

GEOMETRY OF ORBITS OF PERMANENTS AND DETERMINANTS

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Abstract

Let \mathfrak{v} be a complex vector space of dimension m and let $E := \mathfrak{v} \otimes \mathfrak{v}^* = \text{End } \mathfrak{v}$. Consider $\det \in Q := S^m(E^*)$, where \det is the function taking determinant of any $X \in \text{End } \mathfrak{v}$. Fix a basis $\{e_1, \dots, e_m\}$ of \mathfrak{v} and a positive integer $n < m$ and consider the function $\mathfrak{p} \in Q$, defined by $\mathfrak{p}(X) = x_{1,1}^{m-n} \text{perm}(X^o)$, X^o being the component of X in the right down $n \times n$ corner, where any element of $\text{End } \mathfrak{v}$ is represented by a $m \times m$ -matrix $X = (x_{i,j})_{1 \leq i,j \leq m}$ in the basis $\{e_i\}$ and perm denotes the permanent. The group $G = \text{GL}(E)$ canonically acts on Q . Let \mathcal{X}_{\det} (resp. $\mathcal{X}_{\mathfrak{p}}$) be the G -orbit closure of \det (resp. \mathfrak{p}) inside Q . Then, \mathcal{X}_{\det} and $\mathcal{X}_{\mathfrak{p}}$ are closed (affine) subvarieties of Q which are stable under the standard homothecy action of \mathbb{C}^* on Q . Thus, their affine coordinate rings $\mathbb{C}[\mathcal{X}_{\det}]$ and $\mathbb{C}[\mathcal{X}_{\mathfrak{p}}]$ are nonnegatively graded G -algebras over the complex numbers \mathbb{C} .

The aim of this talk is to study some geometric results about the varieties \mathcal{X}_{\det} and $\mathcal{X}_{\mathfrak{p}}$ and to study $\mathbb{C}[\mathcal{X}_{\det}]$ and $\mathbb{C}[\mathcal{X}_{\mathfrak{p}}]$ as G -modules. The work is motivated by the geometric approach initiated by Mulmuley-Sohoni to solve the Valiant's conjecture in Geometric Complexity Theory.