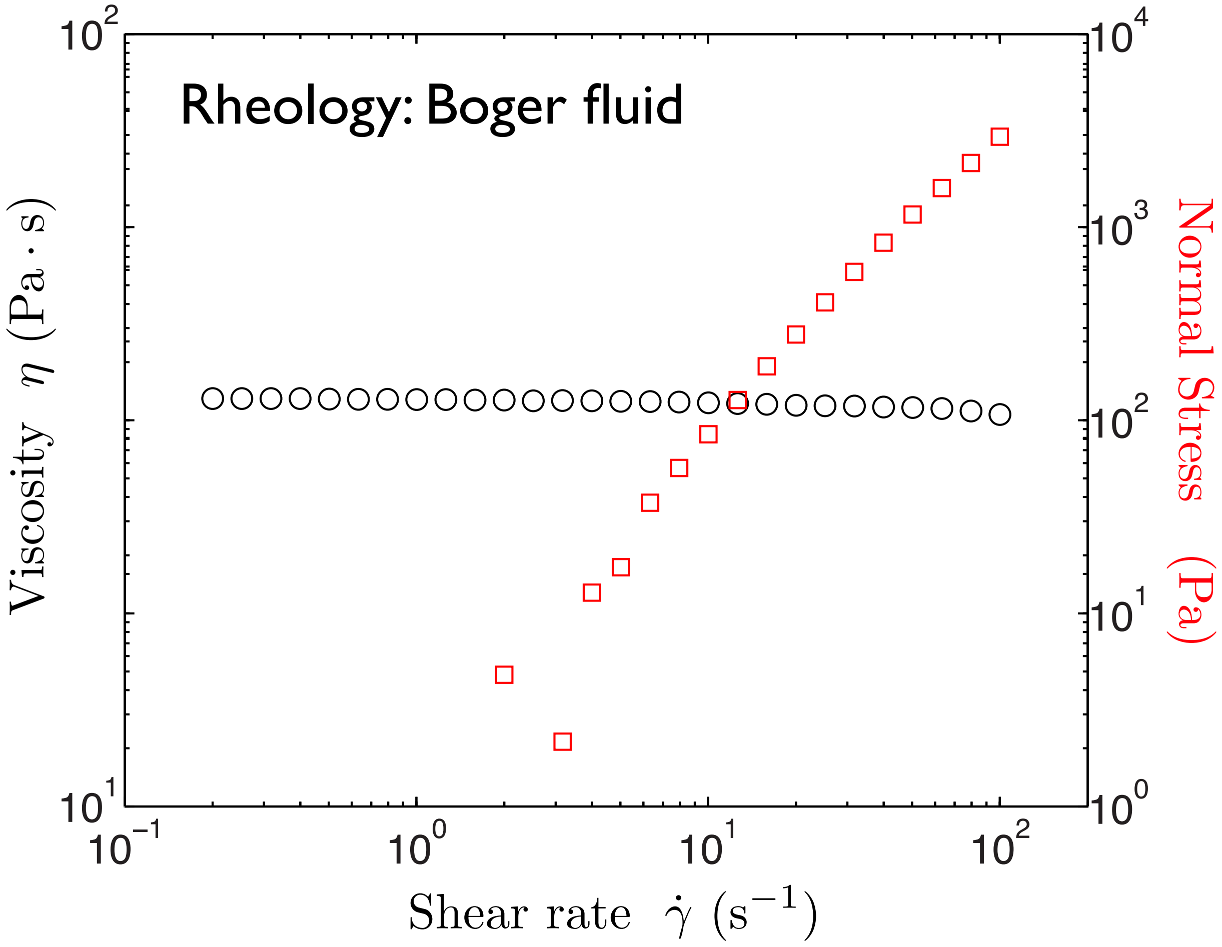


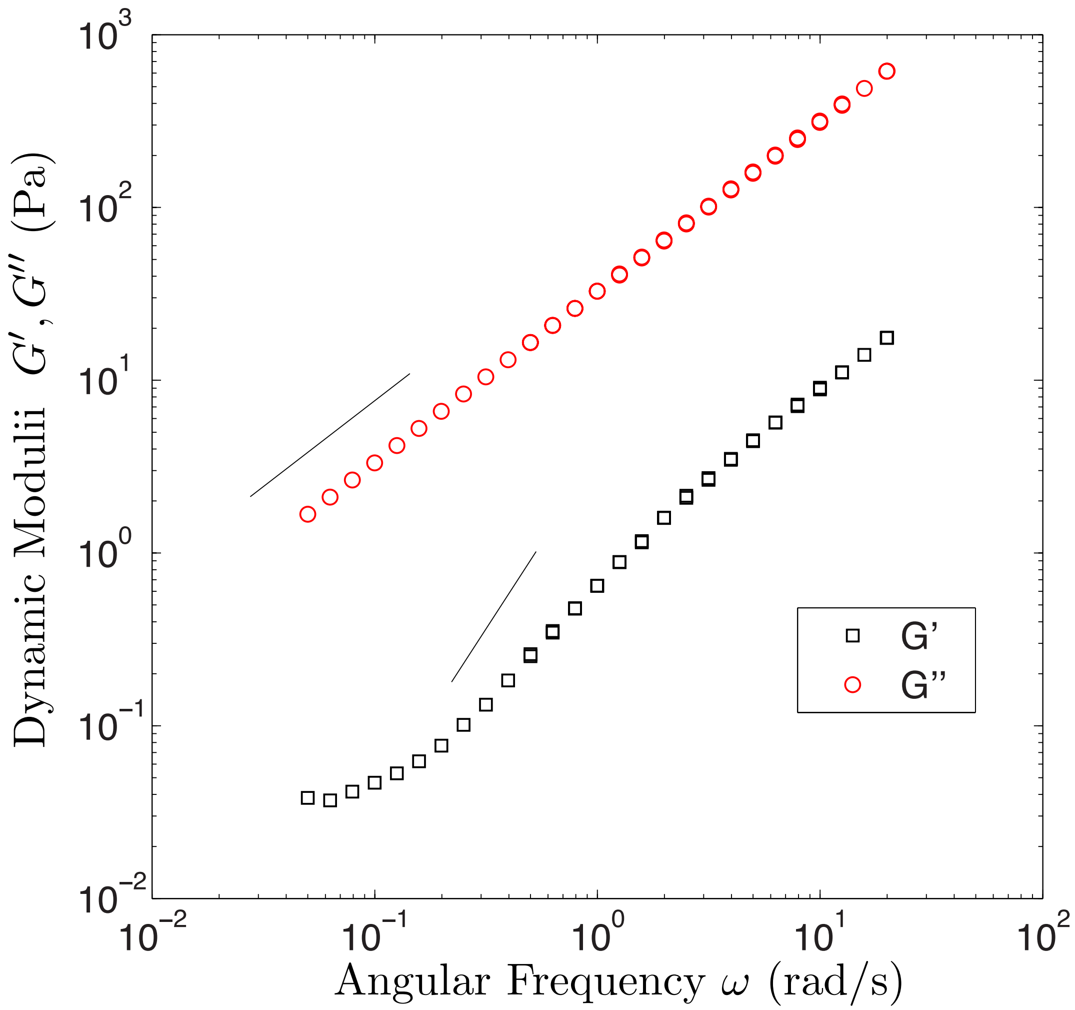
d : diameter of the cross-section of helical fiber

L : arc-length within a pitch

Rotating helix in Viscoelastic fluids

Polyisobutylene (PIB) suspended in Polybutene (PB)

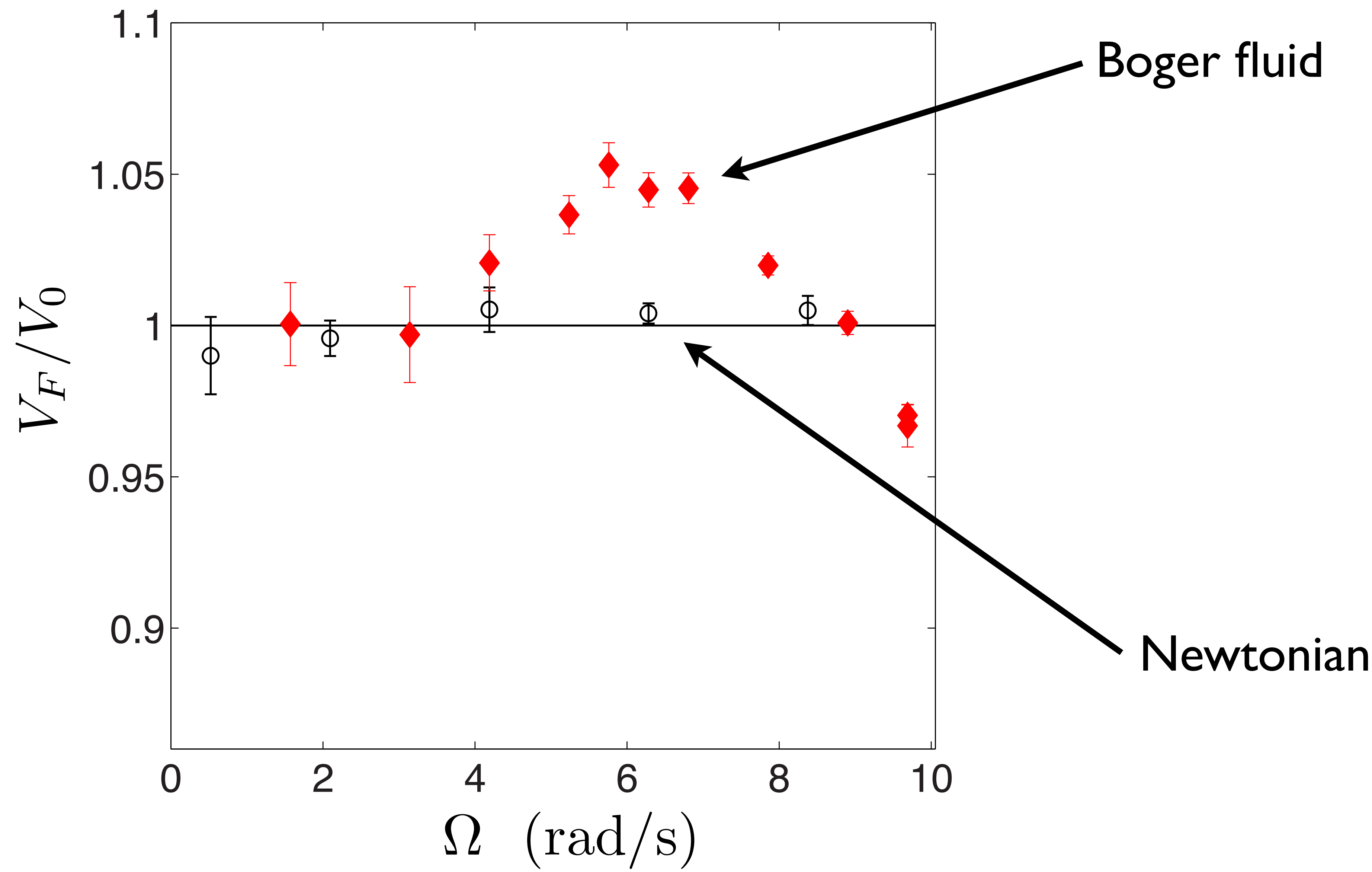




Relaxation time: $\tau = 0.4$ s

Rotating helix in Viscoelastic fluids

free swimming speed of rotating helix in viscoelastic fluids



Next: fluid with stronger elasticity

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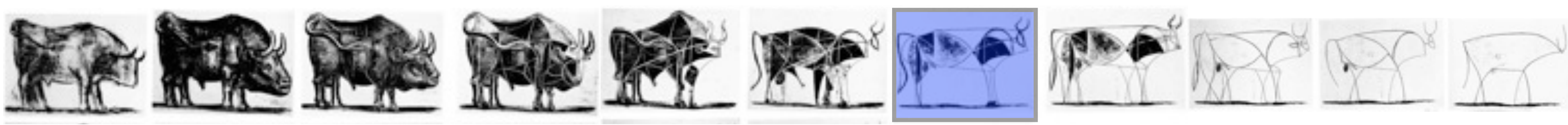
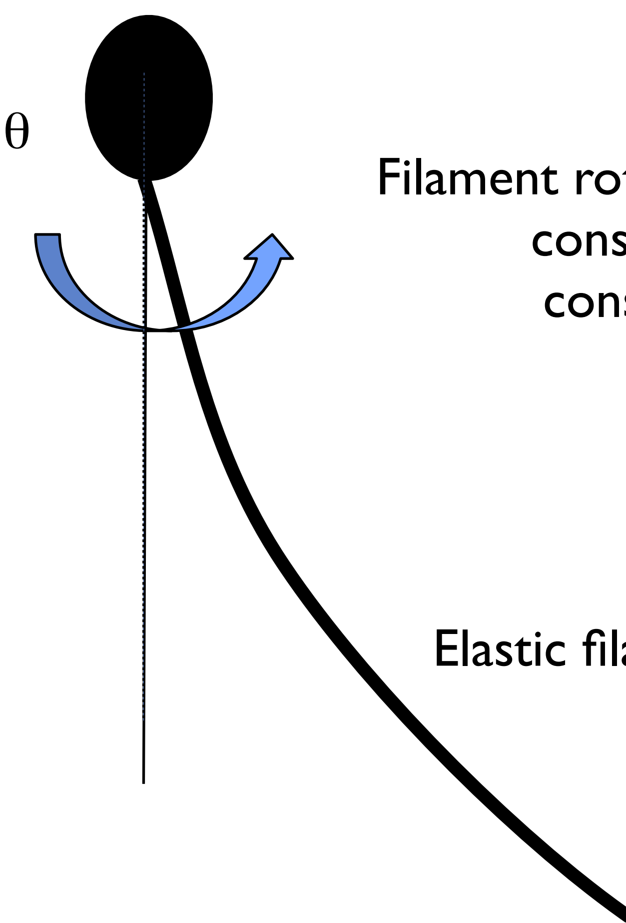
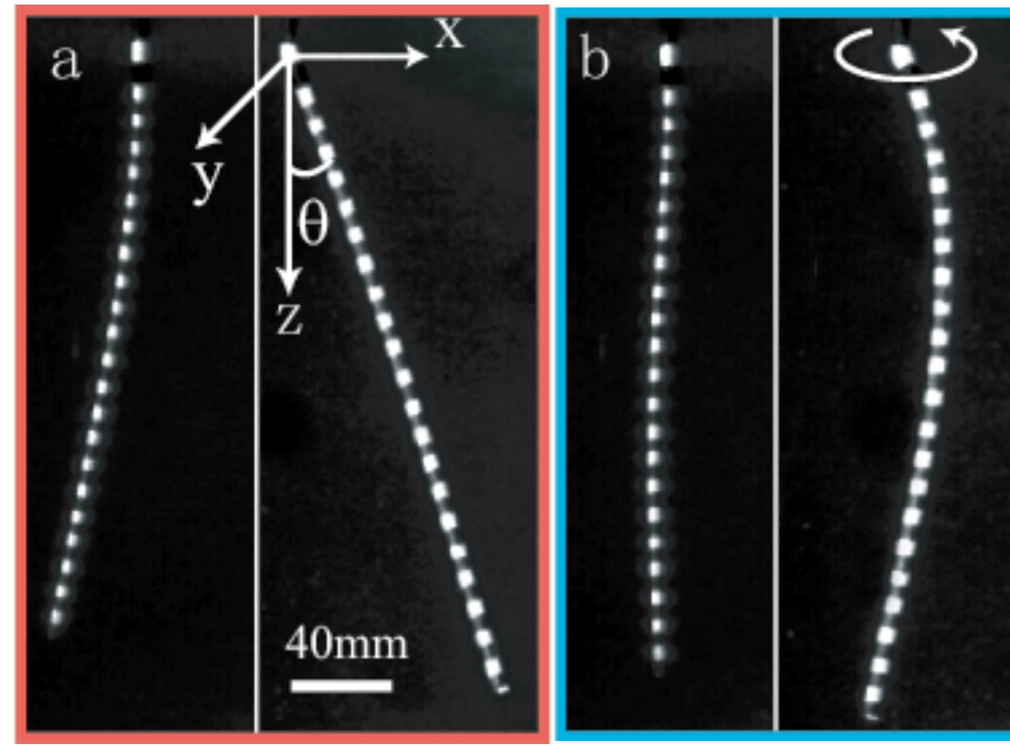
Elastic filament interacting with a viscous flow

- Motivation
  - Understand simplified behavior of flexible filaments in viscous flows
  - Look at a simpler model for microrobotic propulsion
  - Understand previous numerical experiments
    - (Manghi, Schlagberger & Netz, *PRL* 2006)





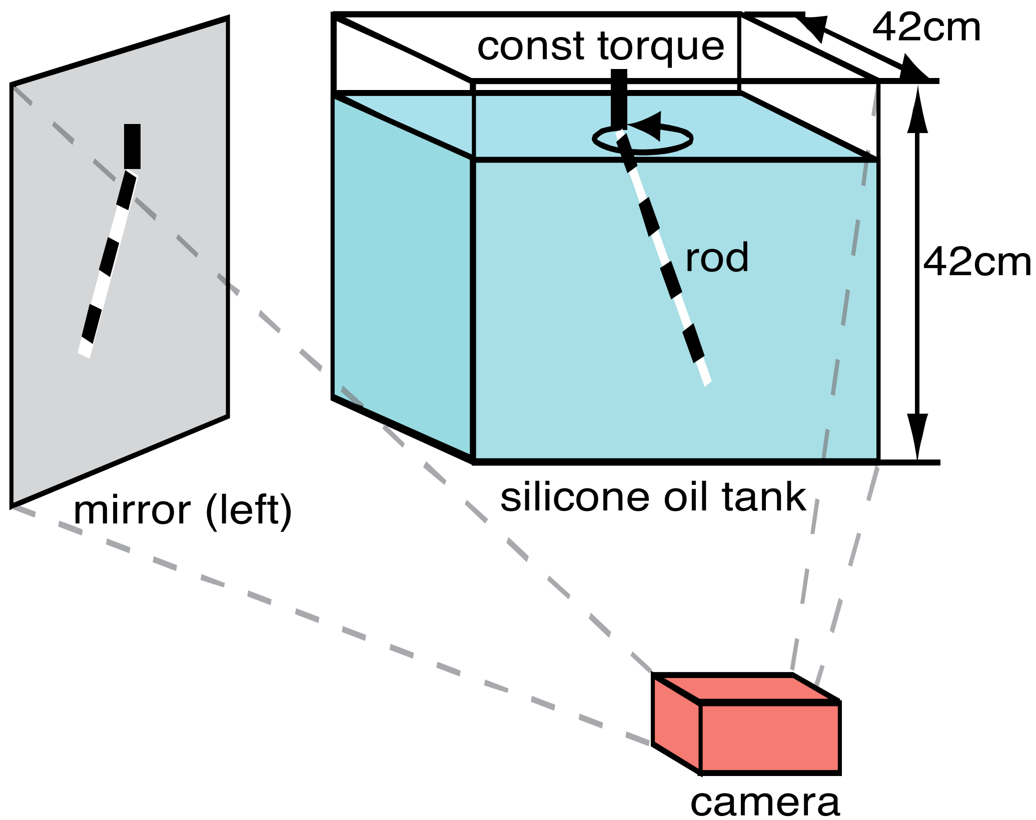
Shape Bifurcation of an Elastic Rotating Rod



before bifurcation

after bifurcation

(Video Sped up x10)



Experiment Parameters :

bending modulus $A \approx 3 \times 10^{-3}\ \mathrm{N/m^2}$

radius of rod $a = 2.5\ \mathrm{mm}$

viscosity $\eta \approx 110\ \mathrm{N \times s/m^2}$

length of rod $L = 200 - 280\mathrm{mm}$

tilted angle $\grave{e} \approx 20^\circ - 30^\circ$

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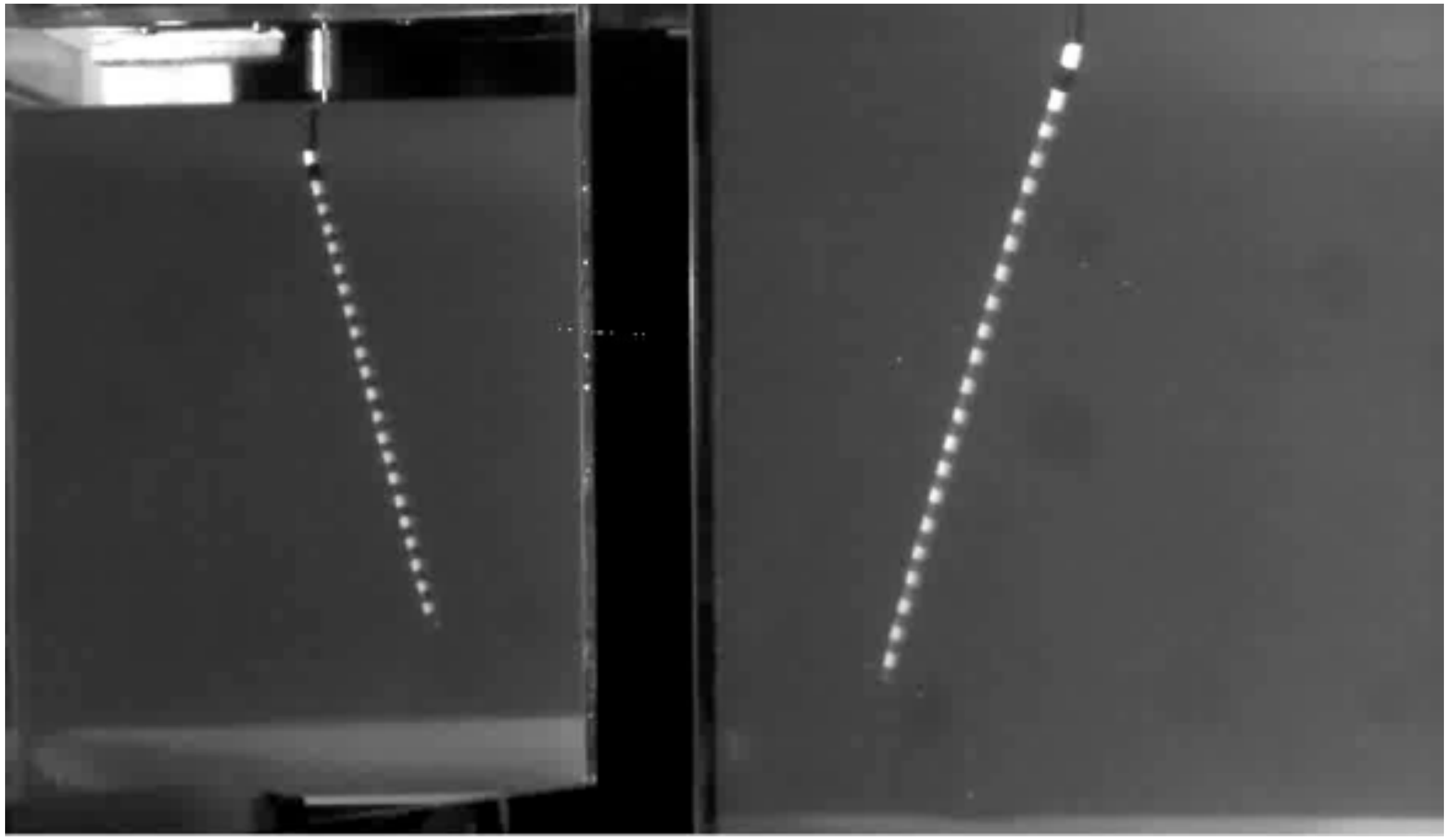

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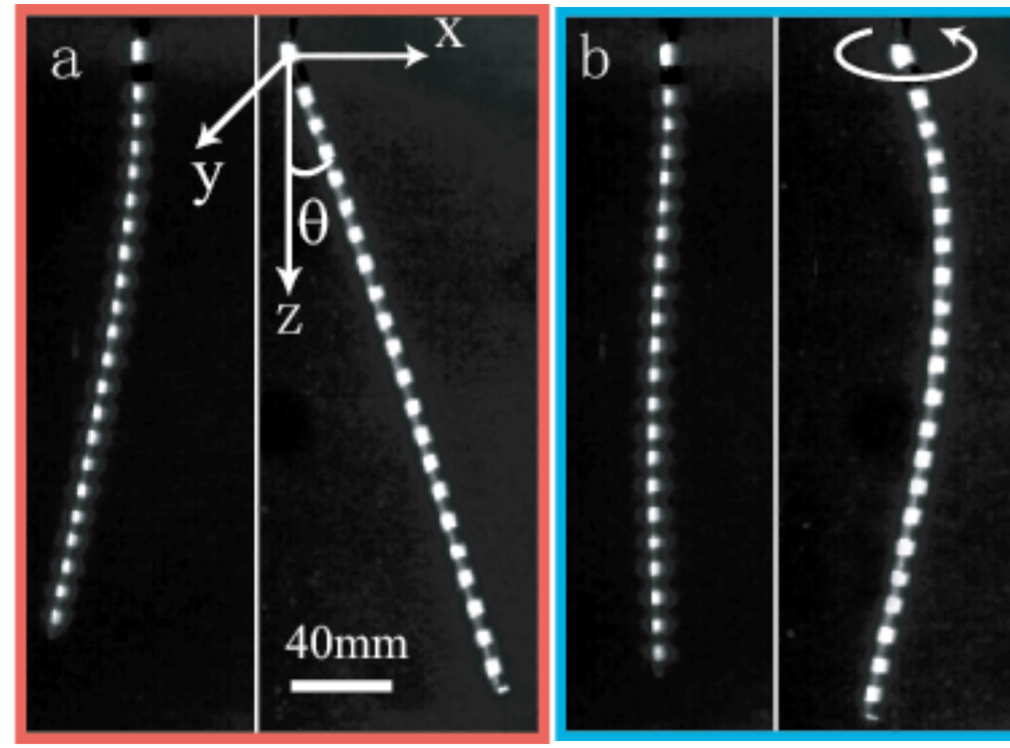

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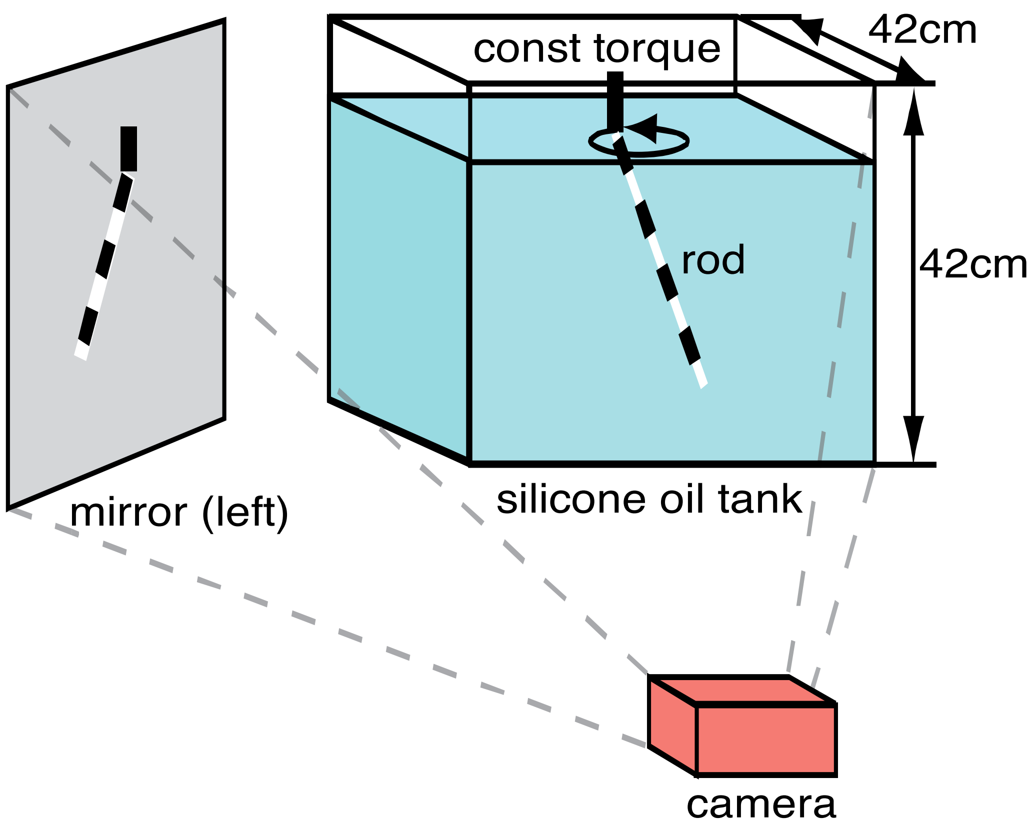

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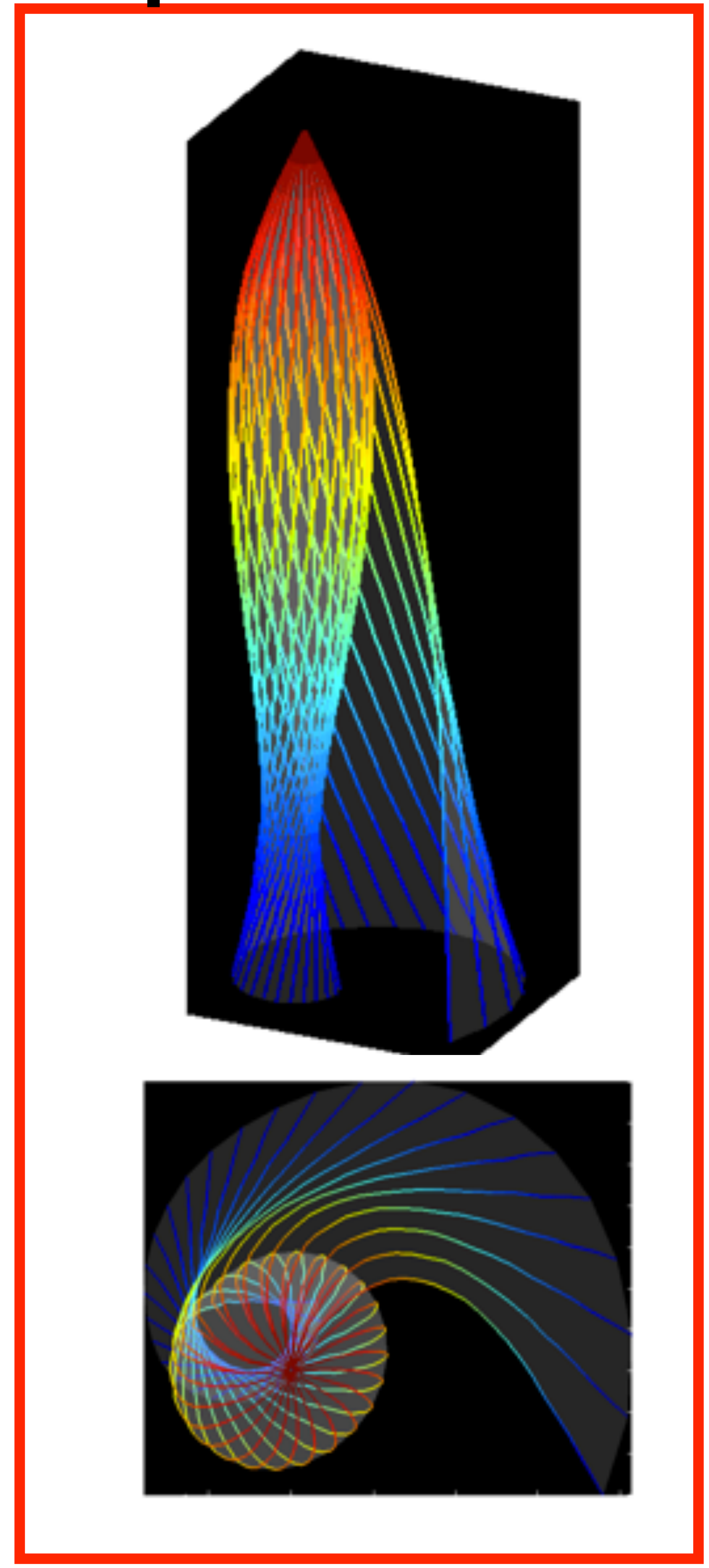
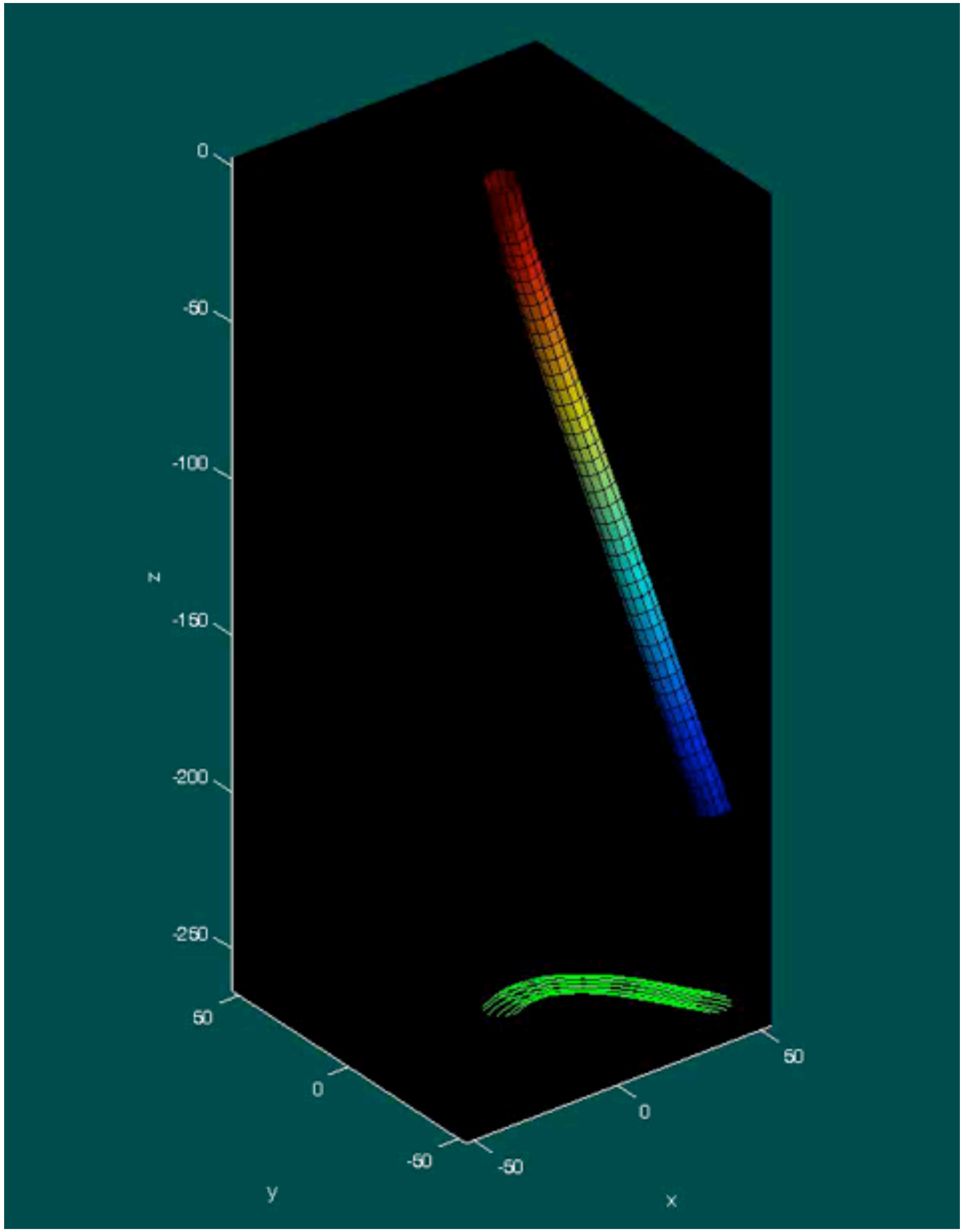
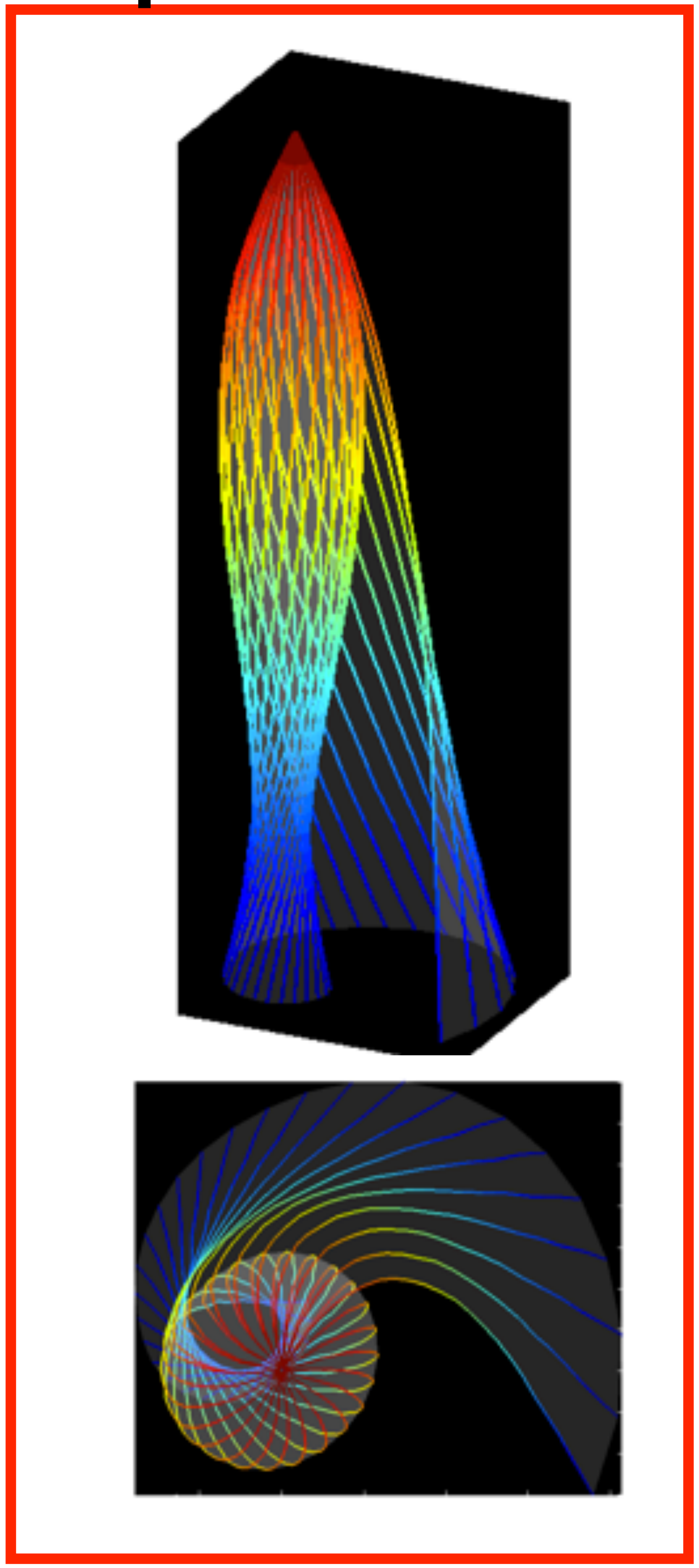
Reconstructed motion from expt. data



Motion around the bifurcation

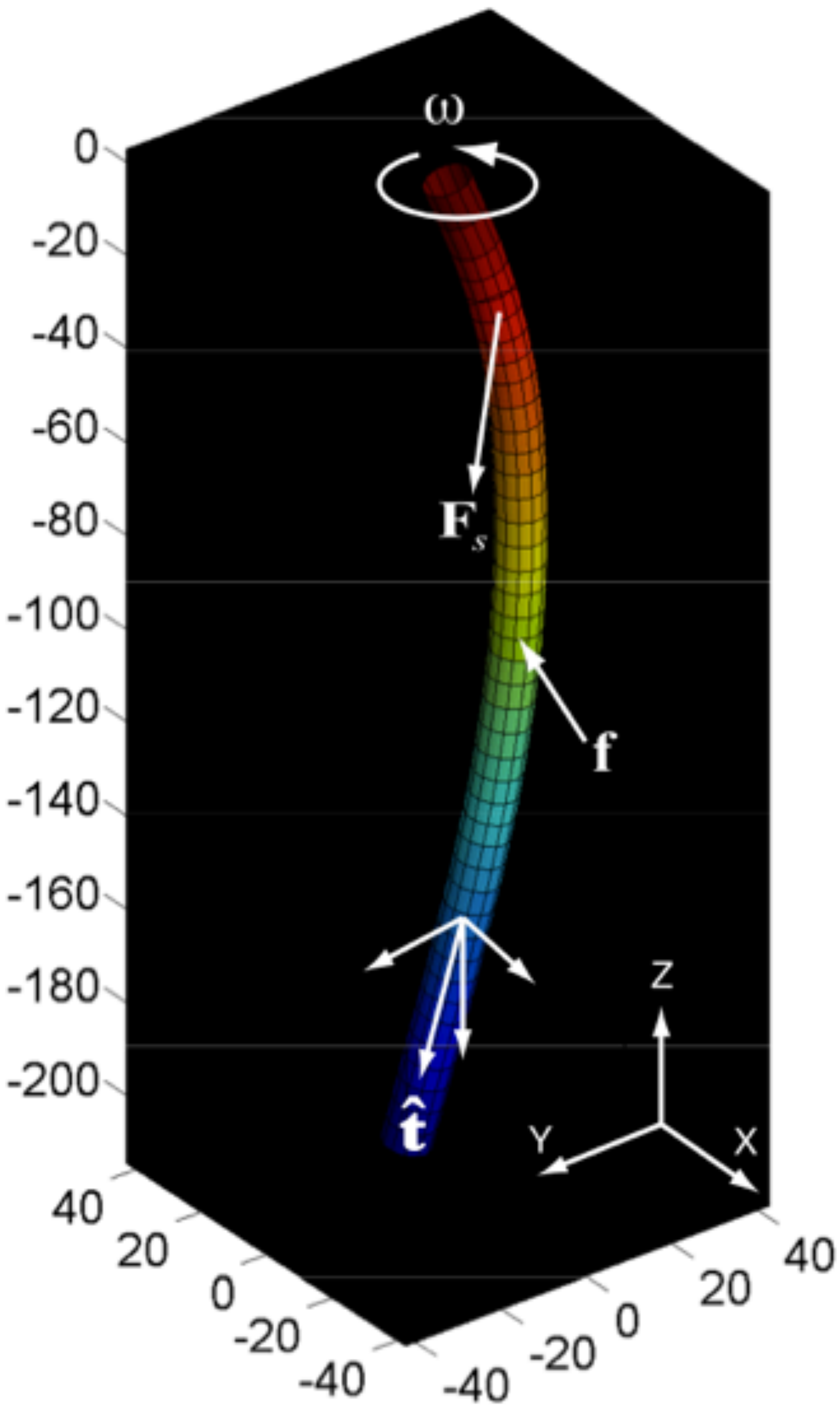
Reconstructed motion from expt. data





Motion around the bifurcation

Theory



- Stokes flow (reversible flow)

$$\mu \nabla^2 \mathbf{v} = \nabla p$$
$$\nabla \cdot \mathbf{v} = 0$$

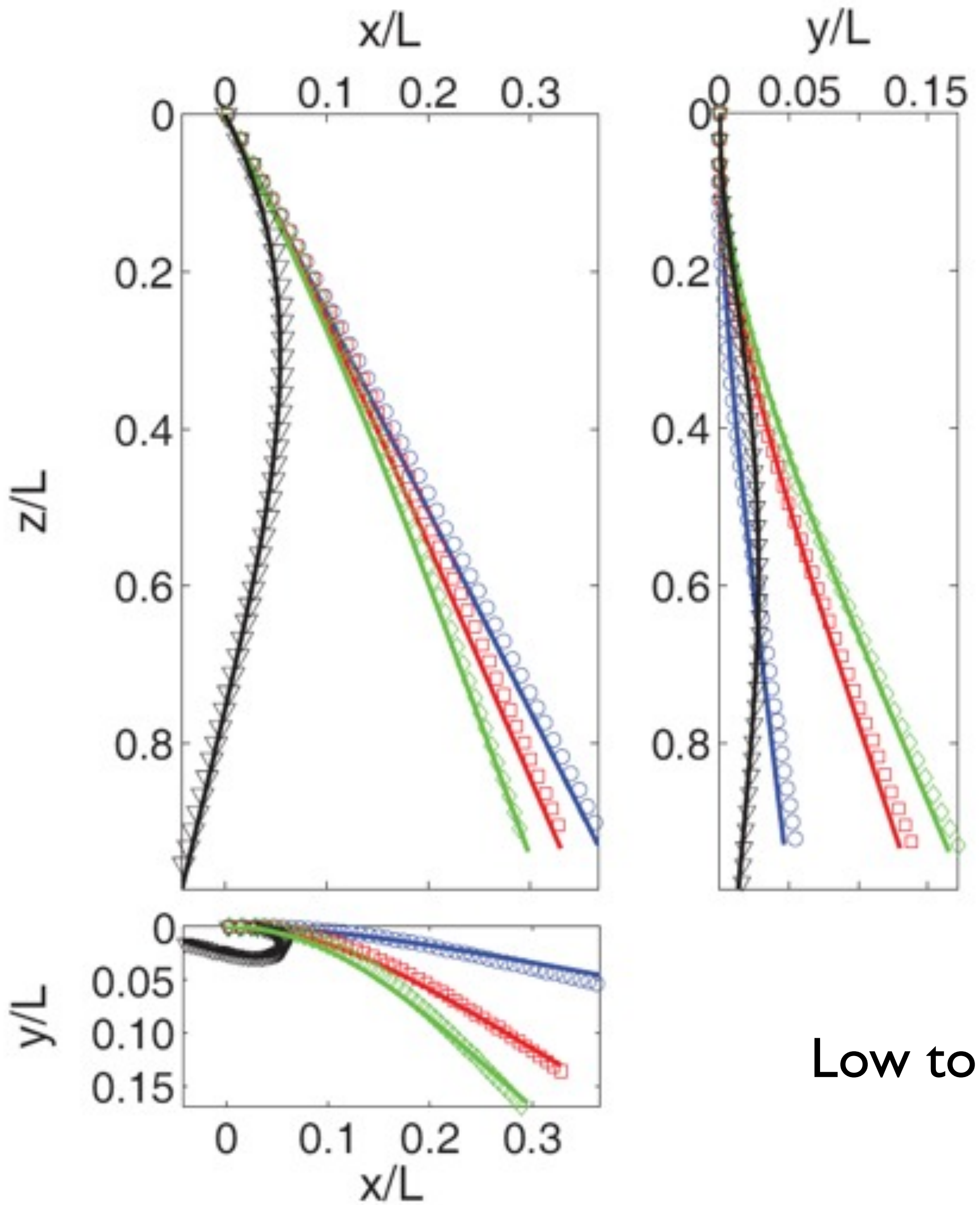
  - Analyze in the rotating frame (steady)
  - No forces due to non-inertial frame
- disregard twist
  - Only bending stiffness
- resistive force theory:

$$f_n = \mu C_n u_n, \quad f_s = \mu C_s u_s,$$

- Hydrodynamic forces are local
- balance of forces and moments
- Key non-dimensional parameters:

$$\theta, \quad \chi = \frac{\mu \omega L^4}{A} = (L/l_v)^4, \quad M = M_{mot} L/A$$

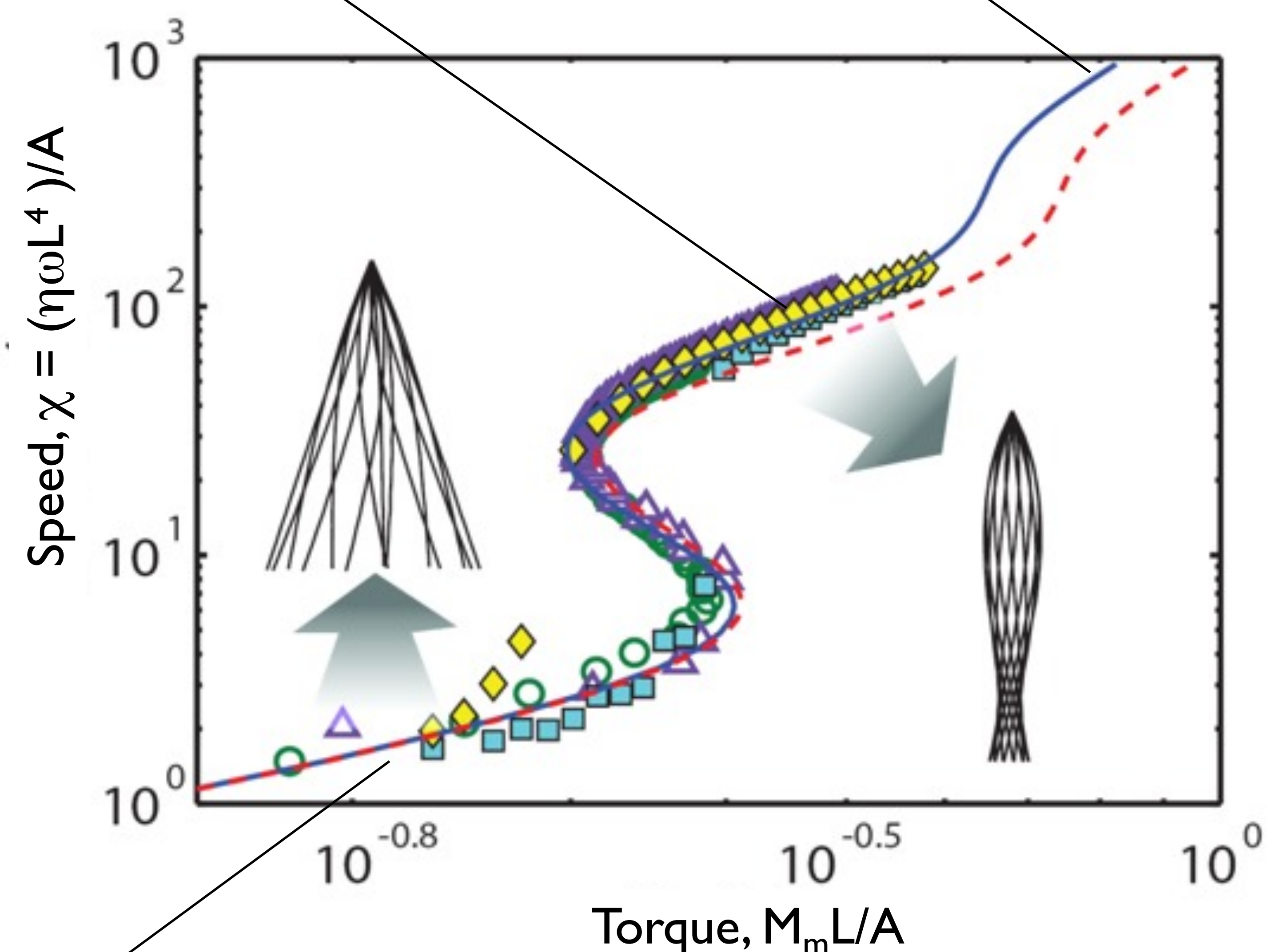
Comparison between theory and experiment



(Qian et al. PRL 2008)

Intermediate regime: $M_mL/A \sim \chi^{1/2}$

High torque: $M_mL/A \sim \chi^{1/4}$



Low torque: $M_mL/A \sim \chi$

	Constant torque	Constant velocity
Ascending	□	○
Descending	◇	△

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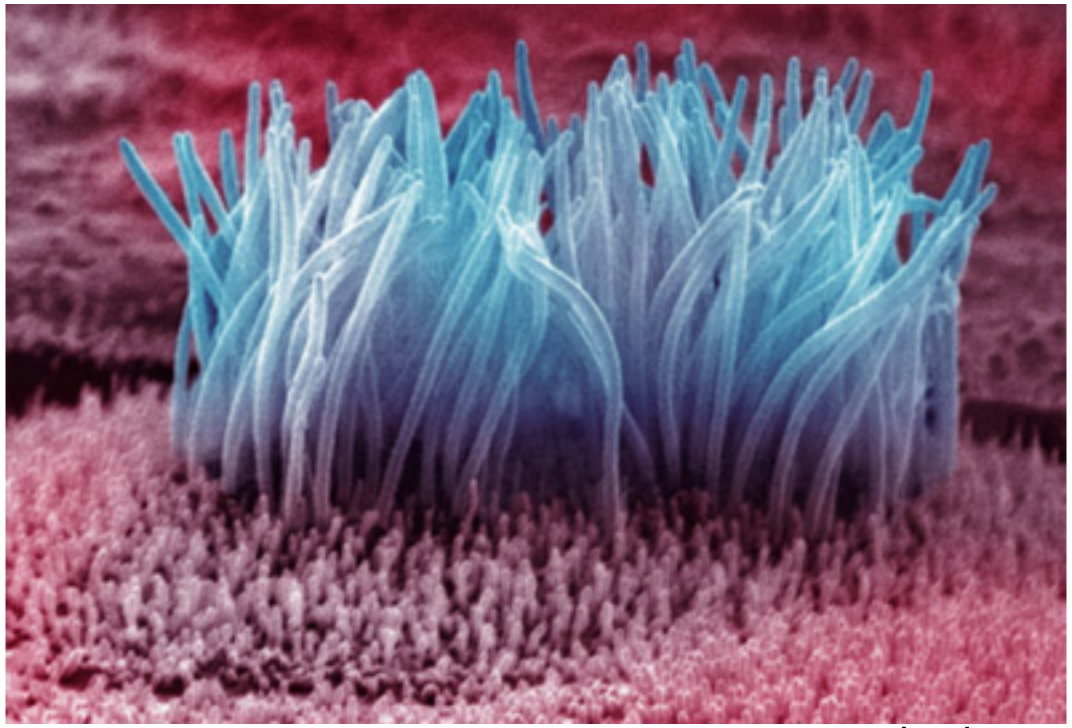
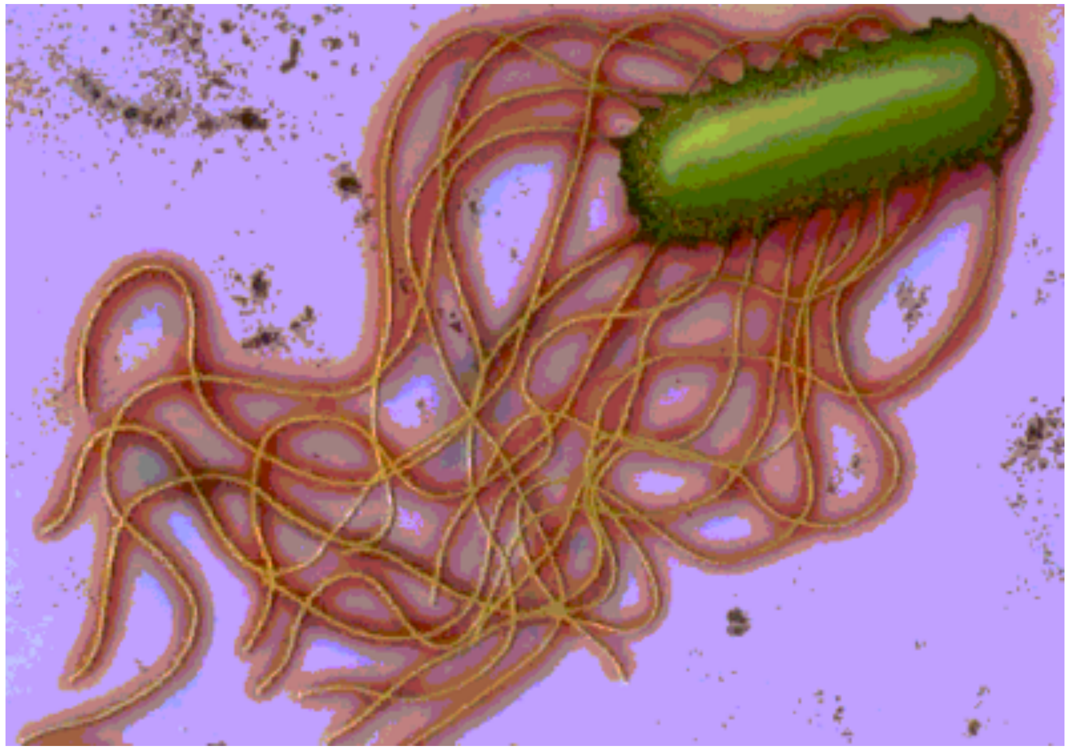
Hydrodynamic interactions

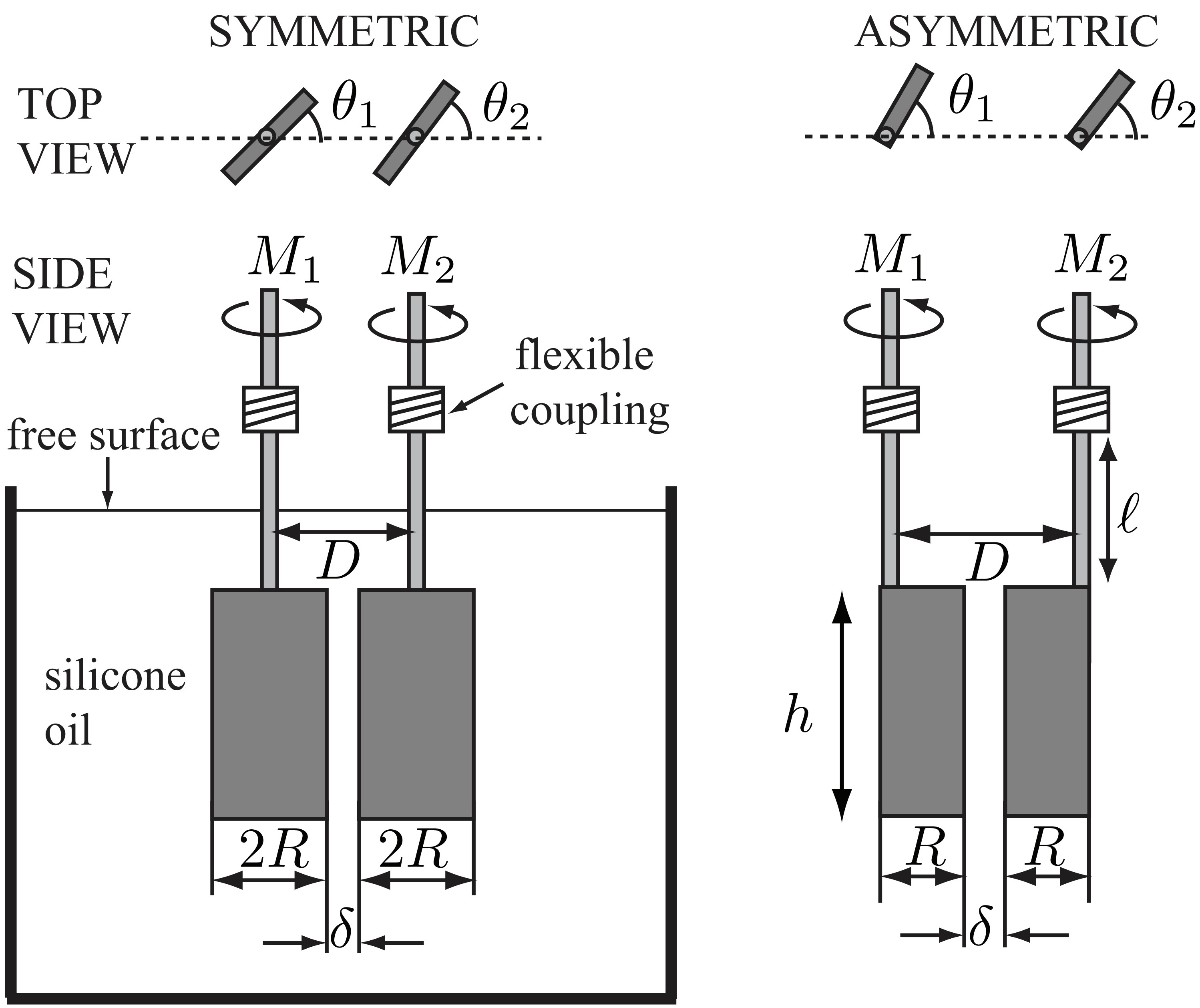
- Cilia and flagella occur in many biological (and engineering) systems.
- Collective motion important for transport and motility
- Interesting questions for flagella and cilia:
  - synchronization through hydrodynamic interactions
  - synchronization vs. chaotic motion
  - role of compliance
  - time-scales for interactions
  - effects of multiple filaments
  - effects of long range hydrodynamic forces



webmd.com

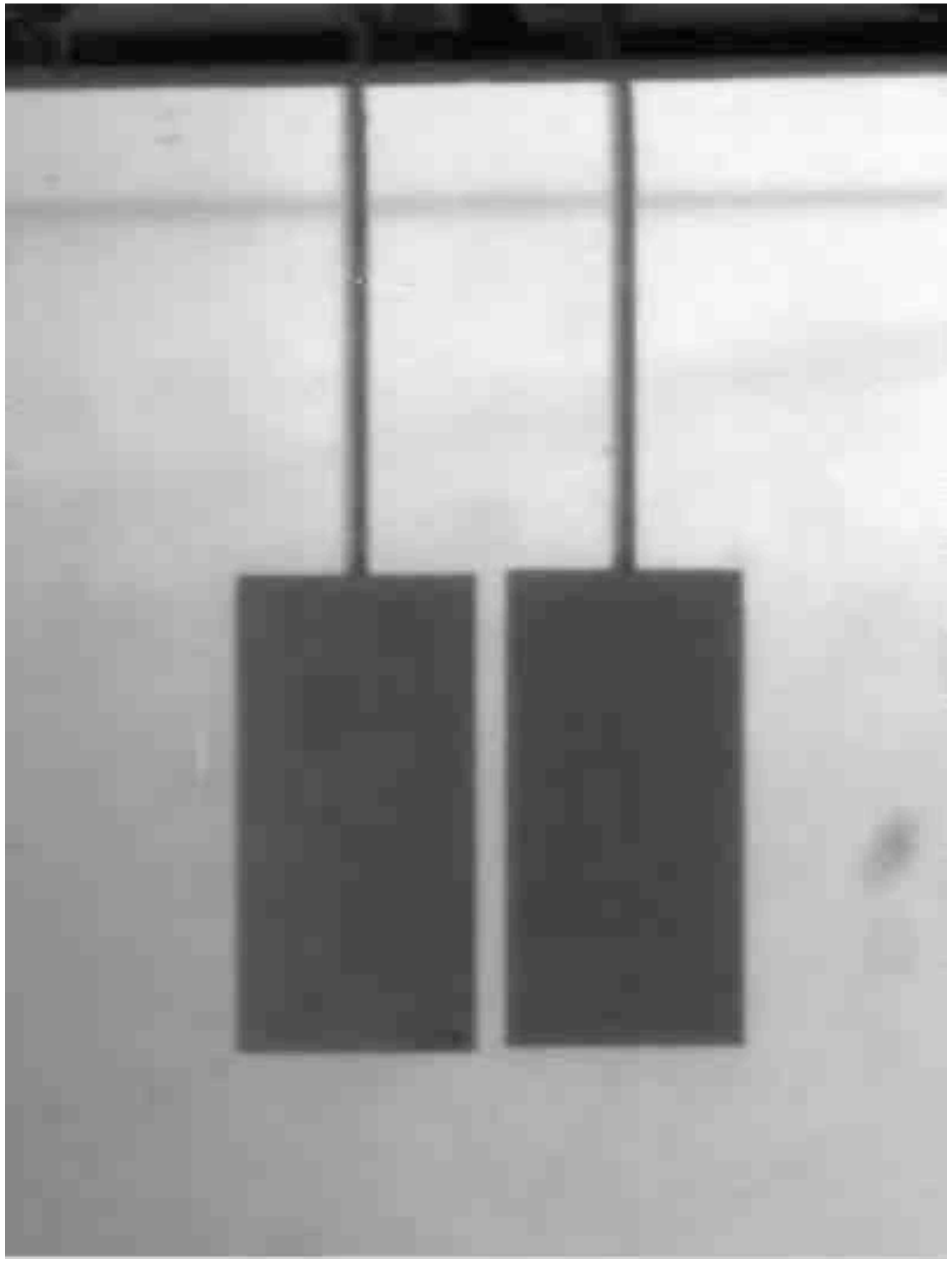


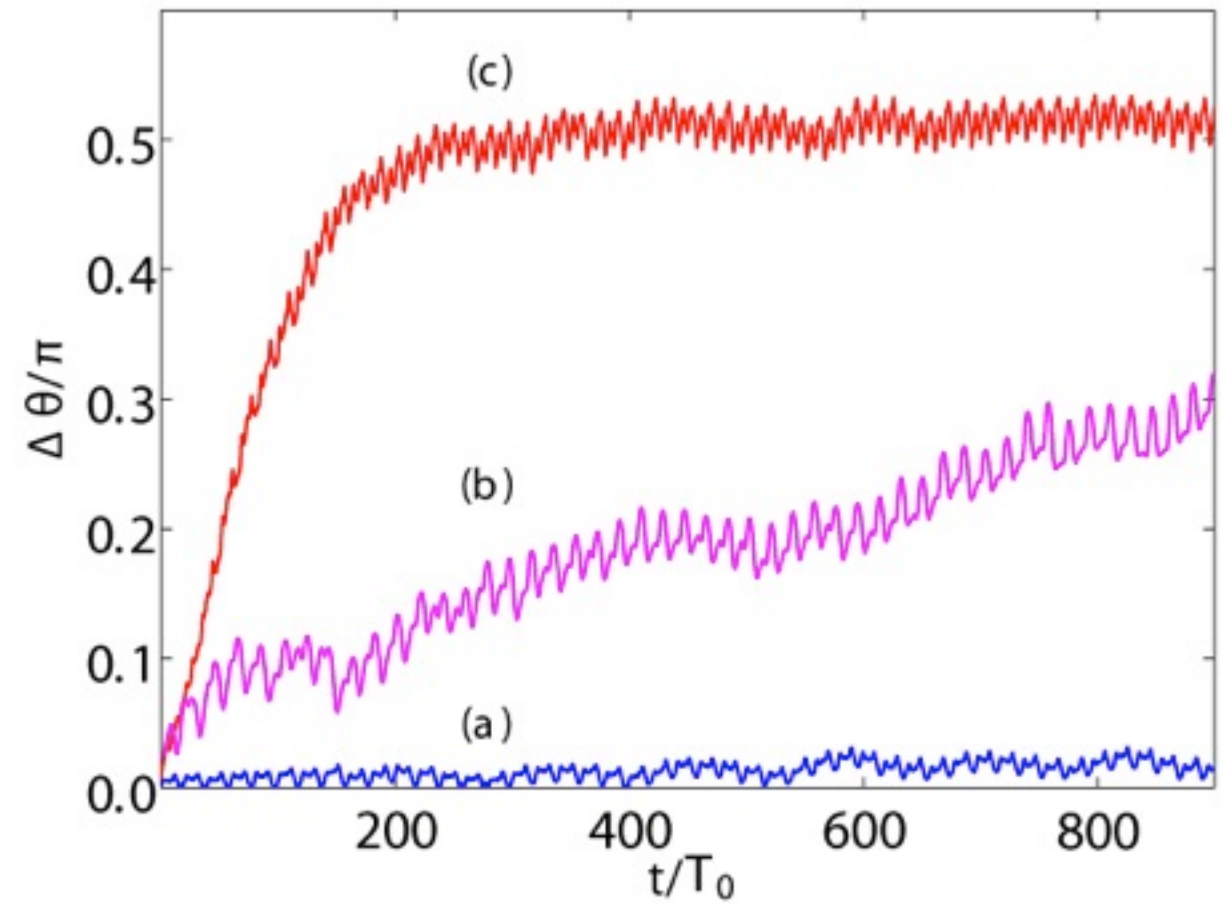
Two-paddle model system



$$Re = \mathcal{O}(10^{-3})$$

Two-paddle interactions





$$T_o = 6\pi\mu R^3/M$$

a) Rigid shafts – no synchronization
b) Rigid shafts – torque mismatch
   – Phase wandering
c) Flexible shafts – synchronization!

(Qian et al, PRE 2009)

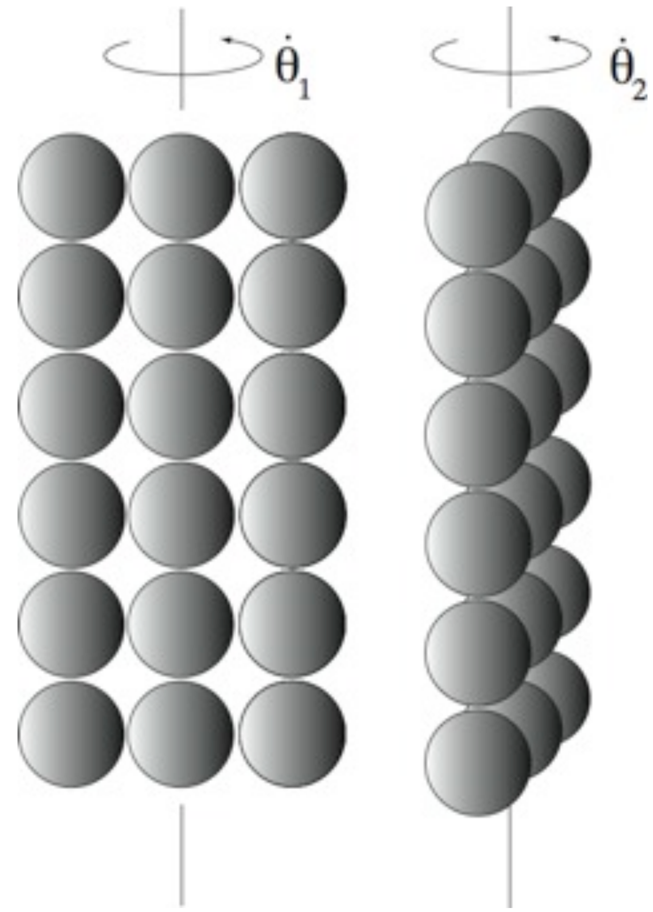
Numerical simulation - Regularized

$$\mu \nabla^2 \mathbf{v} - \nabla p + \mathbf{F} = 0$$

$$\nabla \cdot \mathbf{v} = 0$$

$$\mathbf{F} = \frac{15\epsilon^4}{8\pi} \frac{1}{(r^2 + \epsilon^2)^{7/2}}$$

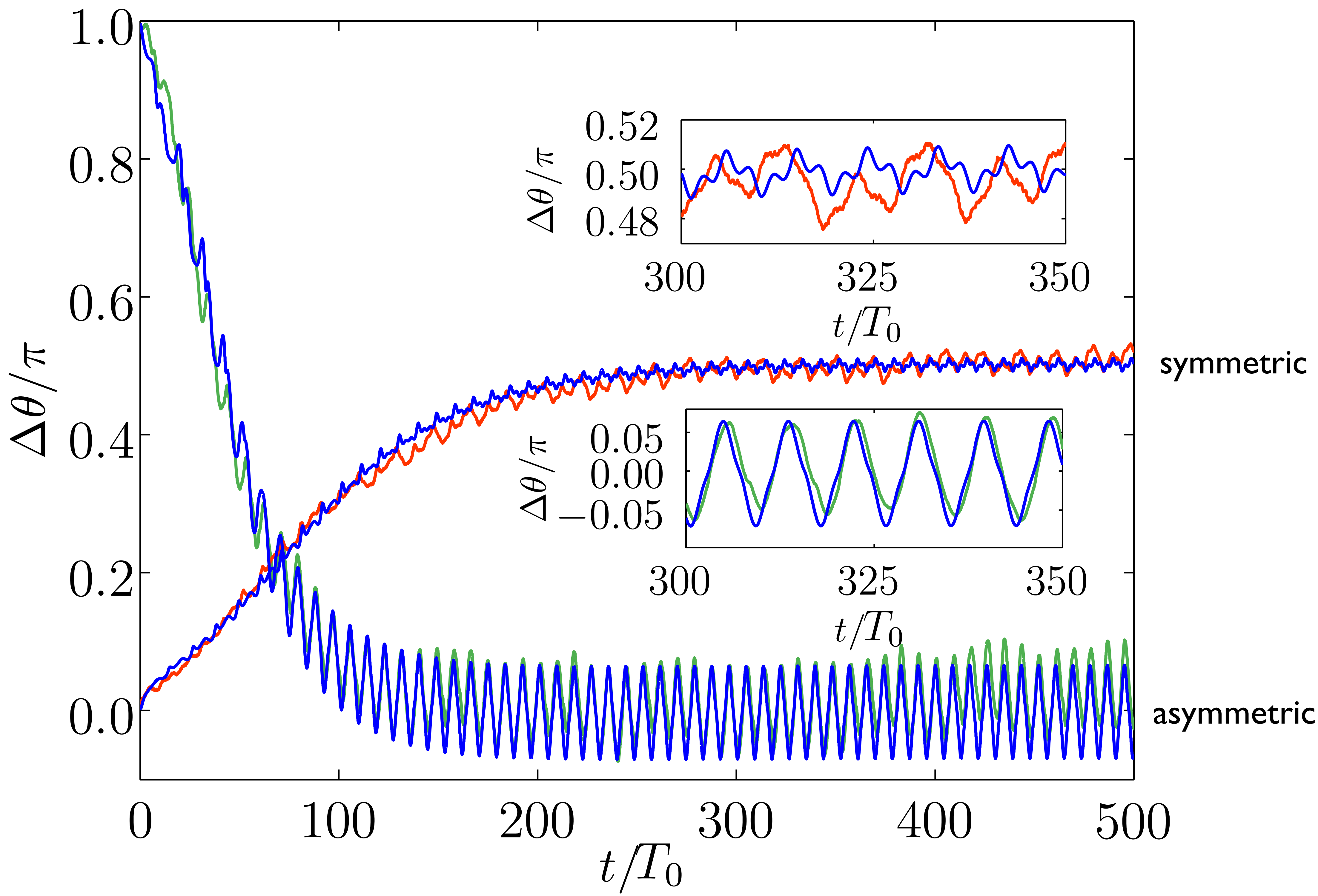
(Cortez, 2001)



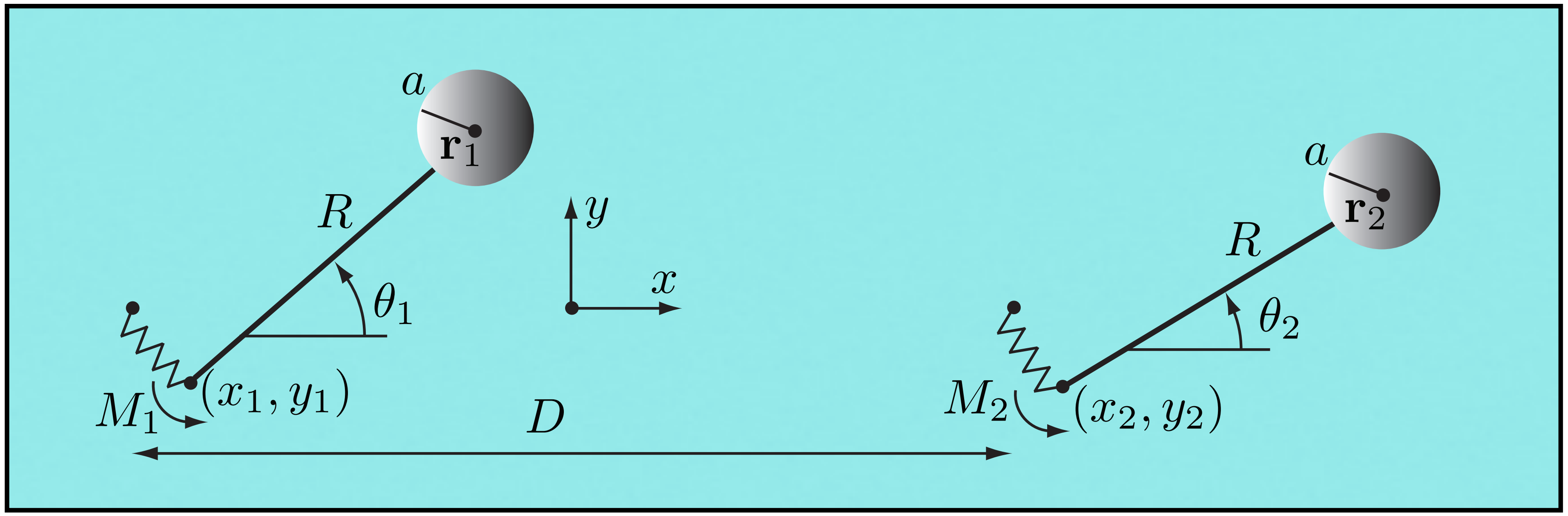
$$\begin{pmatrix} M_1 \\ M_2 \end{pmatrix} = \begin{pmatrix} A(\theta_1, \theta_2) & B(\theta_1, \theta_2) \\ B(\theta_1, \theta_2) & A(\theta_1, \theta_2) \end{pmatrix} \begin{pmatrix} \dot{\theta_1} \\ \dot{\theta_2} \end{pmatrix}$$

Kim & Powers (2004) - rigid helices – no synchronization
Reichert & Stark (2005) - flexible coupling leads to synchronization

Simulation and experiment



Analytical approach



velocity on ball *i*, induced by ball *j*

ignore "far field" term

$$v_i = \frac{F_i}{6\pi\mu a} + \frac{1}{8\pi\mu}\sum_{i\neq j}\left[\frac{F_j}{|r_{ij}|} + \frac{(F_j \cdot r_{ij})r_{ij}}{|r_{ij}|^3}\right]$$

$$a \ll R \ll D$$

Sum of forces and moments; for two balls:

Niedermeyer et al. (Chaos, 2008)

Qian et al. (PRE, 2009)

$$-\frac{\Delta M}{M_1} + \Delta\dot{\theta} + \frac{9}{8}\frac{a}{D}\Delta\dot{\theta}\cos\Delta\theta + \frac{3}{8}\frac{a}{D}\cos(2\bar{\theta}) = 0$$

$$\frac{\Delta M}{M_1} + 2 - 2\dot{\bar{\theta}} - \frac{3}{4}\frac{a}{D}\dot{\bar{\theta}}\left[-3\cos\Delta\theta + \cos(2\bar{\theta})\right] = 0$$

multiple-scale (slow) evolution equation for phase difference: $(\mu\omega/k \ll 1)$

$$\frac{\langle\Delta\theta\rangle}{dt} = -\boxed{\frac{9}{2}\frac{M_1}{kR^2}\frac{a}{D}}\sin\langle\Delta\theta\rangle + \frac{\Delta M}{M_1}$$

1/T time constant for synchronization

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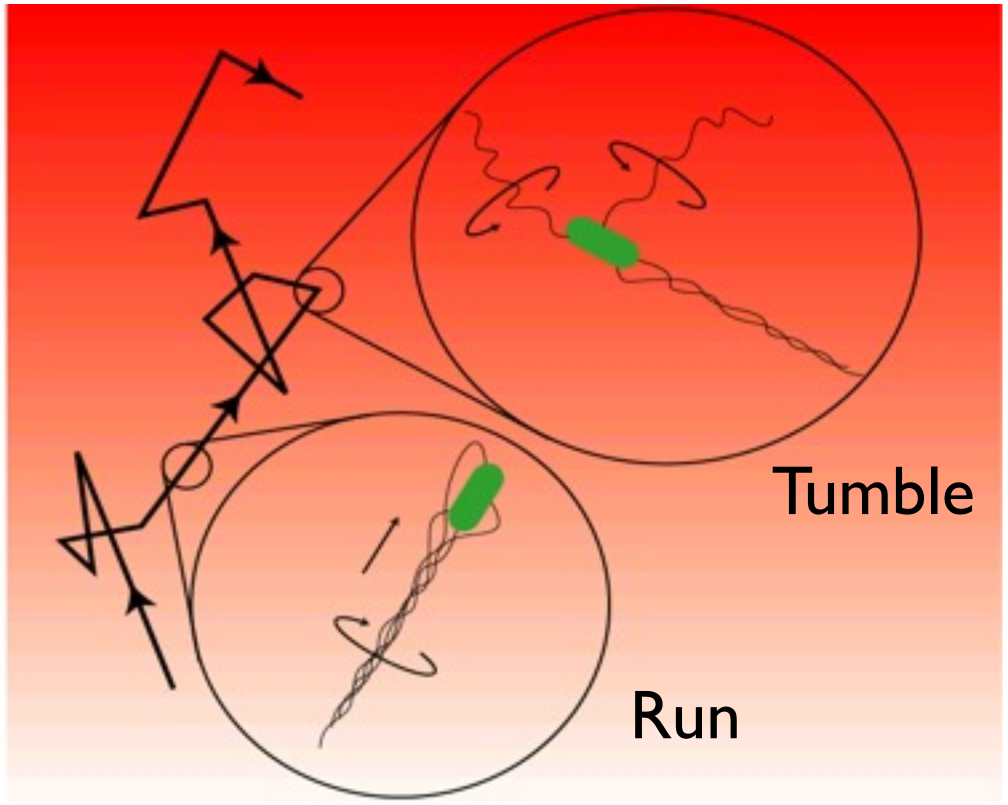
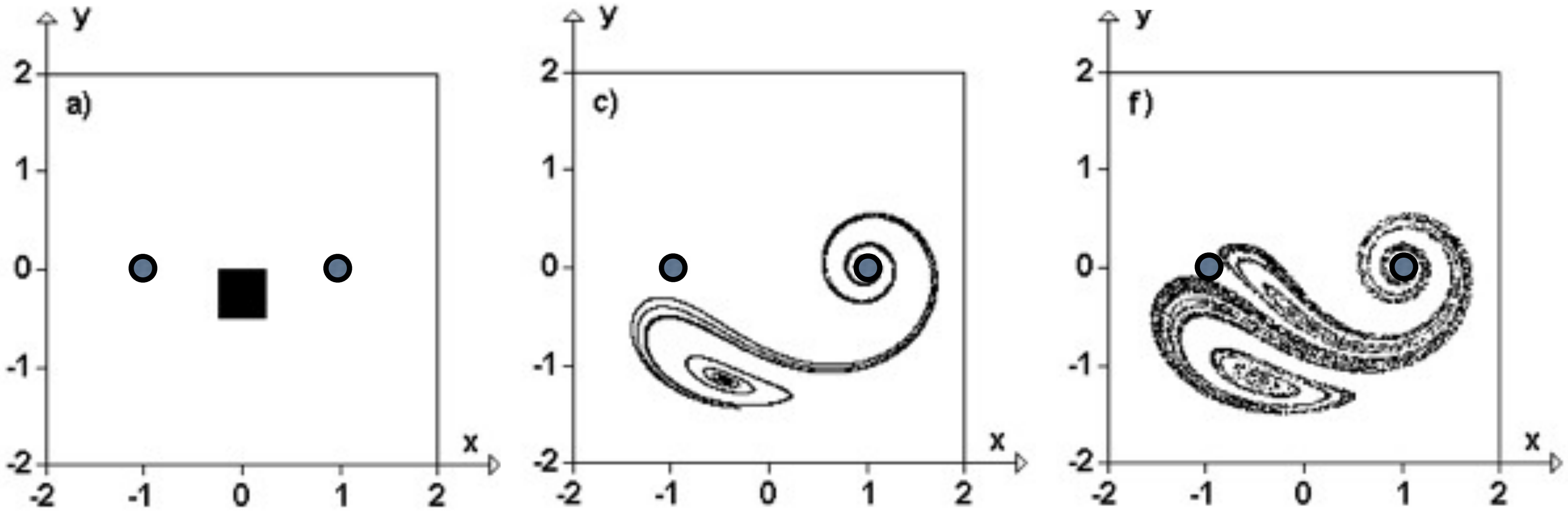
  - etc.
- What next?

Motile bacteria as chaotic mixers

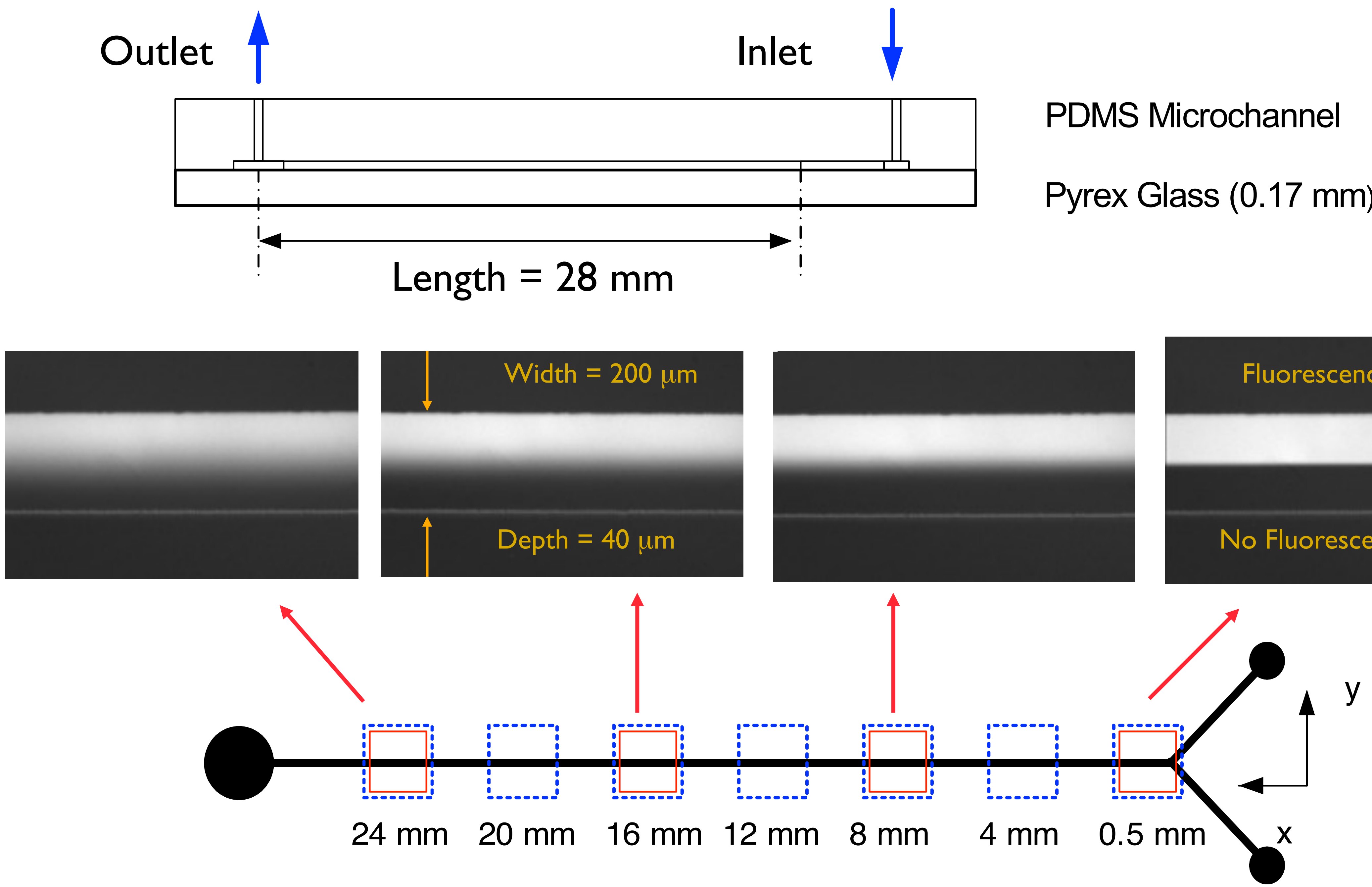
- Chaotic advection (Aref 1984)
  - Viscous laminar flow
  - Random switching of "blinking vortices"
- Bacteria have flagella that alternate rotation direction randomly
  - Cell bodies execute random walk
    - Chaotic mixing in zero Re flow?



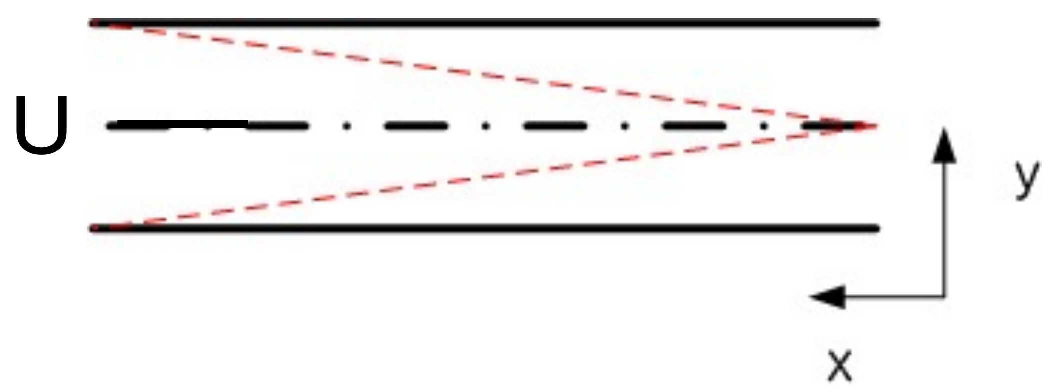


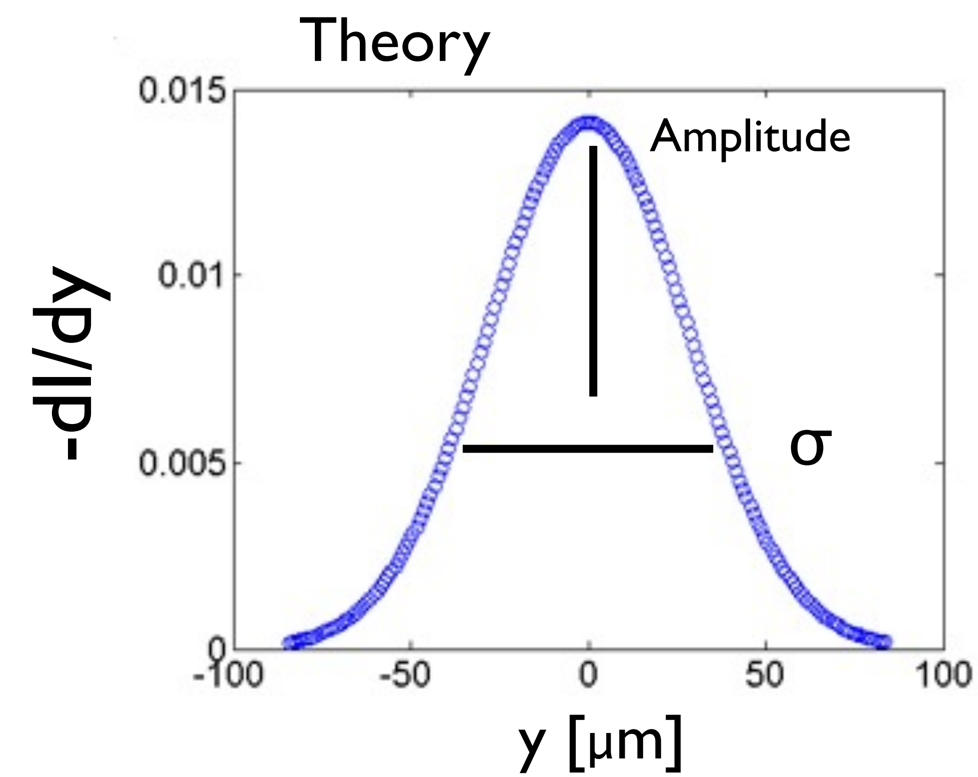
(Images courtesy of Károlyi György)

Mixing using freely swimming bacteria



Theory for standard diffusion.



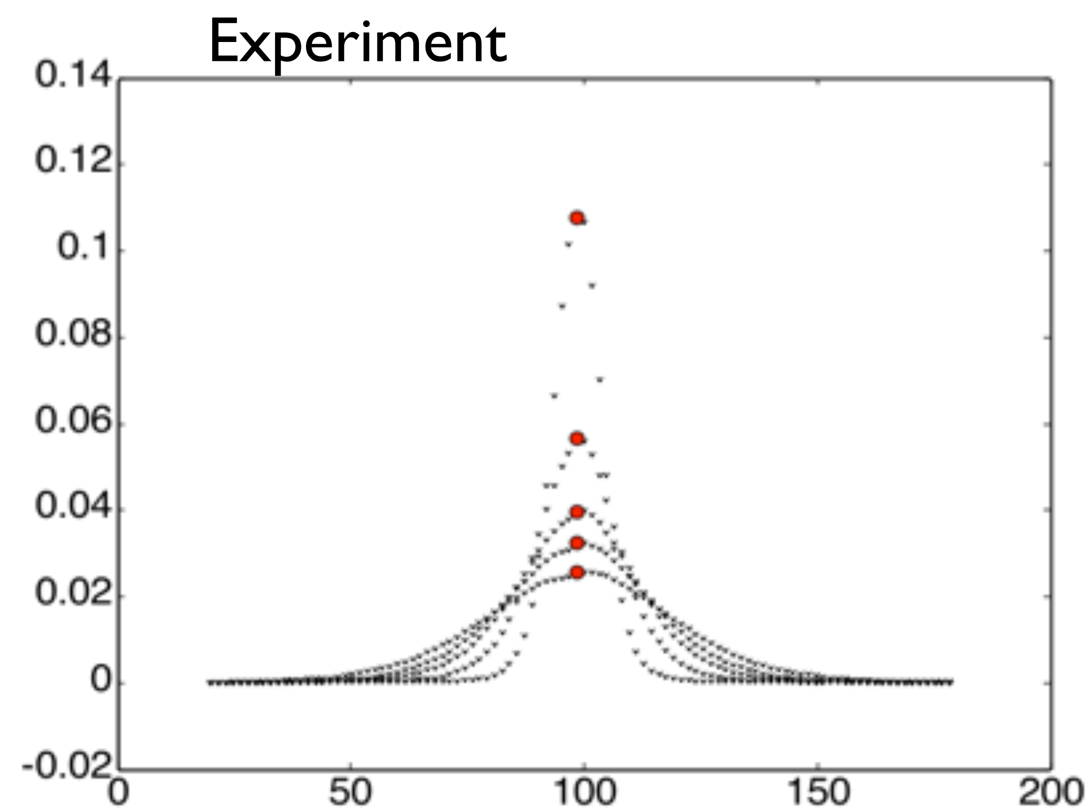


Diffusion Equation: $$U\frac{\partial I}{\partial x} = D\frac{\partial^2 I}{\partial y^2}$$

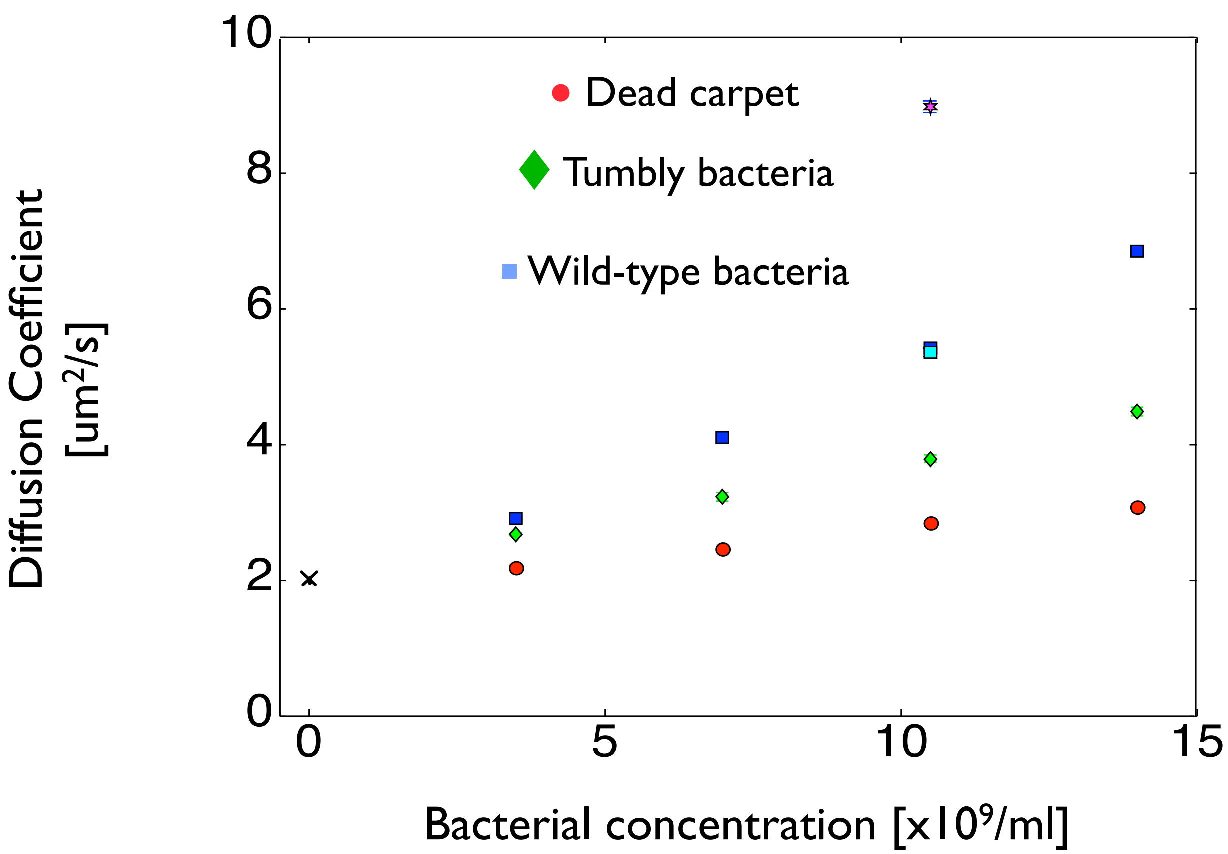
Intensity: $$I(y,x) = \frac{1}{2} I_0 erfc \frac{y}{2\sqrt{D\tau}}$$

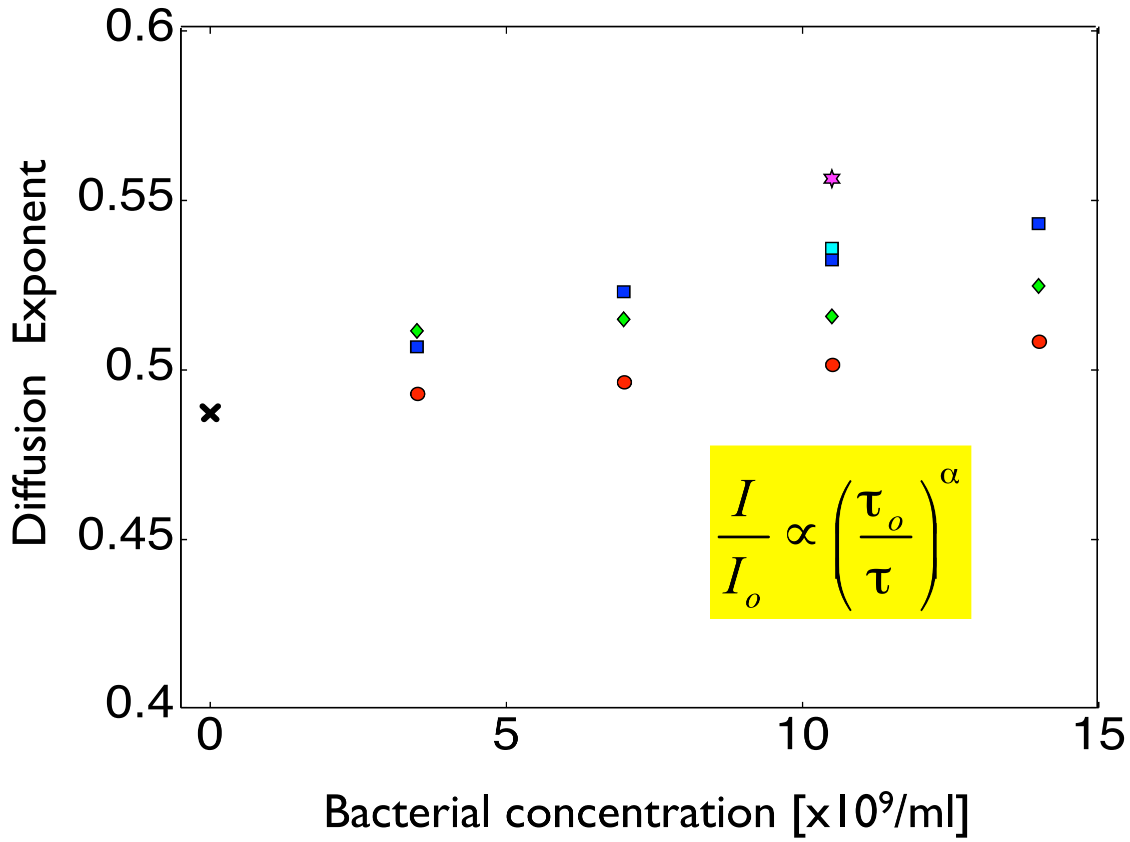
Intensity Gradient: $$-\frac{dI}{dy} = \frac{1}{2}\frac{I_0}{\sqrt{\pi D\tau}} \exp^{-(\frac{y}{2\sqrt{D\tau}})^2}$$

Similarity Variable: $$\tau = \frac{x}{U}$$



Enhanced diffusion due to bacteria

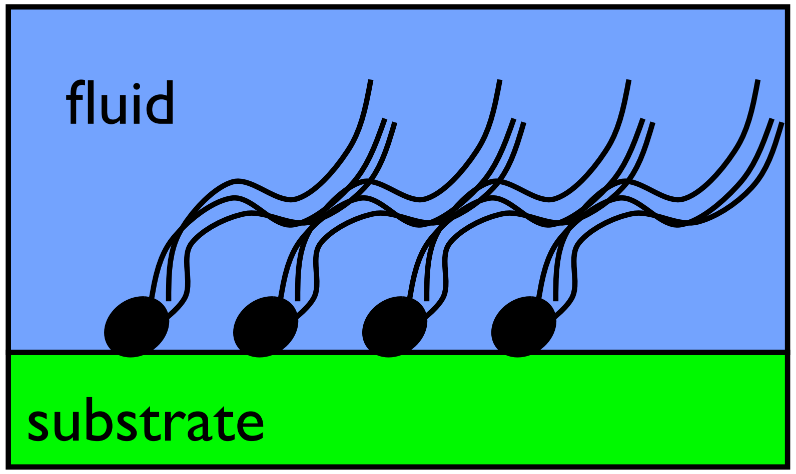


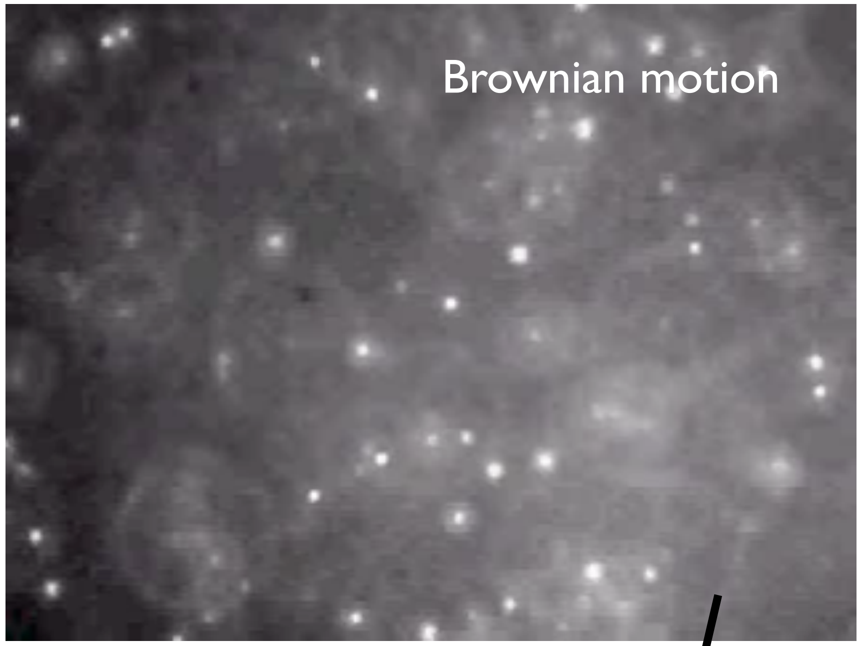


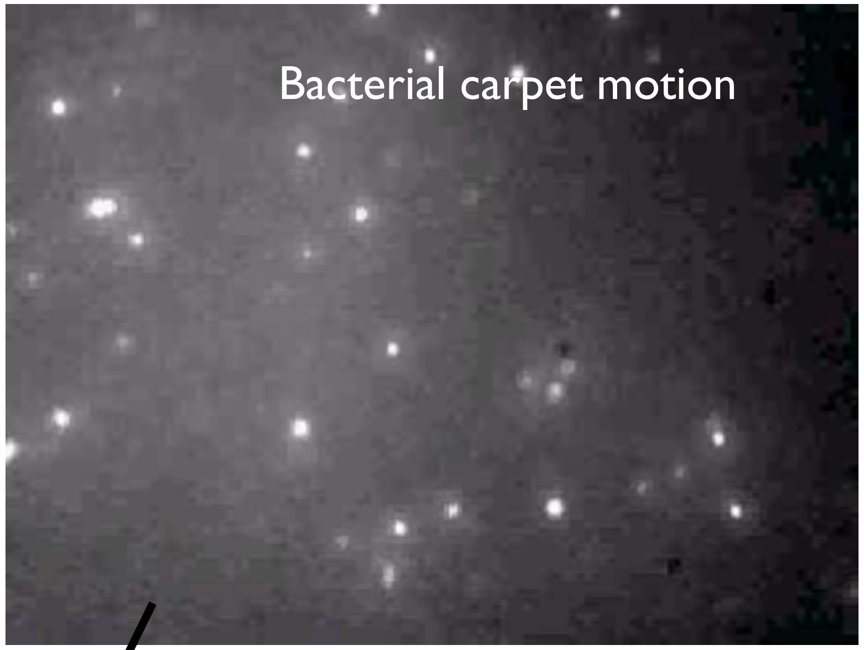
- Diffusion of small molecule rises from 20 – 80 um^2/s
  - Large molecules and 1 um beads show 50x enhancement
- Diffusion faster than standard "Fickian" diffusion
  - Introduction of new time scale

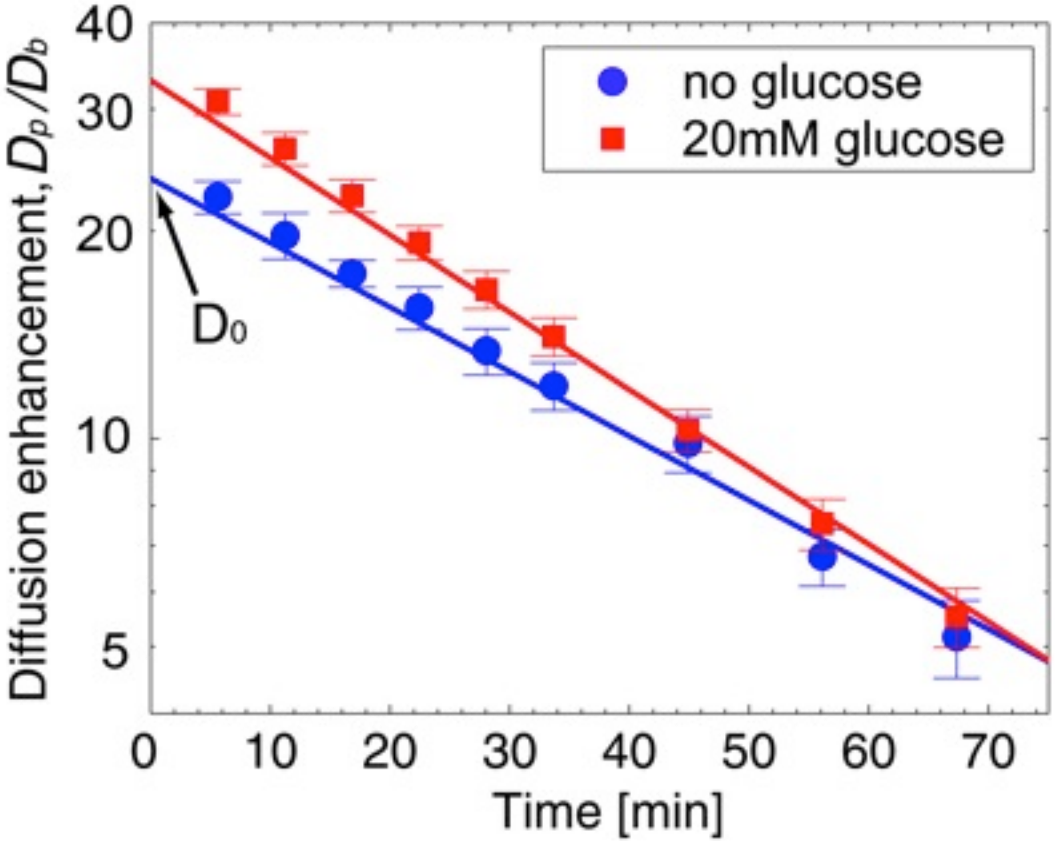
(Kim & Breuer, Phys Fluids 2004)

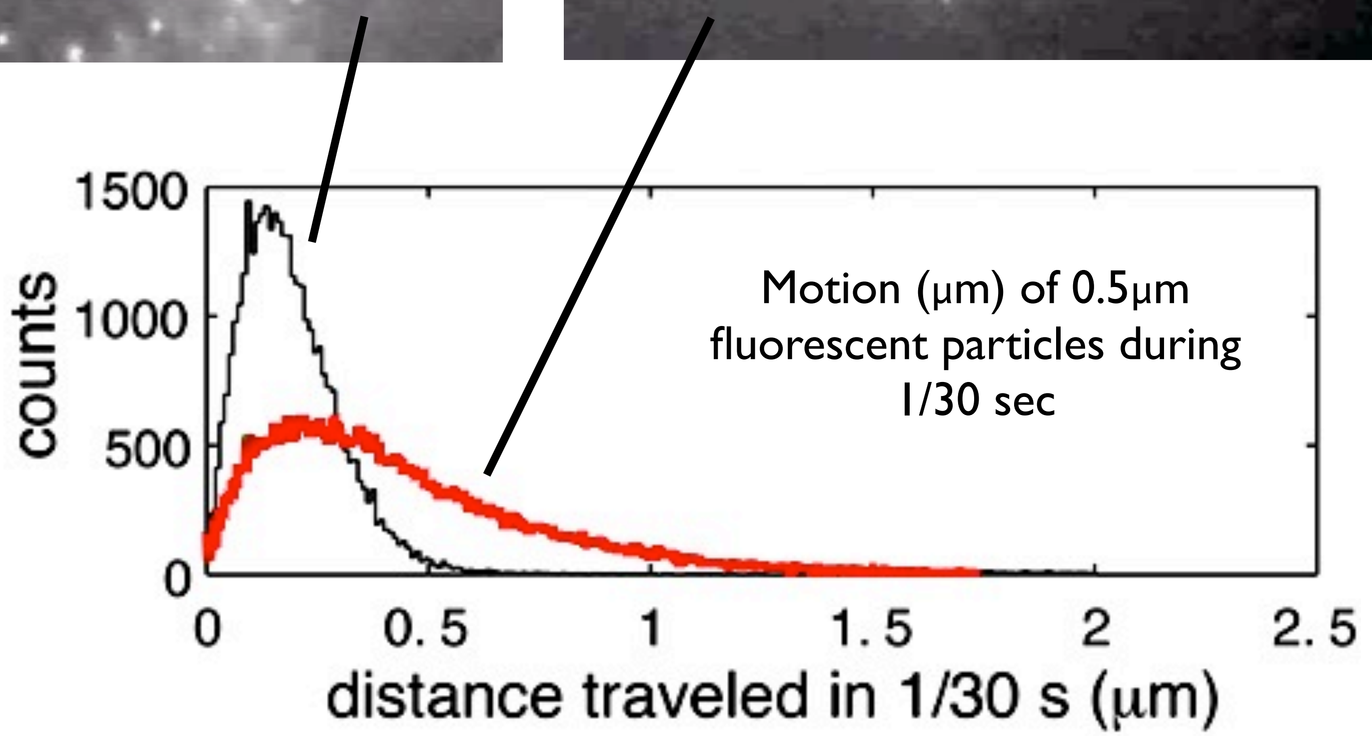
Bacterial carpets and enhanced mixing











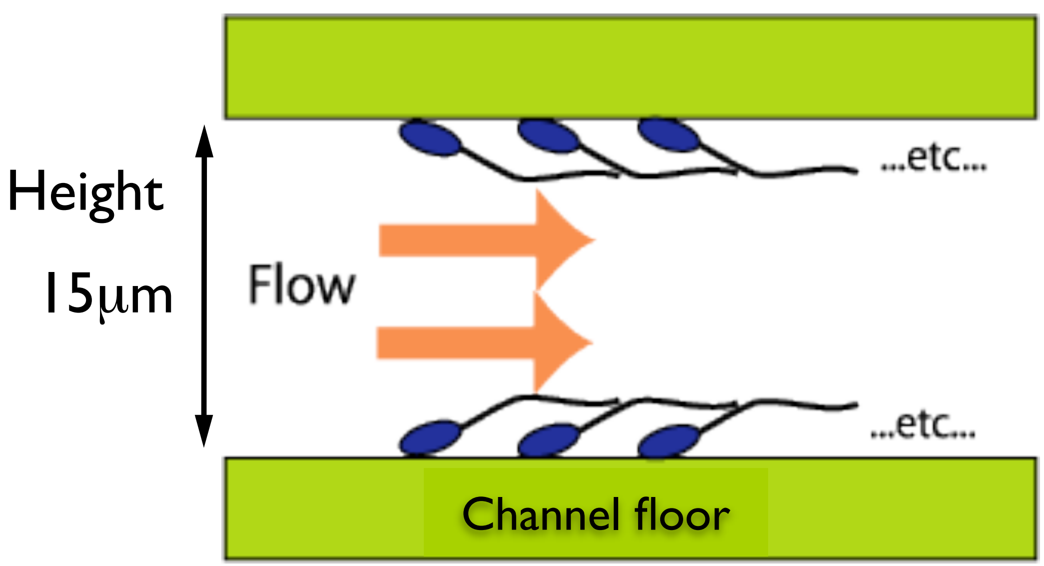
(Darnton, Turner, Breuer & Berg Biophys J. 2003)

Bacterial pumps

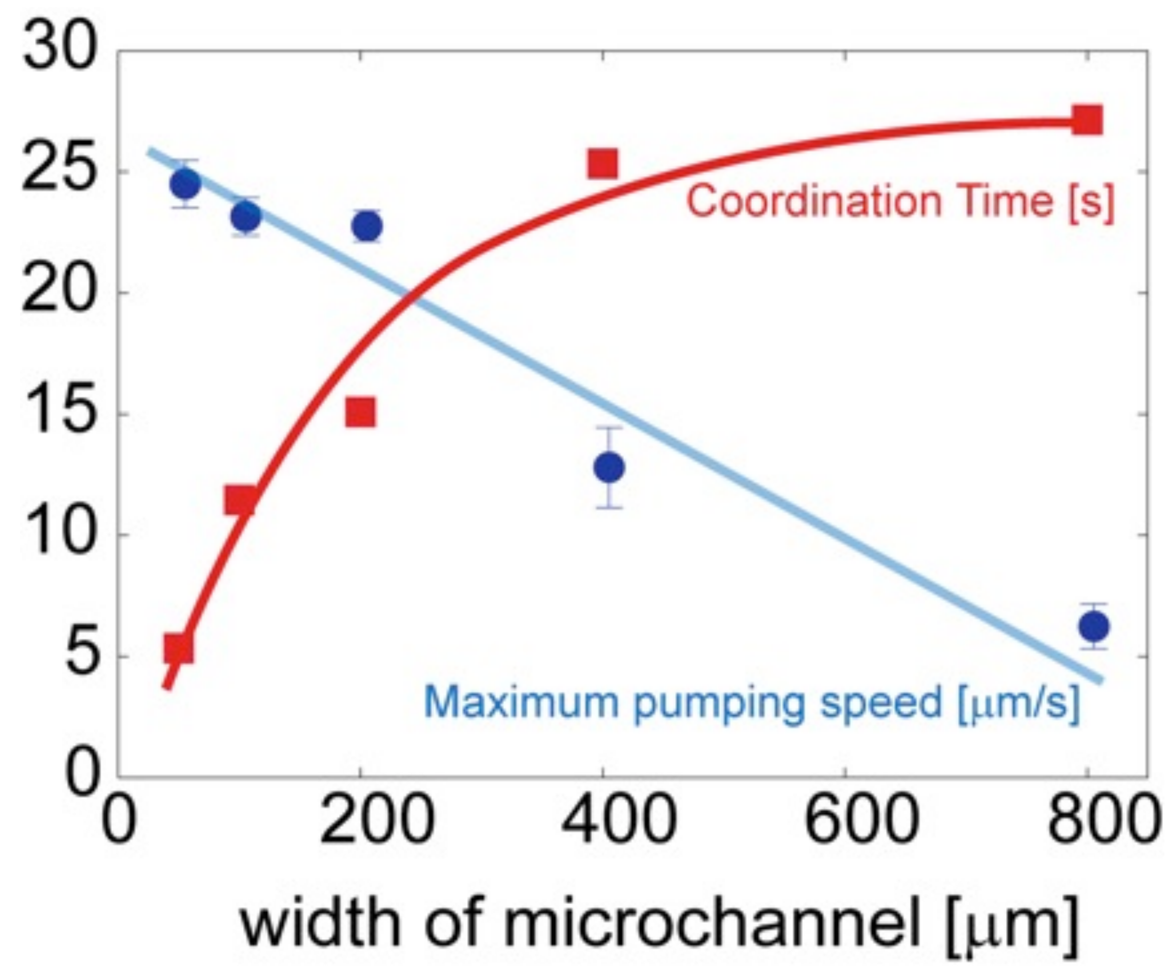
Top View

Width
~500μm

Side View



(Kim & Breuer, Small 2008)



- Pumping arises spontaneously
- Direction determined by "last flush"
- "Crystallization" process

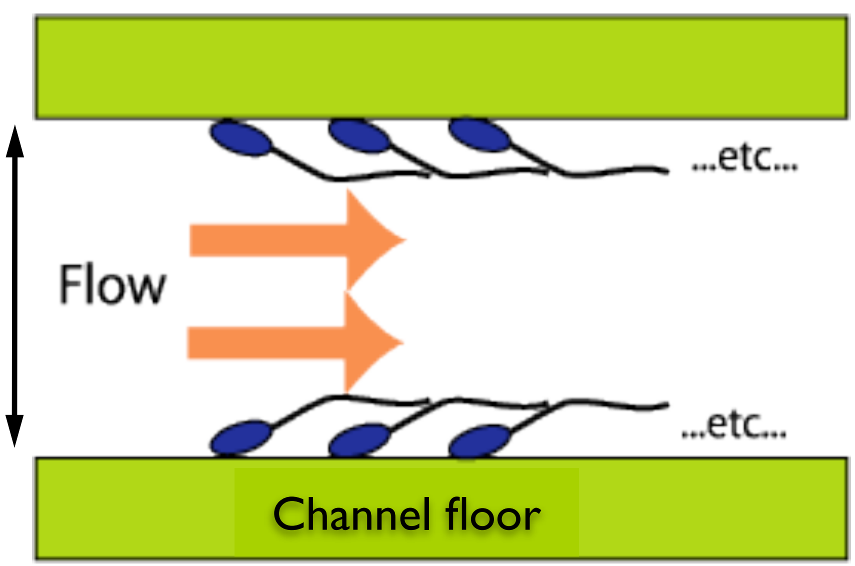
Bacterial pumps

Top View

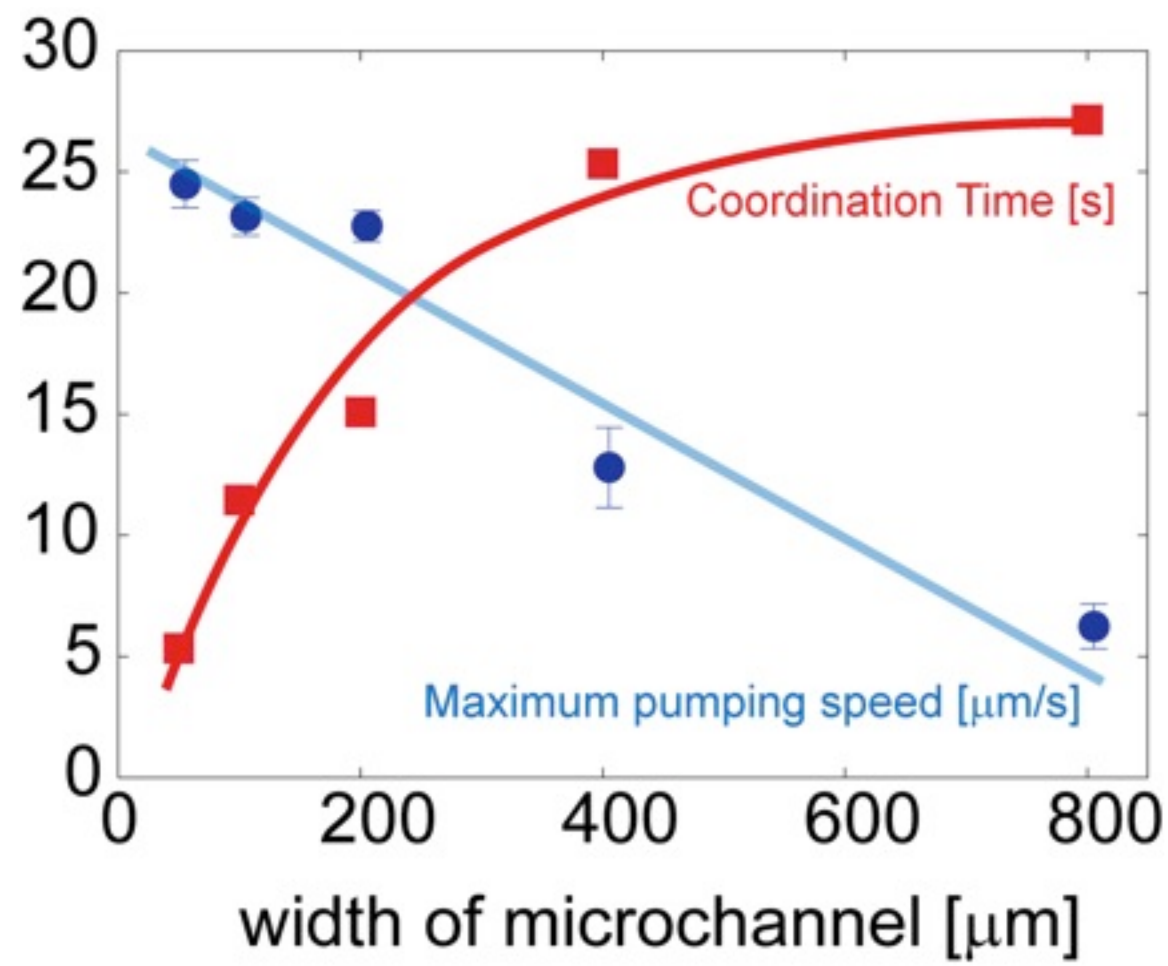
Width
~500μm

Side View

Height
15μm



(Kim & Breuer, Small 2008)



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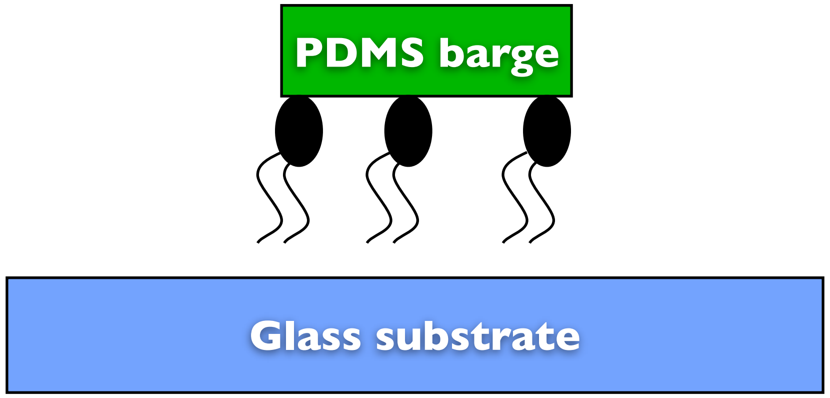
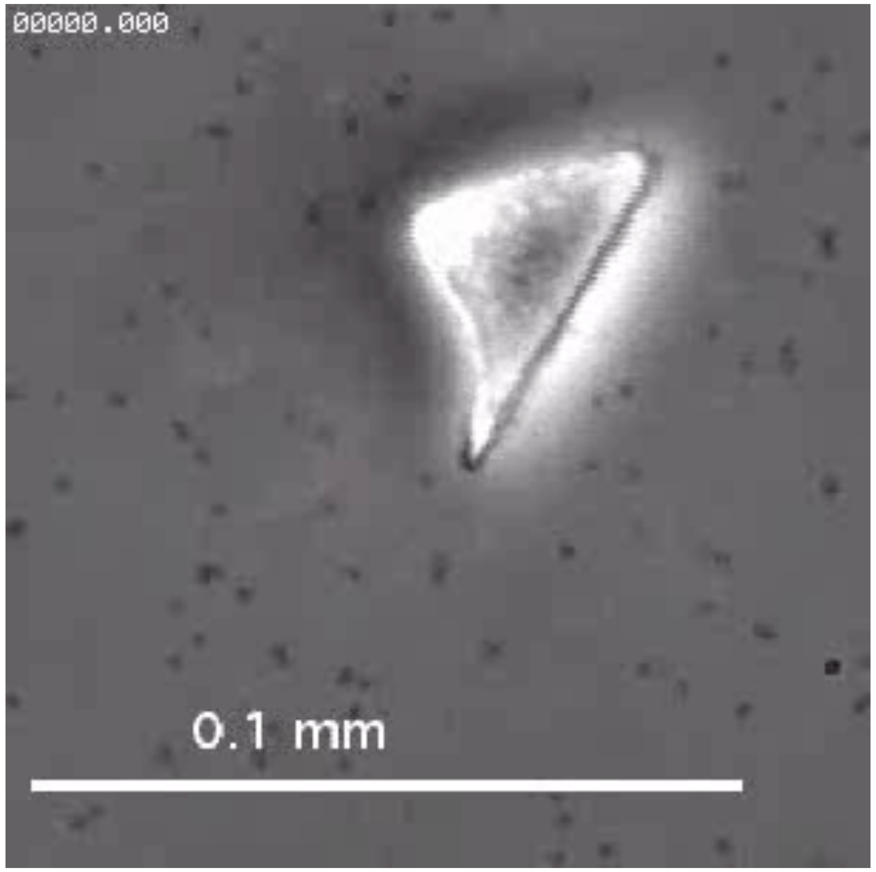
Bacterial barges

- Rotation 33 deg/s
- Surface area 1400 um^2
- Long axis 65 um
- Bacteria 300



- Translates 5 um/s
- Rotation 4.6 deg/s
- Surface area 4350 um^2
- Long axis 92 um
- Bacteria 1100



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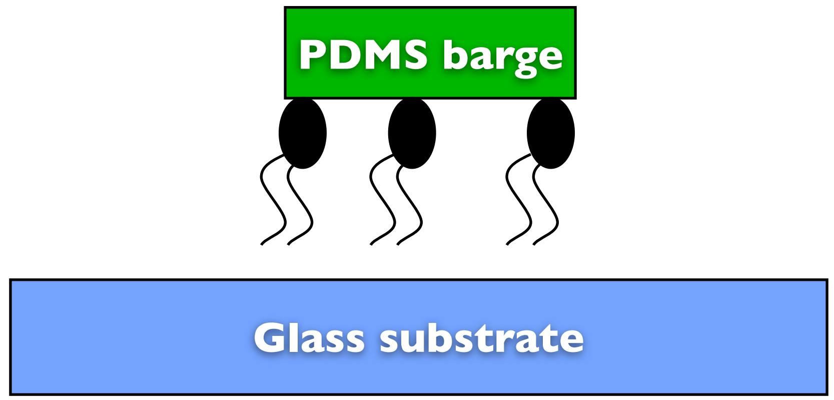

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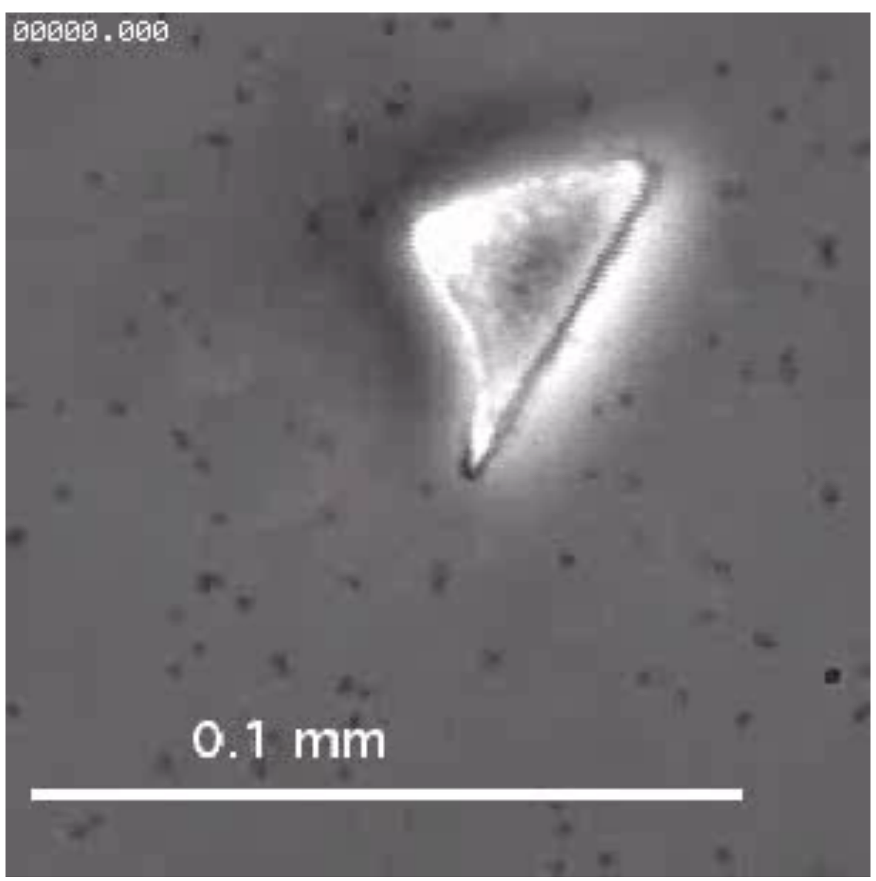

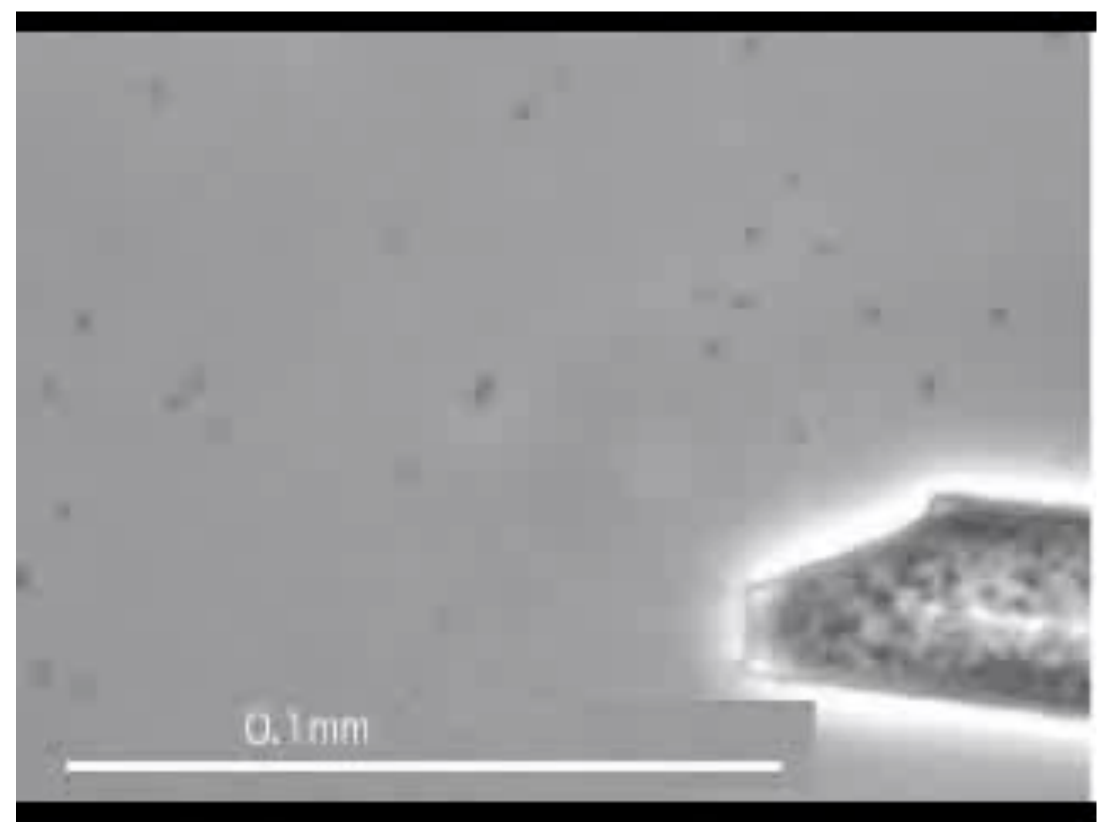

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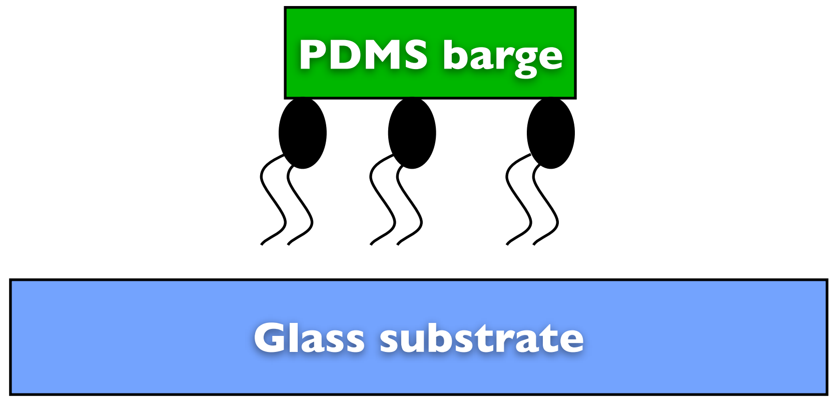

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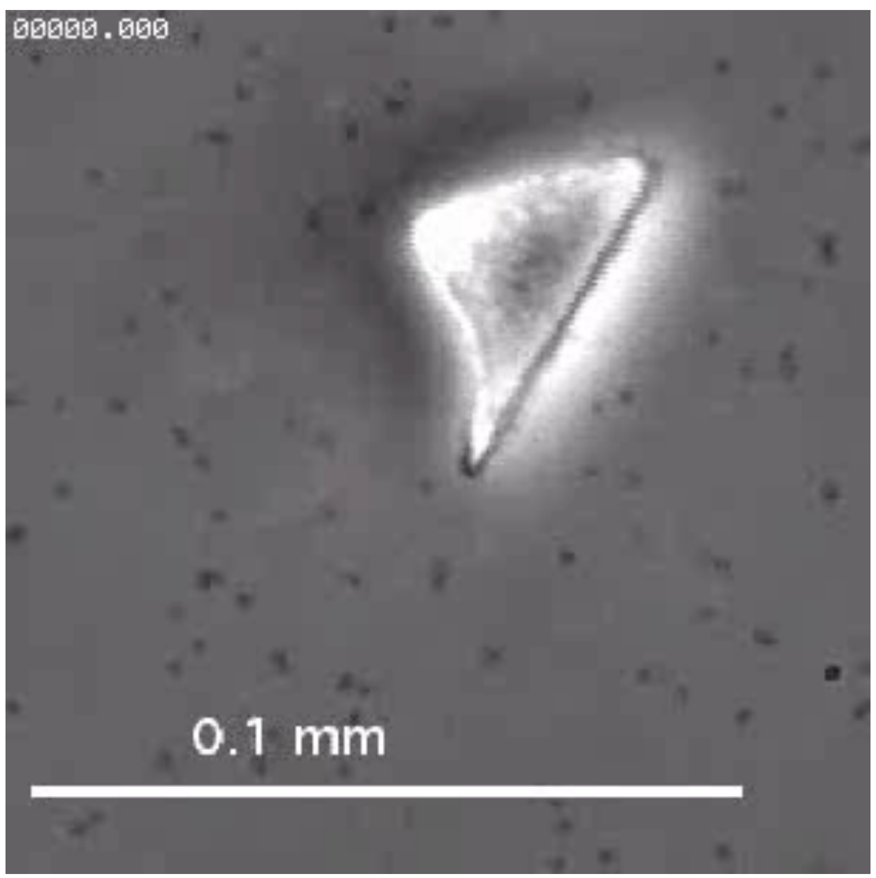

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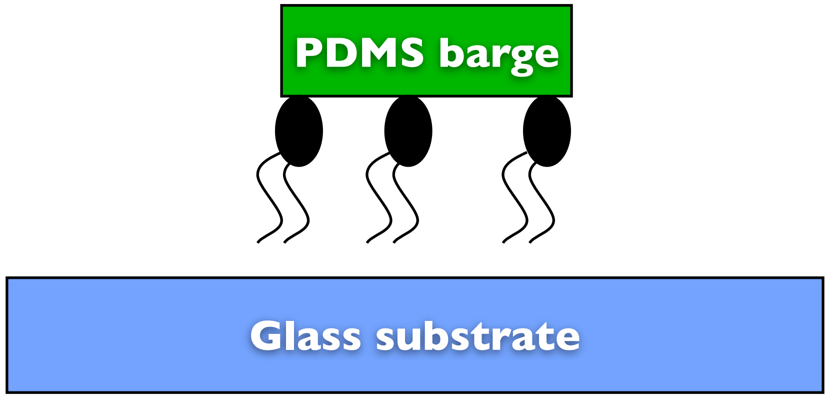

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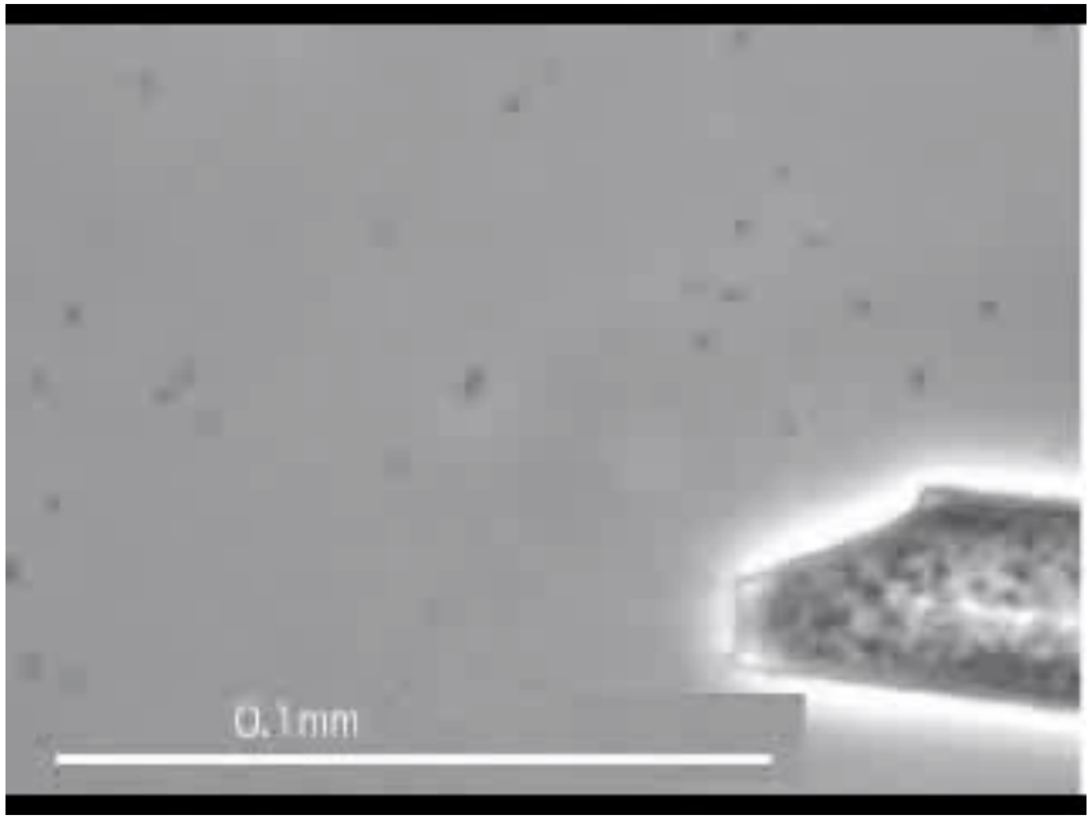
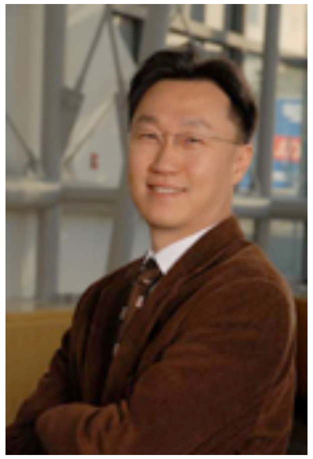


Summary:

- Force-free swimming speed measured:
  - newtonian fluids
  - viscoelastic fluids underway
- Fluids-filament interactions:
  - coiling of an elastic filament due to viscous stresses
- Synchronization of adjacent filaments
  - hydrodynamic interactions
  - elastic compliance necessary
  - multiple paddles illustrate signficantly more complex (& chaotic) behavior



- Acknowledgements: Tom Powers, Bin Liu, Min Jun Kim, Qian Bian, Dave Gagnon
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