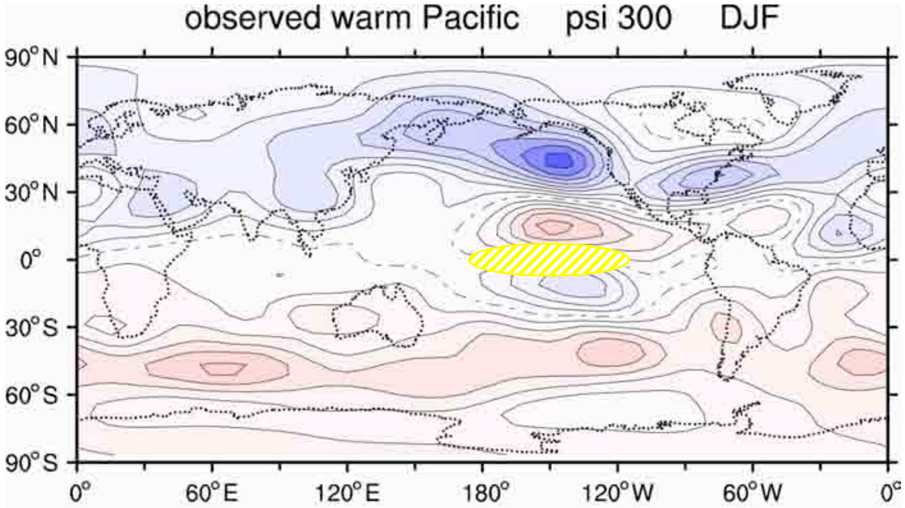
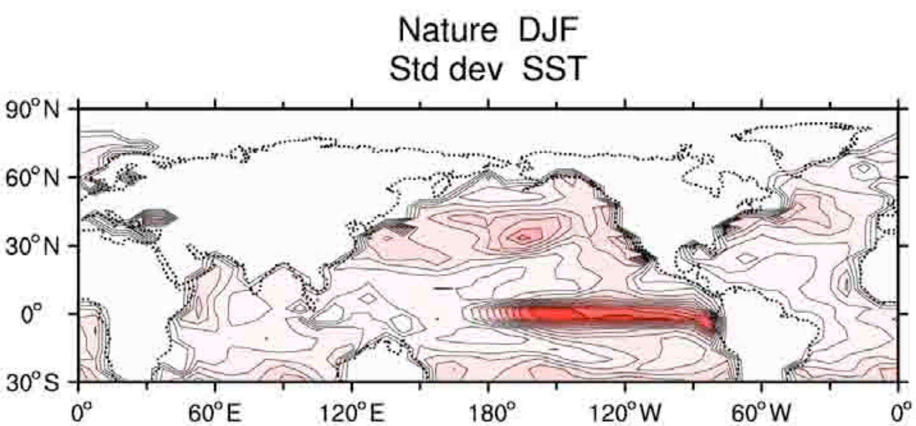




July - September 1987 OLR

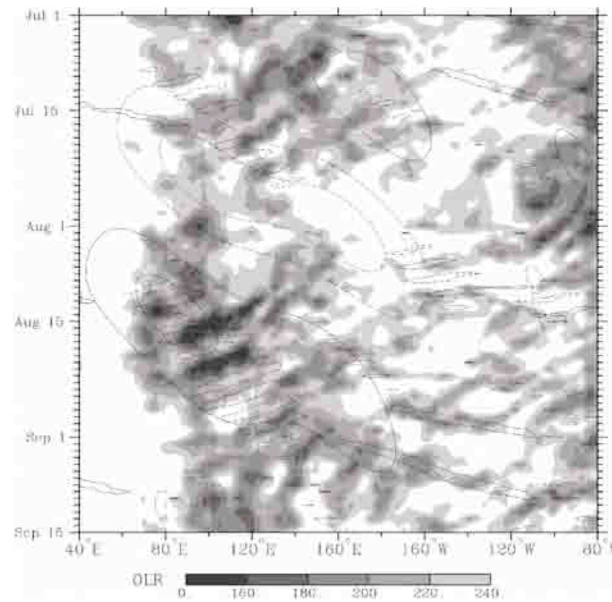


FIG. 6. Time–longitude plot of total OLR (shading, as indicated), filtered ISO, and MRG–TD OLR (contours, solid negative, contour interval 10 W m^{-2}, zero contour omitted), and filtered Kelvin wave OLR (contoured at −12 W m^{-2} only), averaged from 2.5° to 15°N, from 1 Jul to 15 Sep 1987.

Straub & Kiladis (2003)

Atmospheric Response Operators from the Fluctuation Dissipation Theorem: Validation and Applications

Grant Branstator, NCAR

Andrey Gritsun, RAS

Andrew Majda, CIMS

- Data availability
- “Friendlier” conditions
- Generalization to functionals of state

FDT Generalized to Functionals of State

Suppose have discretized dynamical system

$$\frac{d\phi}{dt} = F(\phi, \Lambda, f); \phi\big|_{t=0} = \phi_0; \phi, \phi_0 \in \Phi \qquad (\#)$$

Interested in

$$\langle A(\phi) \rangle = \int A(\phi)\rho(\phi)d\phi, \text{ for stationary PDF } \rho$$

Suppose f is changed by a small δf. Want

linear U] $\delta \langle A(\phi) \rangle = U\delta f$

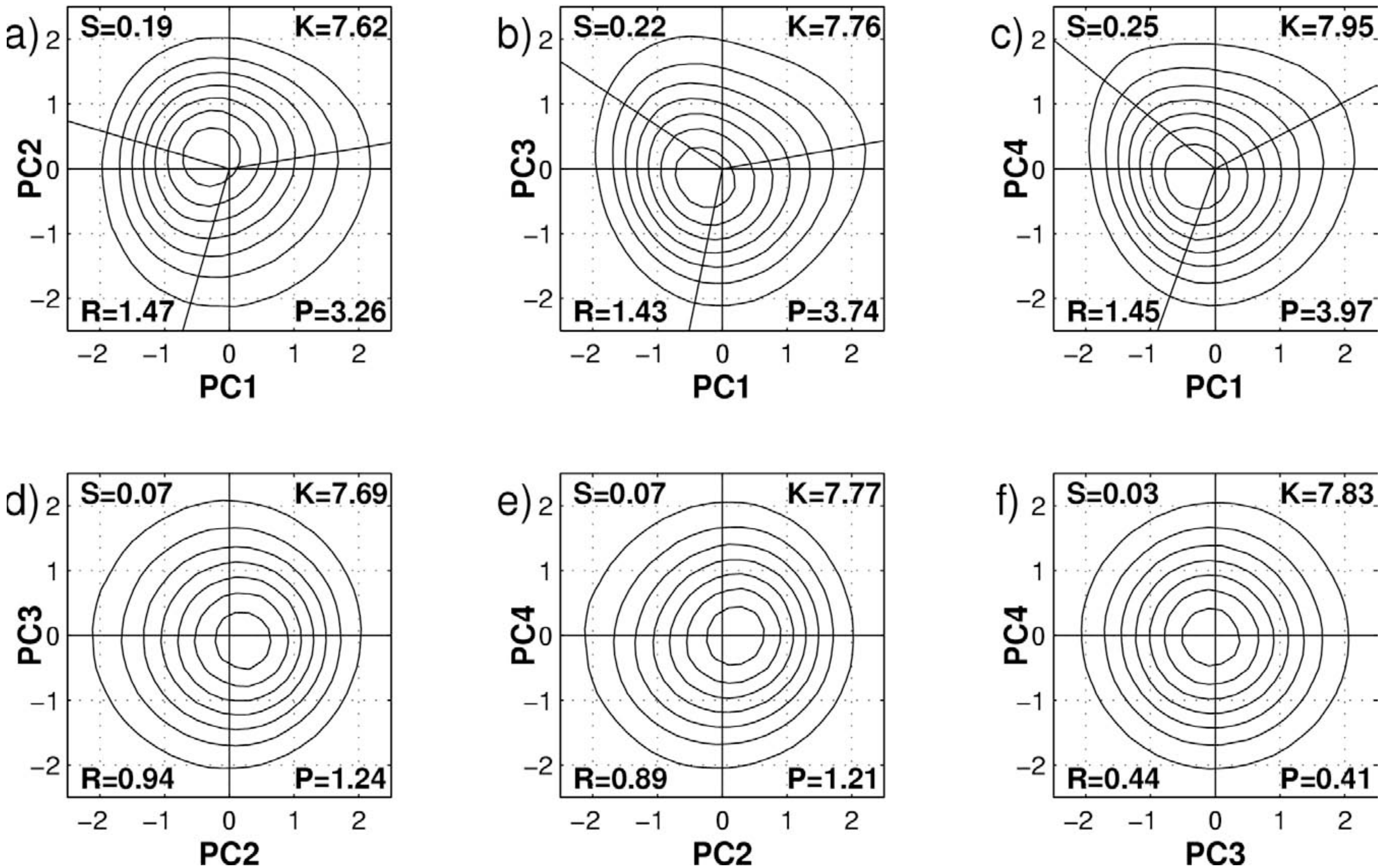
If (*Deker & Haake, 1975; Risken, 1984*)

* (#) is assumed to include a small noise term so its PDF is not fractal

* (#) has a F - P equation with unique solution,

then

$$U_{ij}(t) = \int_0^t \left\langle A(\phi_i(t'+\tau)) \frac{\partial \ln \rho(\phi(t'))}{\partial \phi_j(t')} \right\rangle d\tau$$



Berner & Branstator (2006)

Generalized FDT -- Simplifications

For a system like the earth's atmosphere

$$\rho \cong \rho_0 \exp\left\{-(C^{-1}(0)\phi', \phi')/2\right\}$$

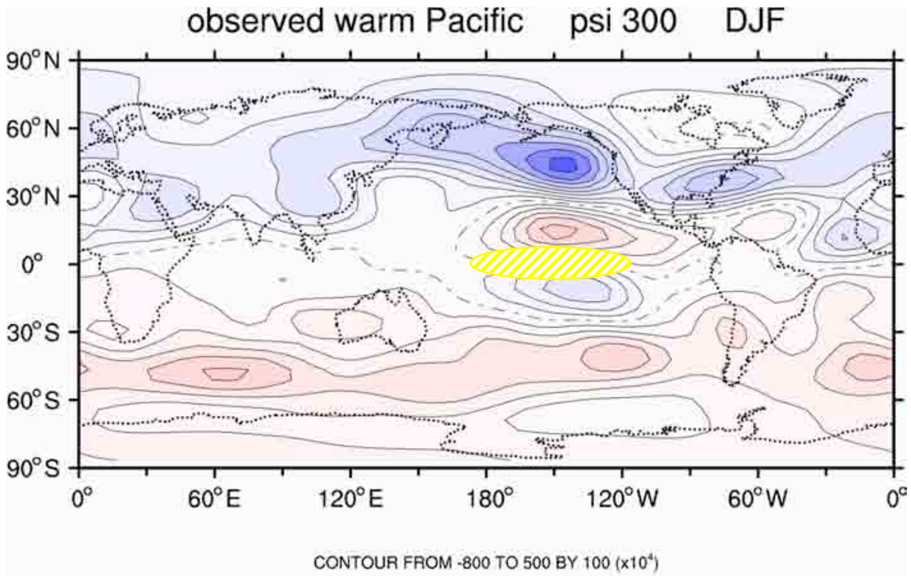
So

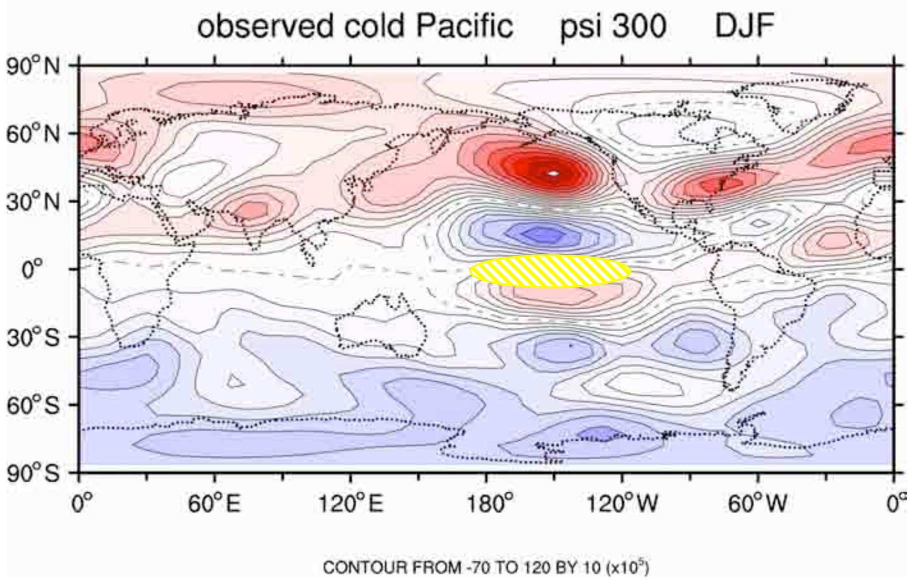
$$\boxed{U(t) \cong \int_0^t \left\langle A(\phi'(t'+\tau))(\phi'(t))^T \right\rangle C^{-1}(0)\,d\tau}$$

Note if $A = I$,

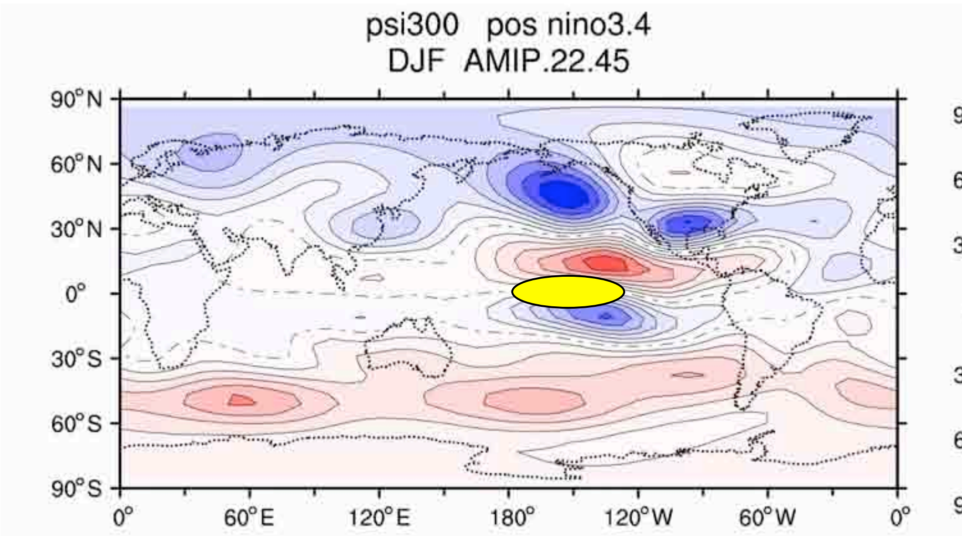
$$U(t) = \int_0^t C(\tau)C^{-1}(0)\,d\tau$$

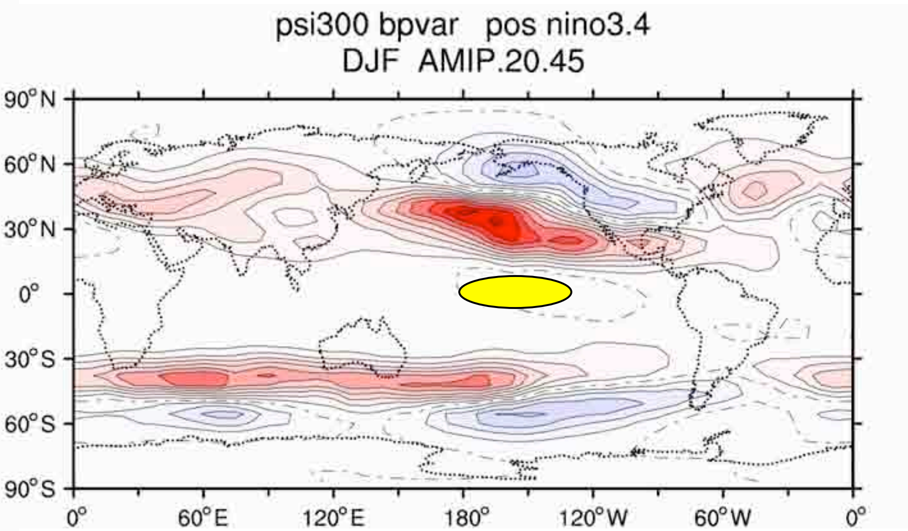
Majda, Abramov & Grote (2005) found third order accuracy for $A = I$ and second order accuracy for quadratic A.

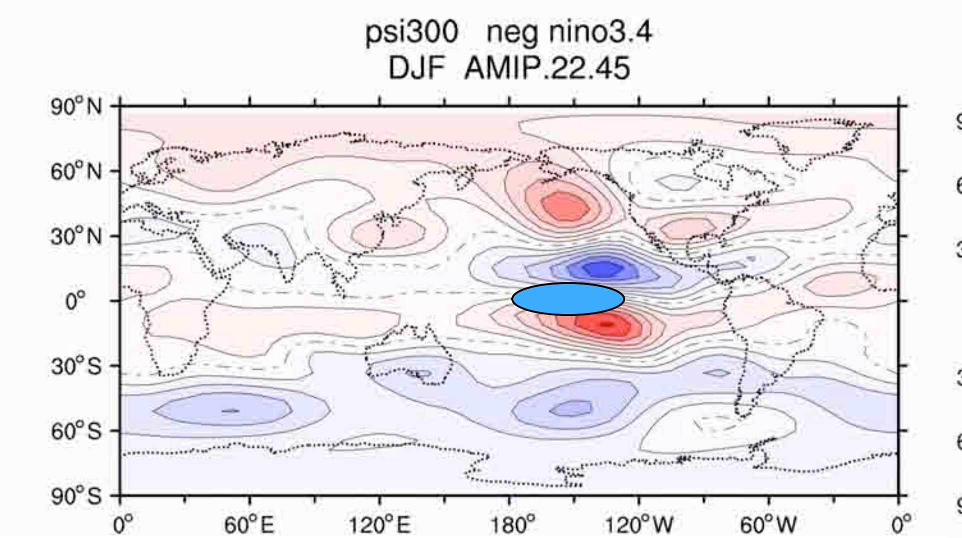

observed warm Pacific psi 300 DJF
90°N
60°N
30°N
0°
30°S
60°S
90°S
0°
60°E
120°E
180°
120°W
60°W
0°
CONTOUR FROM -800 TO 500 BY 100 (x10^4)

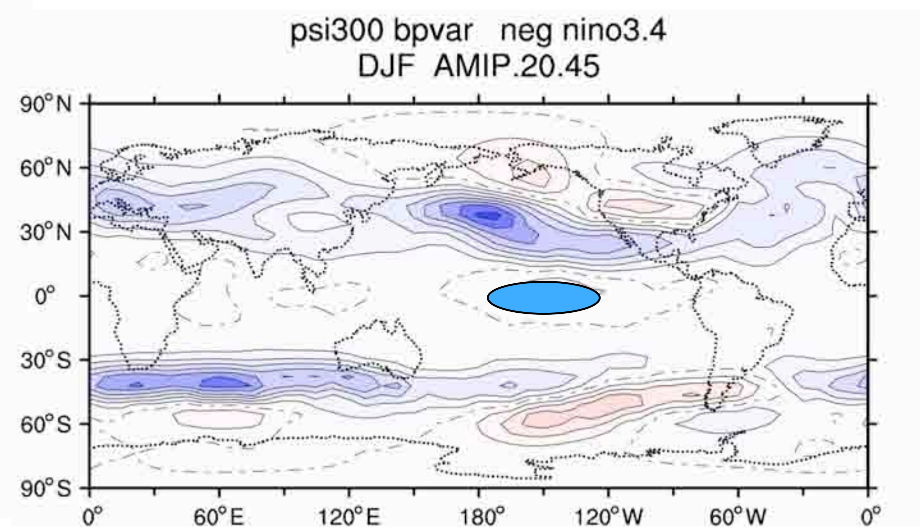

observed cold Pacific psi 300 DJF
90°N
60°N
30°N
0°
30°S
60°S
90°S
0°
60°E
120°E
180°
120°W
60°W
0°
CONTOUR FROM -70 TO 120 BY 10 (x10^5)

$$\hat{\psi}_{300} \qquad \widehat{(\psi'_{300})^2}$$









Application

Atmospheric general circulation model (NCAR's CCM0)

* Avoid sampling limitations when calculating lag covariances
* Enables rigorous testing of the resulting operator

Primitive equations, circa 1980 physical parameterizations

Perpetual January, fixed boundary conditions

$$\left.\begin{array}{l}\text{R15}\\ \\ \text{9 level}\end{array}\right\}\ \text{18352 degrees of freedom}$$

8 million 12hrly simulated states

Reduce Dimensionality

1. Pick fields from

   * ps
   * psi x 9
   * chi x 9
   * T   x 9
   * water vapor mixing ratio x 9

2. Truncate each field using EOFs

   * psi   100x9 (>90%)
   * T   496x9 (100%)

3. Form multivariate (truncated) fields,

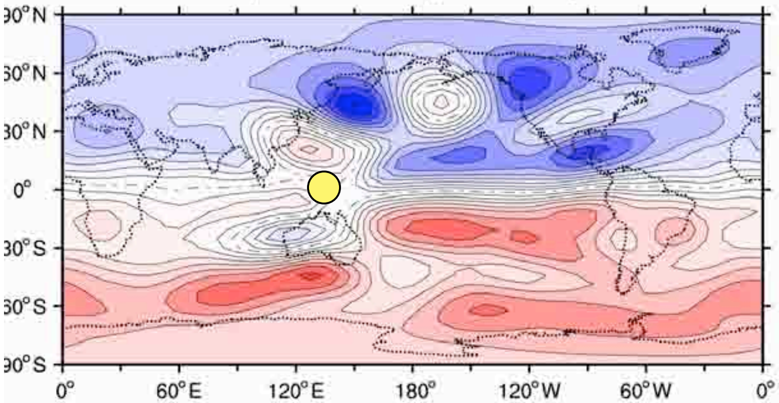
   normalize by std dev & overweight T,

   calculate EOFs

   truncate (1800 EOFs, >95%)

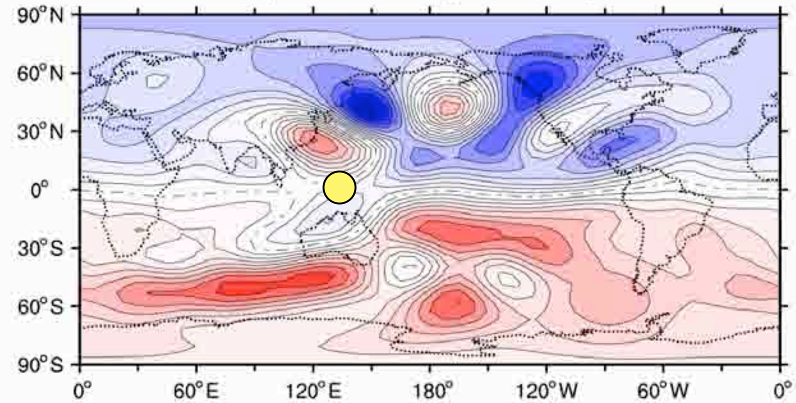
Assume lagged covariances vanish for τ > 30d

CCM0 strong forcing
psi336
(135.00, 0.00) 2.5C/day



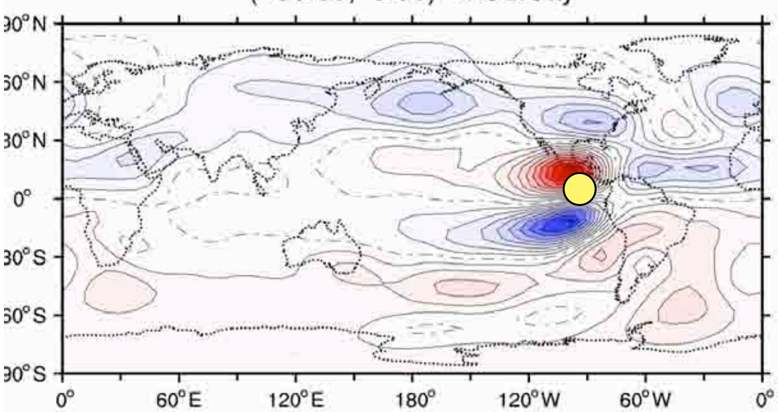
CONTOUR FROM -500 TO 400 BY 50 (x10⁴)
FD vs CCM0 multifield cor= 0.81

FD
psi336
(135.00, 0.00) 2.5C/day



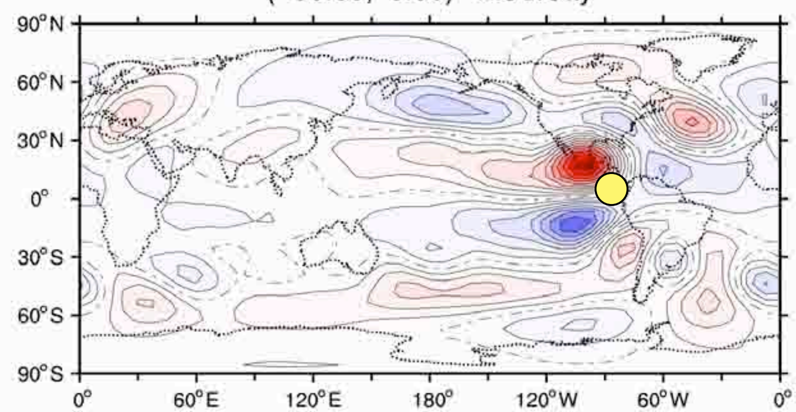
CONTOUR FROM -500 TO 400 BY 50 (x10⁴)
FD vs CCM0 multifield cor= 0.81

CCM0 strong forcing
psi336
(-90.00, 0.00) 2.5C/day



CONTOUR FROM -450 TO 500 BY 50 (x10⁴)
FD vs CCM0 multifield cor= 0.77

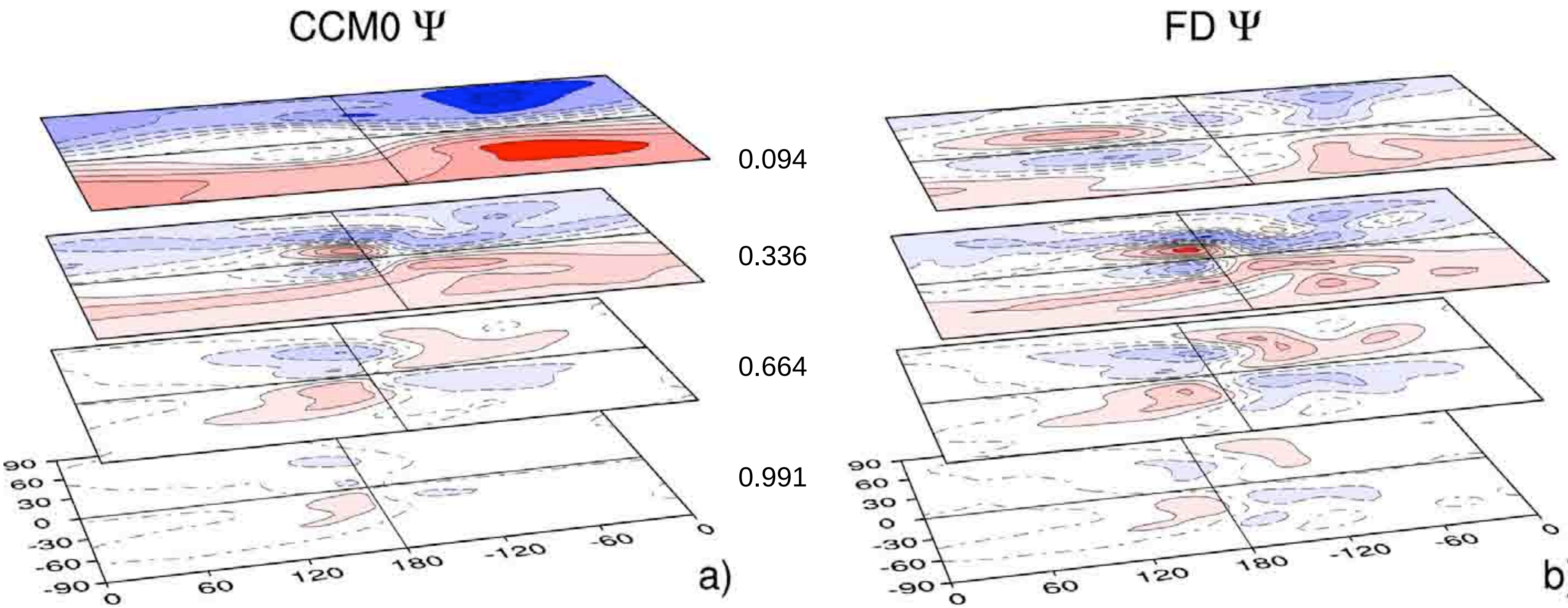
FD
psi336
(-90.00, 0.00) 2.5C/day



CONTOUR FROM -350 TO 500 BY 50 (x10⁴)
FD vs CCM0 multifield cor= 0.77

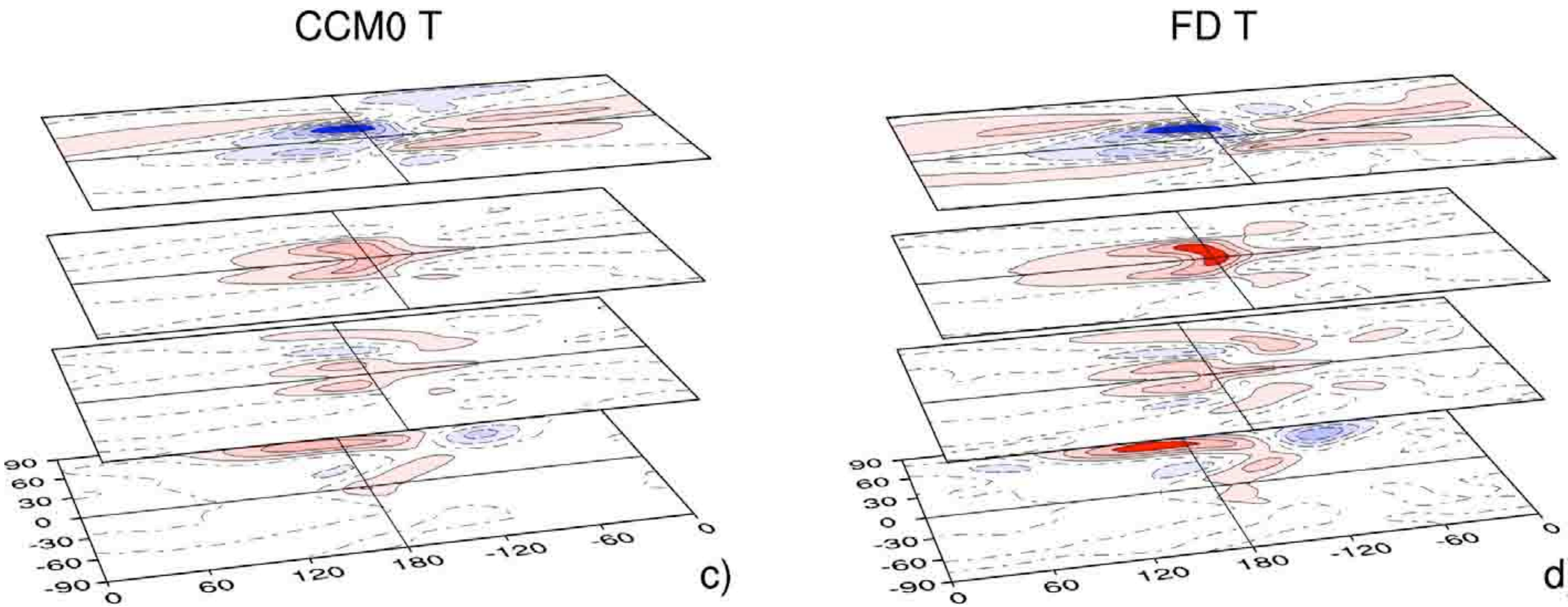
24 case average response to sinusoidal equatorial heating

(streamfunction)

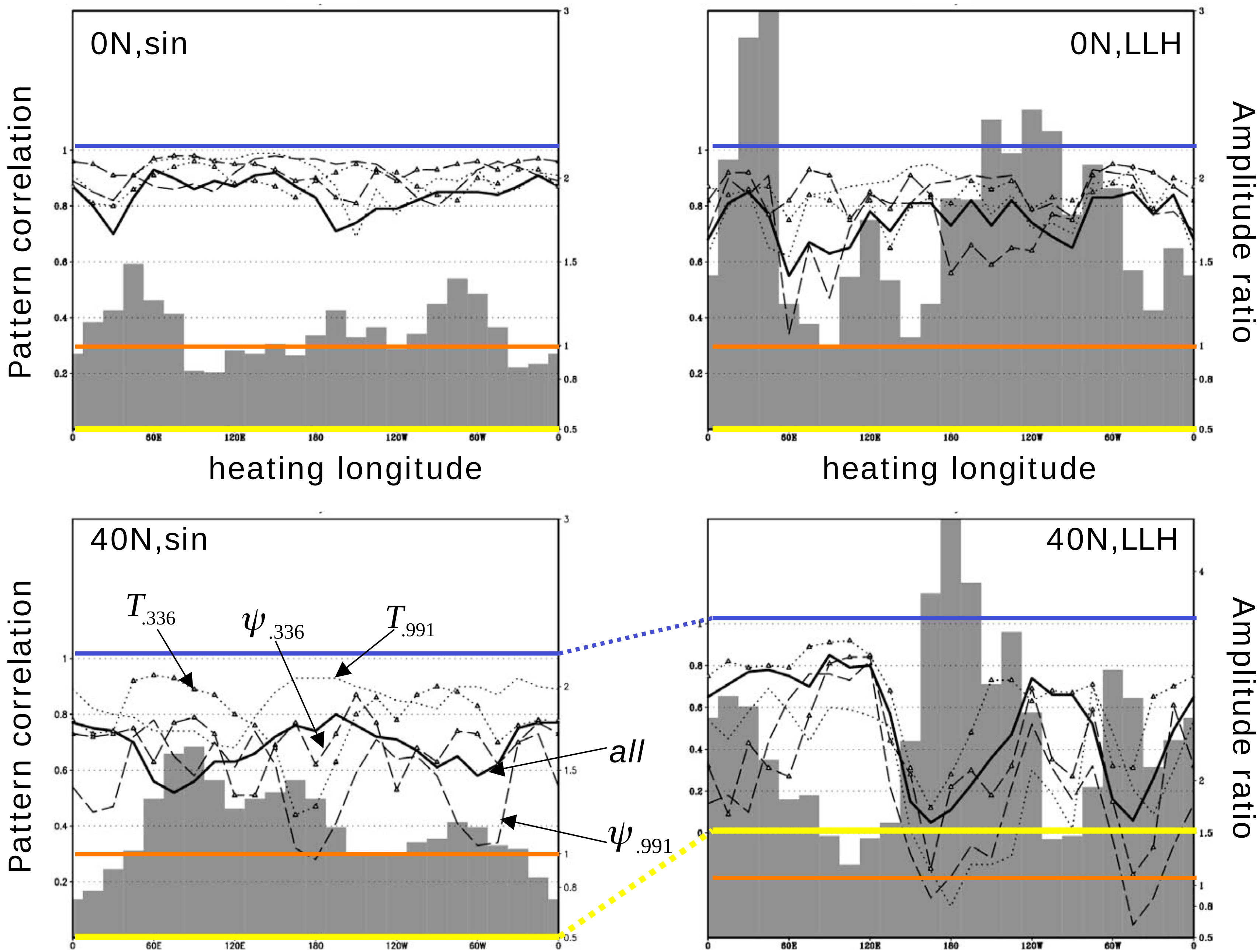


24 case average response to sinusoidal equatorial heating

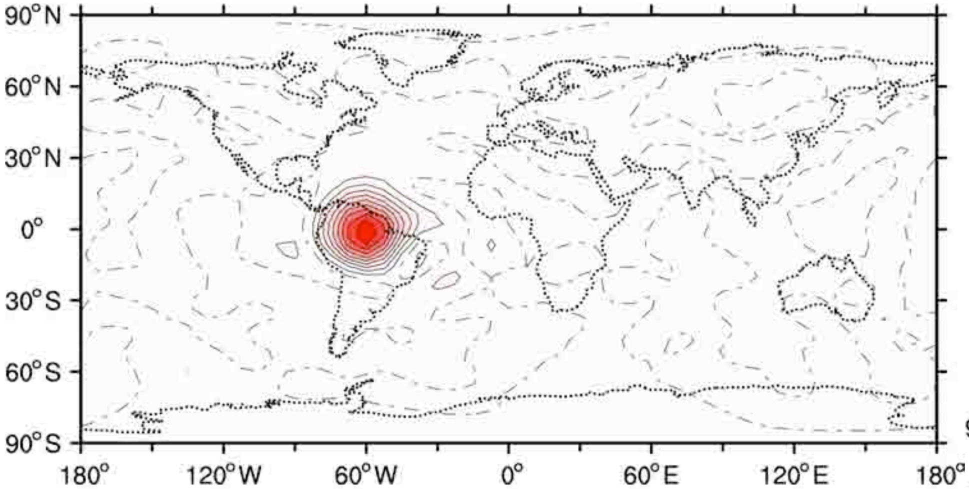
(temperature)



FD skill for individual cases



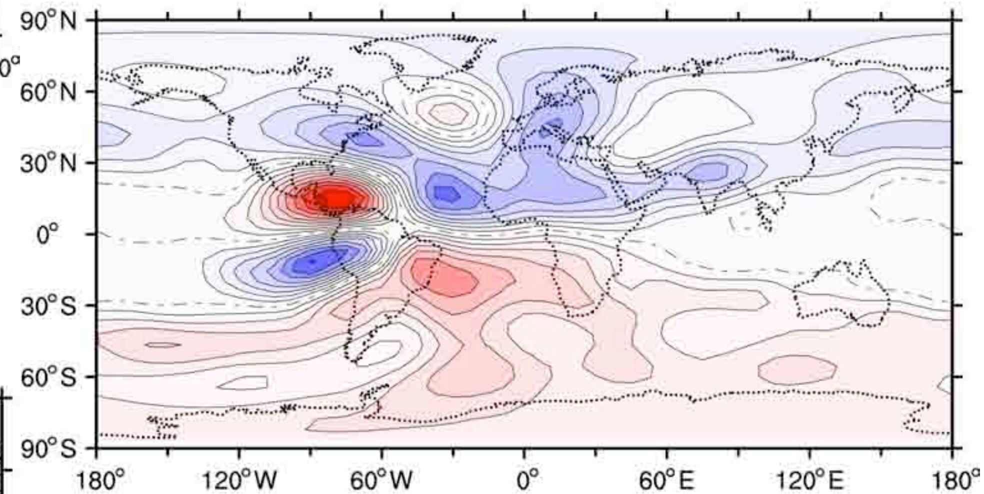
500mb heating



CONTOUR FROM -14 TO 127 BY 14 (x.001)

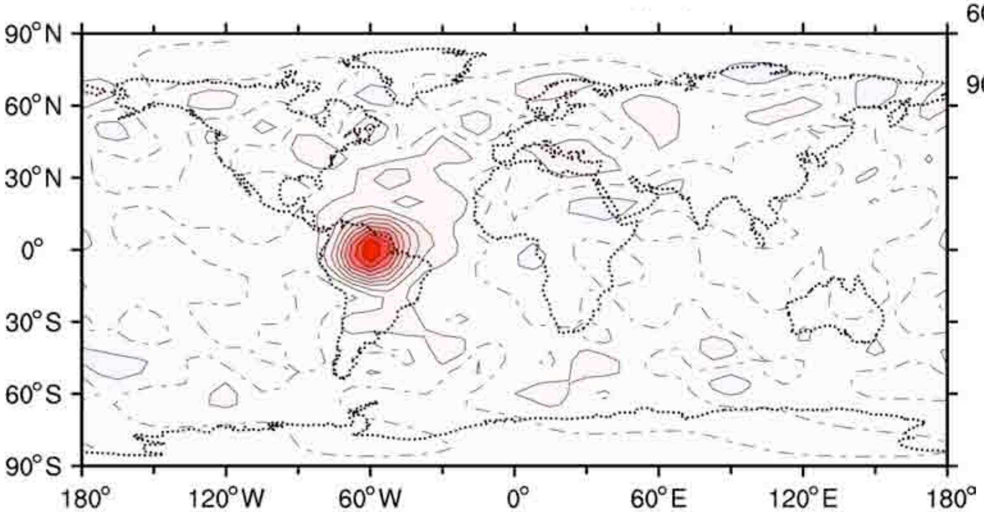
GCM

336mb streamfunction

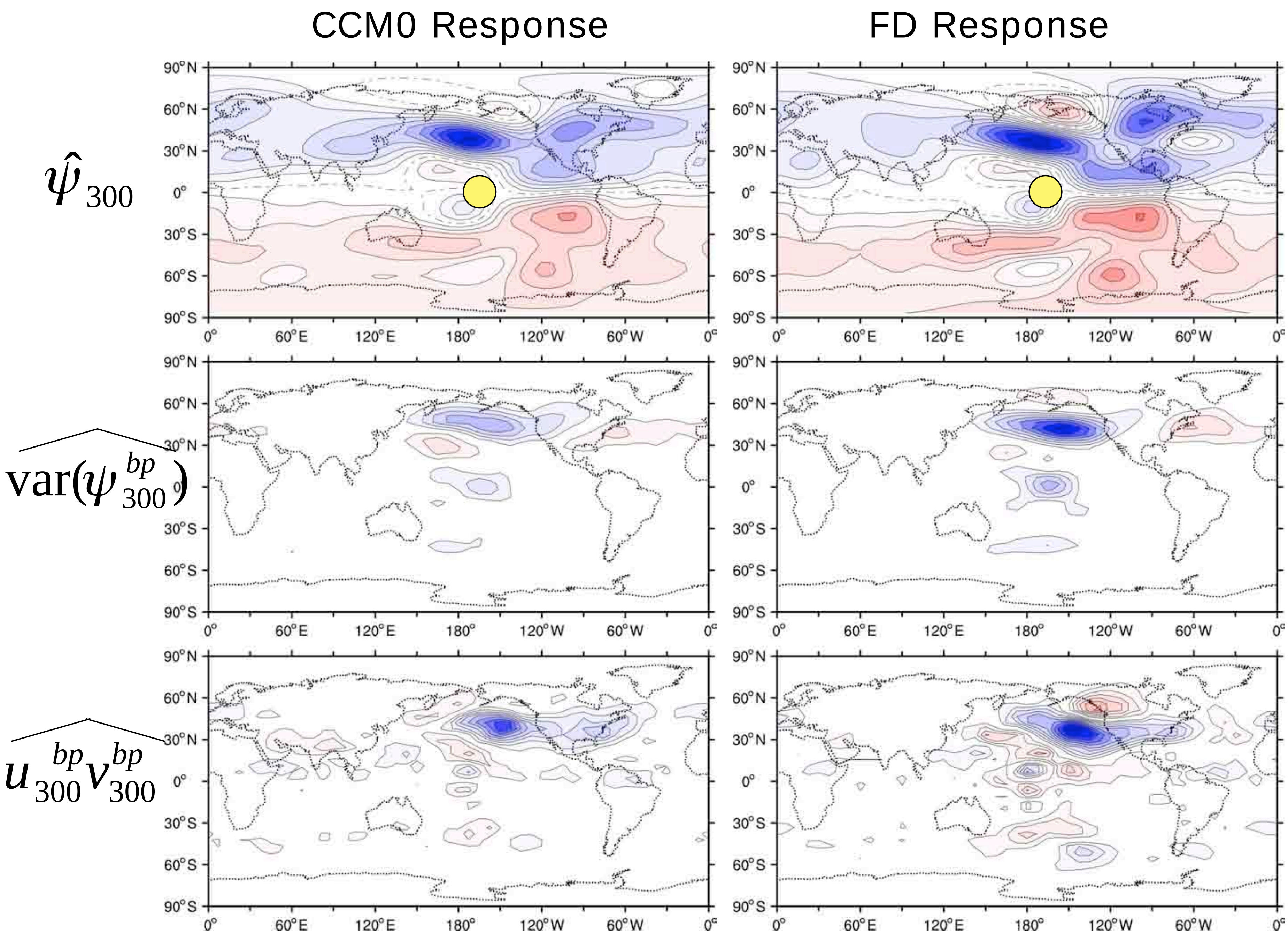


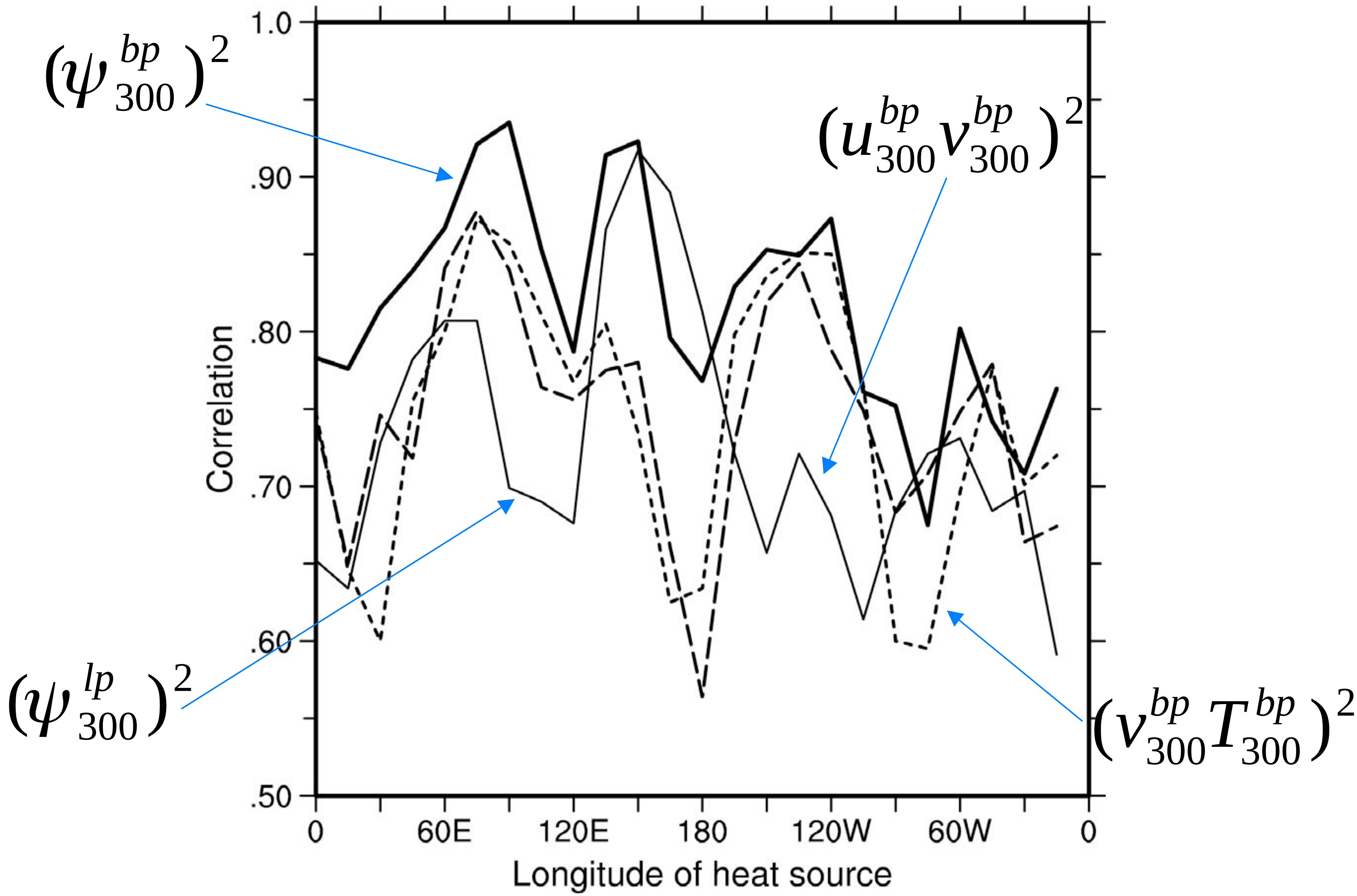
CONTOUR FROM -178 TO 201 BY 22 (x10^{4})

U^{-1}



CONTOUR FROM -9 TO 88 BY 9 (x.001)

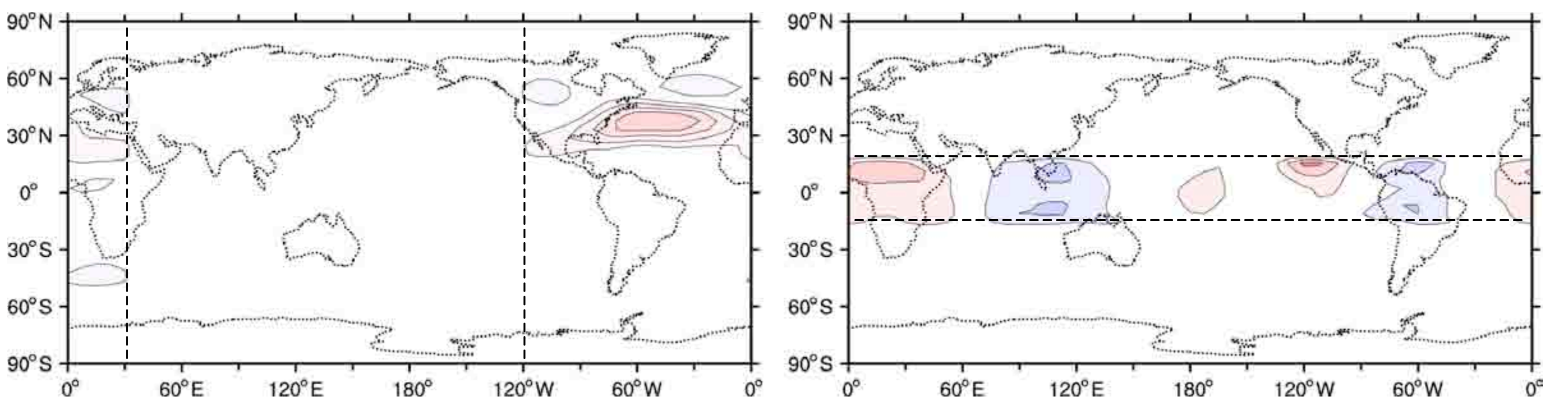

CCM0 Response
FD Response
$\hat{\psi}_{300}$
$\widehat{\mathrm{var}(\psi_{300}^{bp})}$
$\widehat{u_{300}^{bp} v_{300}^{bp}}$
90°N
60°N
30°N
0°
30°S
60°S
90°S
0°
60°E
120°E
180°
120°W
60°W
0°



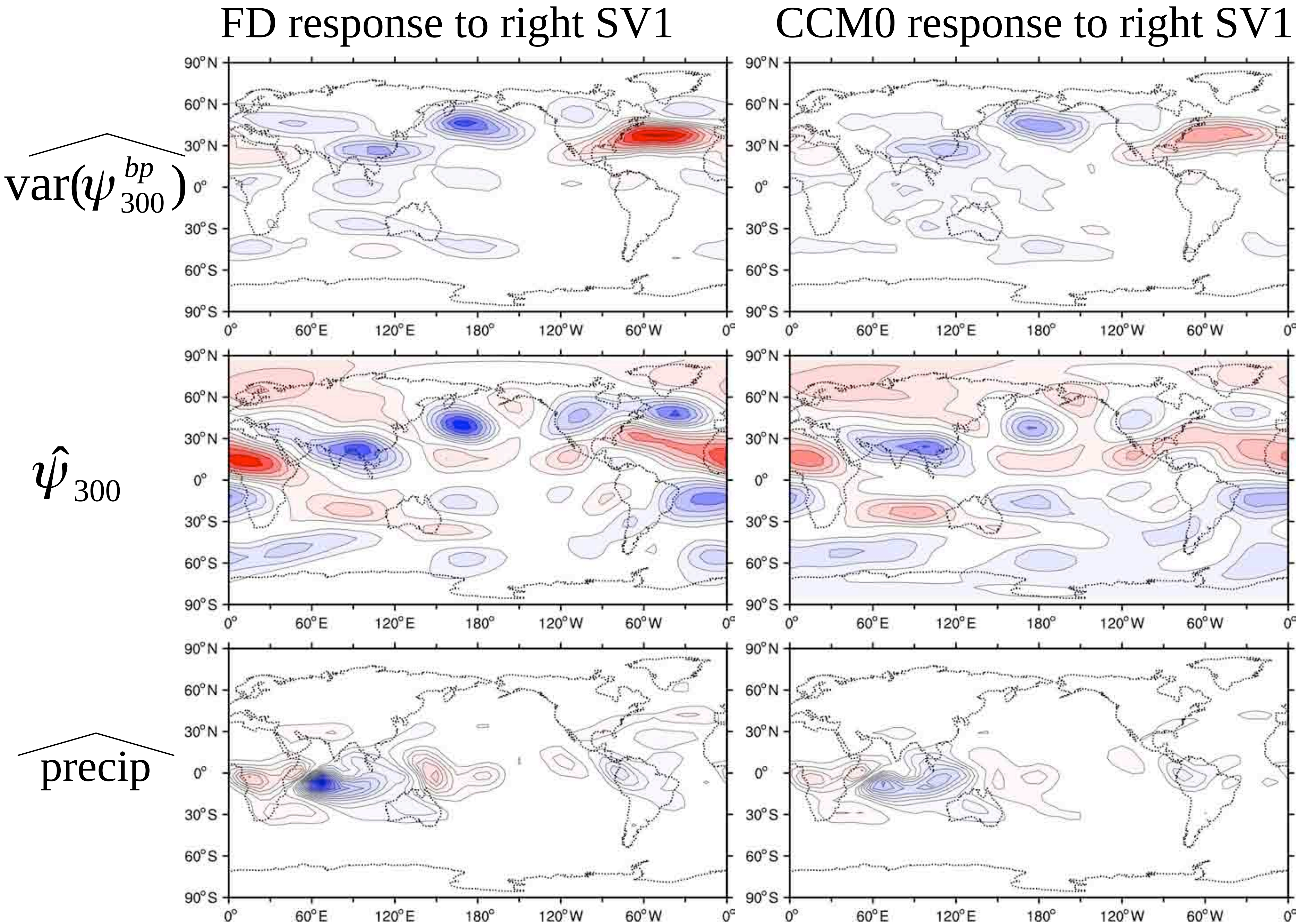
$(\psi_{300}^{bp})^2$
$(u_{300}^{bp} v_{300}^{bp})^2$
$(\psi_{300}^{lp})^2$
$(v_{300}^{bp} T_{300}^{bp})^2$
1.0
.90
.80
.70
.60
.50
Correlation
0
60E
120E
180
120W
60W
0
Longitude of heat source

SVD of transformed FD operator

$(\psi_{300}^{bp})^2$ response | T forcing



left SV | right SV


FD response to right SV1
CCM0 response to right SV1
$\widehat{\mathrm{var}(\psi_{300}^{bp})}$
$\hat{\psi}_{300}$
$\widehat{\mathrm{precip}}$
90°N
60°N
30°N
0°
30°S
60°S
90°S
0°
60°E
120°E
180°
120°W
60°W
0°

Time-dependent Forcing

For f fixed in time

$$R(t) = U(t)f$$

$$\text{with } U(t) = \int_0^t C(\tau)C^{-1}(0)d\tau$$

So for a delta function forcing at t', the response at t is

$$\delta R(t) = \delta U(t-t')f(t')\delta t$$

$$\text{for } \delta U(t-t') = C(t-t')C^{-1}(0)$$

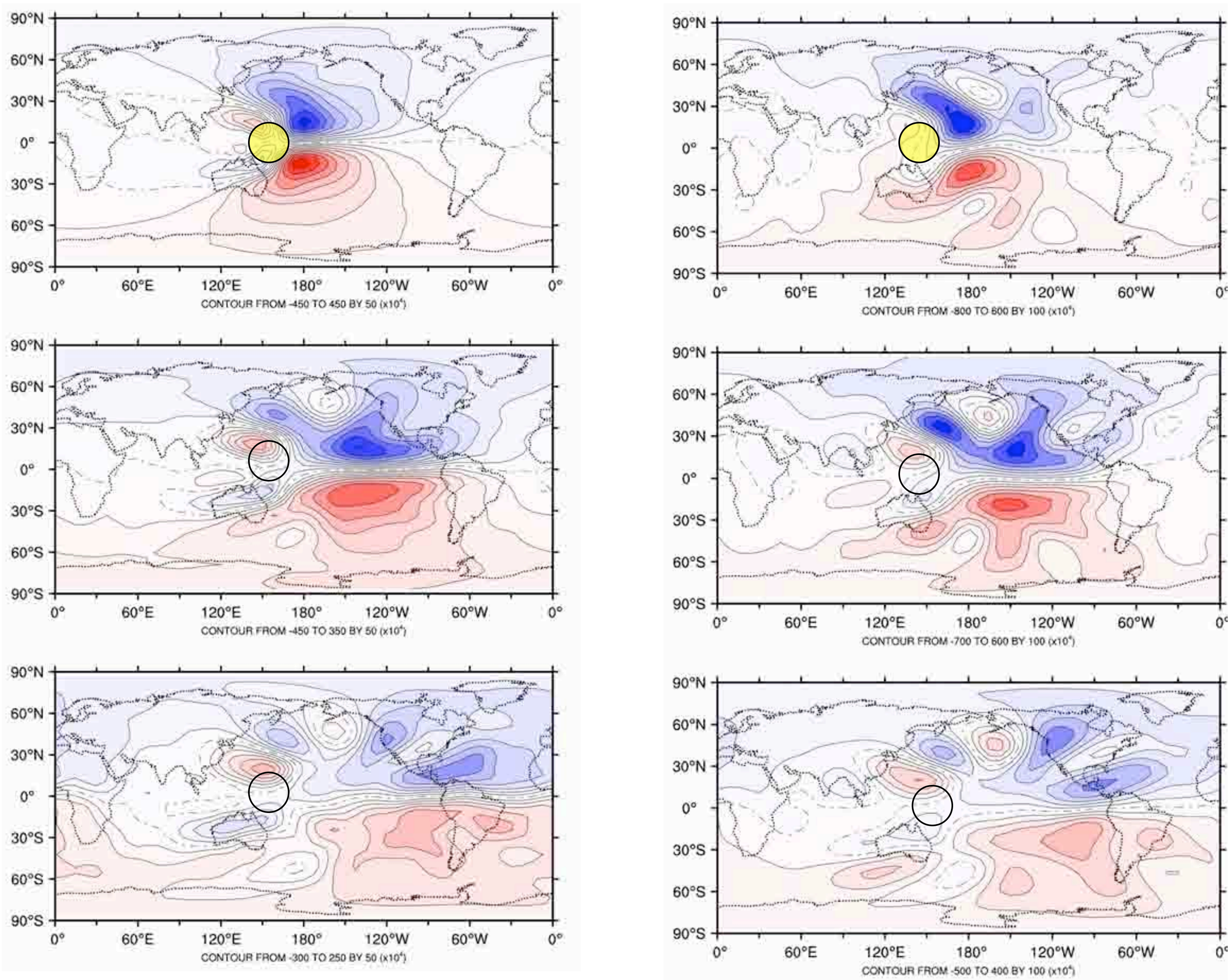
And so for time-dependent f

$$\boxed{R(t) = \int_0^t C(\tau)C^{-1}(0)f(\tau)d\tau}$$

3day pulse forcing

CCM0

FD



day3+0

day3+5

day3+10

July - September 1987 OLR

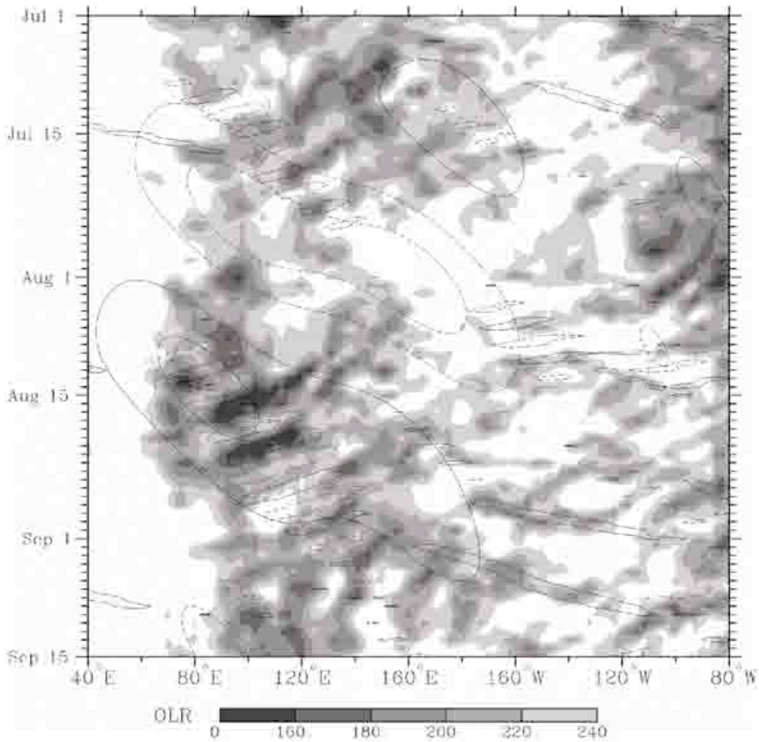
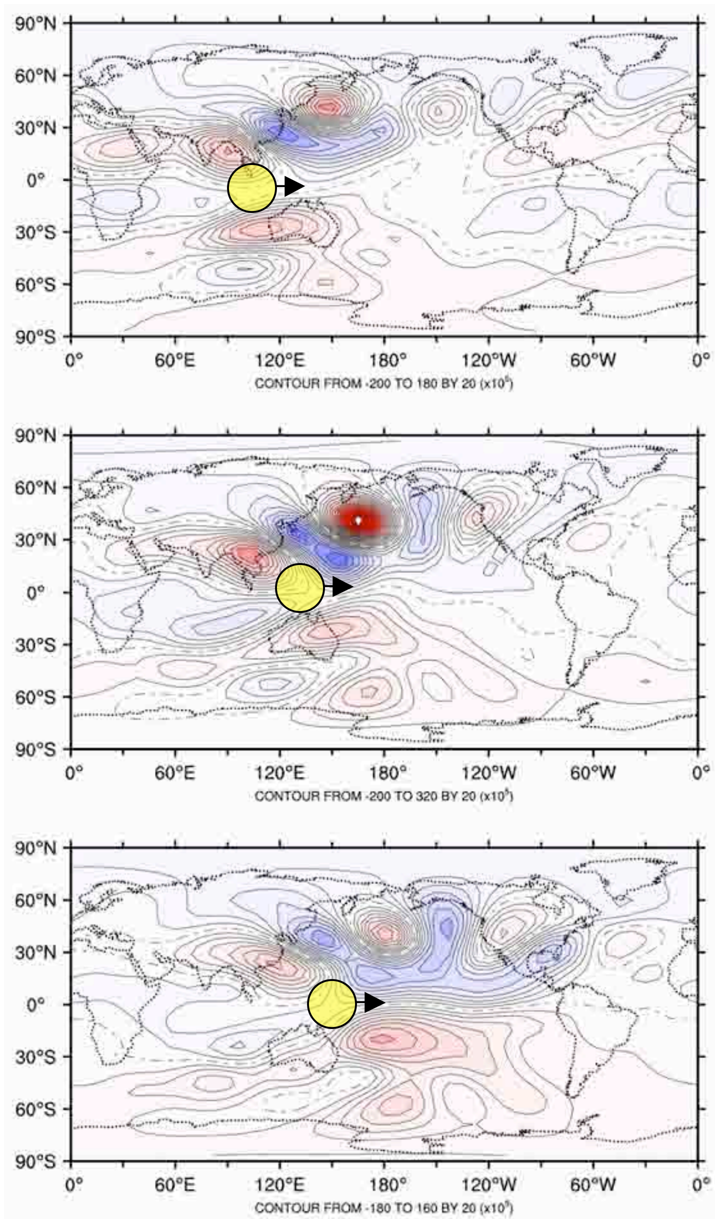


FIG. 6. Time–longitude plot of total OLR (shading, as indicated), filtered ISO, and MRG–TD OLR (contours, solid negative, contour interval 10 W m^{-2}, zero contour omitted), and filtered Kelvin wave OLR (contoured at −12 W m^{-2} only), averaged from 2.5° to 15°N, from 1 Jul to 15 Sep 1987.

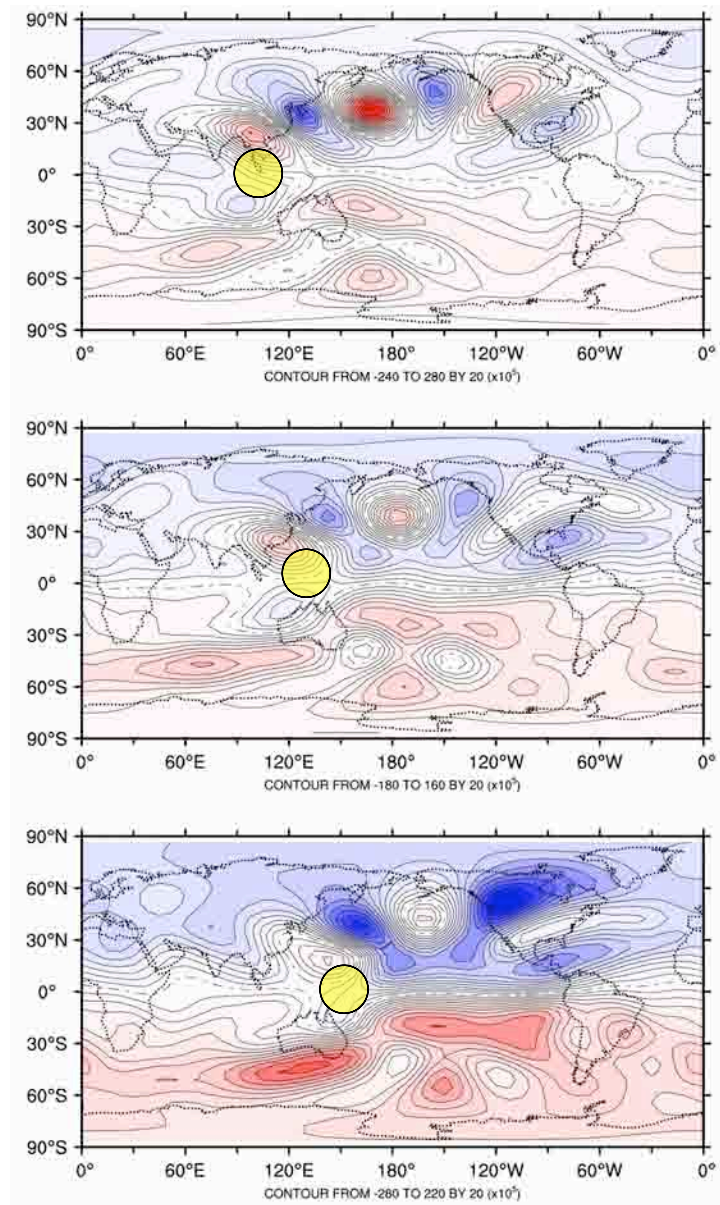
Straub & Kiladis (2003)

FD

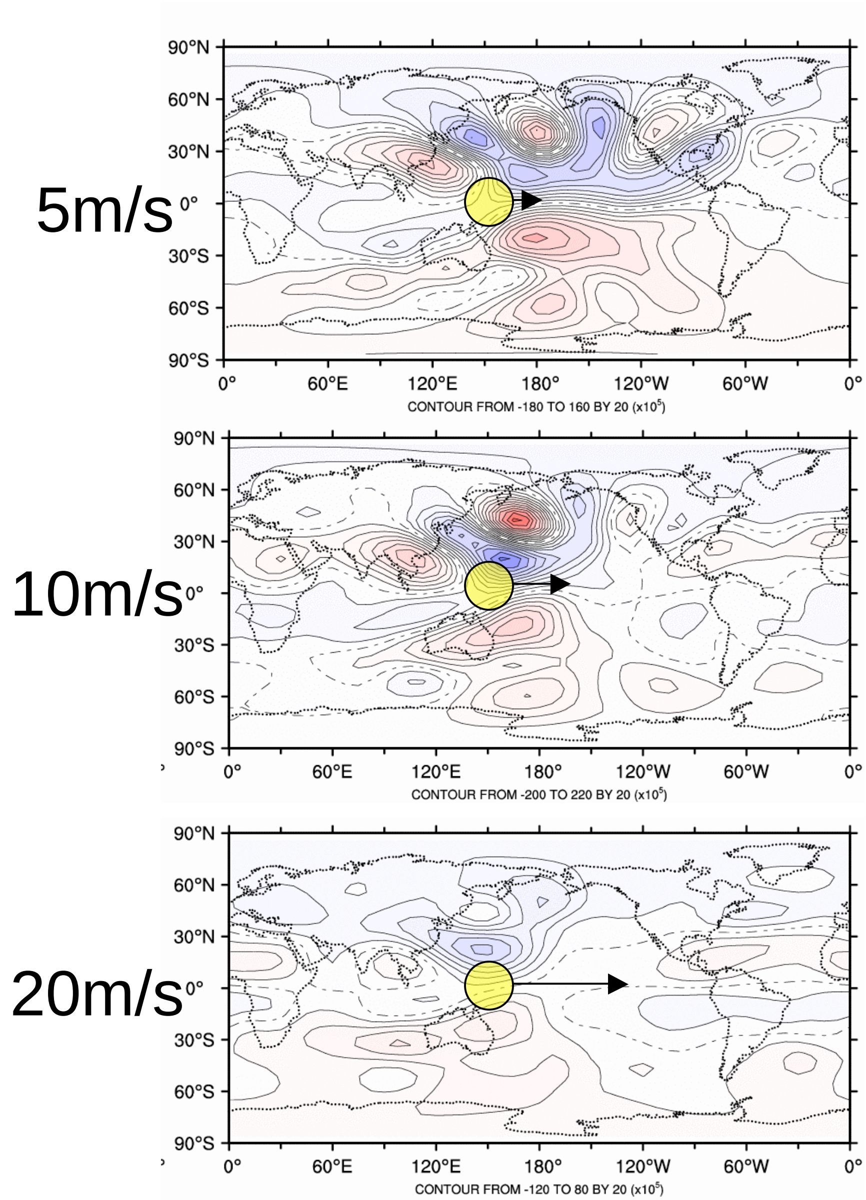
+4deg/d (5m/s) source



steady source



FD
moving source



Summary

The FDT can be used to estimate the response of the mean state and functionals of state.

The FDT can be used to estimate the response to constant and time-dependent forcing.

The FDT gives solutions that are accurate enough to be useful for optimization and inverse problems and for systematic explorations of atmospheric response.

Explorations using FDT operators for CCM0 indicate large sensitivities to forcing position and propagation speed. In particular they show the potential for short-lived or moderately propagating heat sources to affect midlatitudes.