Submittee: Alia Hamieh
Date Submitted: 2017-04-21 14:53
Title: Alberta Number Theory Days IX
Event Type: Conference-Workshop

Location:
BIRS, Banff

Dates:
March 17, 2017 - March 19, 2017

Topic:
Number Theory

Methodology:
Lectures (50-minute, 40-minute, 30-minute, 20-minute)

Objectives Achieved:
This was the ninth edition of Alberta Number Theory Days. Previous conferences took place in Lethbridge (2008), Calgary (2009), and BIRS (2010, 2011, 2013, 2014, 2015, 2016).

This friendly meeting gathers the number theorists of the Alberta Universities to interact and exchange ideas once a year.

Its purpose is to solidify the relations between Alberta Number Theorists, motivate local young researchers, and create relations with out of province researchers.

This year, the conference had a total of thirteen talks.
Three were plenary lectures given by external speakers (from outside of Alberta) who are very well established researchers. Four talks were given by faculty from universities in Alberta and the remainder of the talks given by graduate students and postdocs from PIMS-affiliated universities.

Our out-of-province speakers were: Imin Chen (Simon Fraser University), Lee Troupe (University of British Columbia), Katherine Stange (University of Colorado) and Ursula Whitcher (American Mathematical Society). These are all young and brilliant leading experts in their respective fields of expertise. In addition, they are all excellent speakers and their talks were motivating for the students in the audience.

Contributing to the spirit of fostering open collaboration between institutions, Eric Roettger from Mount Royal University in Calgary gave a talk, expanding the circle of participating schools, and Amy Feaver from The King's University in Edmonton helped with the organization of the conference.

All main Albertan Number Theory centers were equally well represented among participants with 10 people from Edmonton, 9 from Calgary, and 6 from Lethbridge.

Our aim to provide early-career researchers access to a nourishing research environment is reflected in the participant pool in general: Out of the 29 participants, 7 were postdoctoral fellows and 8 were graduate students (masters and doctoral combined).
Among the participants 4 were PIMS postdoctoral fellows: Andrew Fiori (U. Calgary), Alia Hamieh (U. Lethbridge), Shashank Kanade (U. Alberta), and Ha Tran (U. Calgary). Two of these participants were also organizers of the conference.

We strived hard to improve gender representation at our workshop, and
we are extremely happy to report that two of the three plenary lectures delivered by out-of-province mathematicians were given by female mathematicians. Out of the 13 total talks, 6 were delivered by female mathematicians. Overall, out of the 29 participants, 11 were females, and we hope that future editions of ANTD will work towards improving this ratio.

Out of the 29 participants, 19 filled the PIMS demographic survey, whose results are attached separately.

Scientific Highlights:
This year, the scientific highlights were the plenary talks given by Imin Chen (Simon Fraser University), Katherine Stange (University of Colorado) and Ursula Whitcher (American Mathematical Society).

Dr. Imin Chen works in the areas of number theory and arithmetic geometry, with an emphasis on modular curves, Galois representations, modular forms and Diophantine equations. He recieved his Ph.D. from The University of Oxford, and is currently an associate professor at Simon Fraser University. Dr. Chen spoke about the generalized Fermat equation: an equation of the form x^p + y^q = z^r, where p,q,r are positive integers. It is conjectured that these equations have no solutions in non-zero mutually coprime integers x, y, z whenever the integer exponents p, q, r \geq 3. In his talk he gave a survey on current work on this problem, as well as some results that have brought mathematicians closer to understanding this problem. Dr. Chen is very well-spoken and delivered an excellent talk that was well-suited for the diverse audience present.

Dr. Katherine Stange is an assistant professor at the University of Colorado. Before her position in Colorado, she was an NSF postdoc at Harvard University and Stanford University, as well as a PIMS postdoc at Simon Fraser University and the University of British Columbia. She gave an excellent talk, and also had the opportunity to connect with faculty who she met during her term as a PIMS postdoc. Dr. Stange has recently expanded her research interests to \textit{visualizations in number theory}, and gave a very interesting talk which connected Apollonian circle packings to visualizing the class number and Euclidean property of number fields. Her talk was highly technical but also presented in a way that it could be understood by graduate students and postdocs present.

Dr. Ursula Whitcher is an Associate Editor for the American Mathematical Society's \textit{Mathematical Reviews} service and a visiting scholar at the University of Michigan. Her previous appointment was as an Associate Professor at the University of Wisconsin-Eau Claire. At Alberta Number Theory Days, she talked about results on Zeta functions of alternate mirror Calabi-Yau pencils. This is part of a joint project she is working on with Charles Doran, Tyler Kelly,

Adriana Salerno, Steven Sperber and John Voight.

Organizers:
Feaver, Amy, The King's University

Hamieh, Alia, University of Lethbridge

Kanade, Shashank, University of Alberta

Speakers:
A complete list of titles and abstracts for the talks given at ANTD IX are listed below:

Speaker: Imin Chen (Simon Fraser University)\\
Title: On the Generalized Fermat Equation\\
Abstract: A conjectured generalization of Fermat's Last Theorem states that the equation x^p + y^q = z^r has no solutions in non-zero mutually coprime integers x, y, z whenever the integer exponents$p, q, r \geq 3. Since the proof of Fermat's Last Theorem, it was natural to attempt to study this generalization using a similar approach by Galois representations and modular forms. In this talk, I will survey some of the successes of applying this method, current ongoing approaches, and fundamental challenges in carrying out a complete resolution.

Speaker: Lee Troupe (The University of British Columbia)\\
Title: Counting Irreducible Divisors and Irreducibles in Progressions\\
Abstract: Let K/\mathbb{Q} be a number field with ring of integers \mathbb{Z}_K. If K has class number one, the set of irreducible elements of \mathbb{Z}_K coincides with the set of prime elements; in general, this need not be the case. One is led to wonder: Do statements about primes in \mathbb{Z} have analogues for irreducibles in \mathbb{Z}_K, for a general choice of K? This talk concerns two instances where the answer is yes. We will discuss the maximal order of the number of irreducible divisors of an element of \mathbb{Z}_K, and we will provide an asymptotic formula for the number of irreducible elements of norm up to x belonging to a given arithmetic progression.

Speaker: Eric Roettger (Mount Royal University)
Title: More Hodge-Podge Pseudoprimes
Abstract: This talk will give a brief review of basic pseudoprime history. It will also give the principles of a few generalizations of the Lucas functions and how these yield a 'new' type of pseudoprime. Finally, we will conjecture how these new pseudoprimes fit into the more general theory.

Speaker: Renate Schidler (University of Calgary)
Title: A Class of Artin-Schreier Curves with Many Automorphisms
Abstract: Algebraic curves with many points are useful in coding theory, but are also of number theoretic and geometric interest in their own right. Their symmetries are described by their

automorphism group. Other information, such as the number of rational points on the curve and on the associated Jacobian variety over any field, is encoded in their zeta function. Unfortunately, all these objects are generally notoriously difficult to compute.

In this talk, we describe a class of Artin-Schreier curves whose unusually big automorphism group can be explicitly described. The automorphism group contains a large extraspecial subgroup, precise knowledge of which makes it possible to compute the zeta functions of these curves after extending the base field to contain the appropriate field of definition. We find that over fields of square cardinality, these curves are either maximal or minimal, and we classify which curves fall into which category.

This is joint work with Irene Bouw, Wei Ho, Beth Malmskog, Padmavathi Srinivasan and Christelle Vincent.

Speaker: Imin Chen (Simon Fraser University)
Title: On the Generalized Fermat Equation
Abstract: A conjectured generalization of Fermat's Last Theorem states that the equation x^p + y^q = z^r has no solutions in non-zero mutually coprime integers x, y, z whenever the integer exponents p, q, r \geq 3. Since the proof of Fermat's Last Theorem, it was natural to attempt to study this generalization using a similar approach by Galois representations and modular forms. In this talk, I will survey some of the successes of applying this method, current ongoing approaches, and fundamental challenges in carrying out a complete resolution.

Speaker: Forrest Francis (University of Lethbridge)
Title: Special Values Of Euler's Function
Abstract: In 1909, Landau showed that
\[\limsup \tfrac{n}{\phi(n) \log\log{n}} = e^\gamma,\]
where \phi(n) is Euler's function. Later, Rosser and Schoenfeld asked whether there were infinitely many n for which {n}/{\phi(n)} > e^\gamma \log\log{n}. This question was answered in the affirmative in 1983 by Jean-Louis Nicolas, who showed that there are infinitely many such n both in the case that the Riemann Hypothesis is true, and in the case that the Riemann Hypothesis is false.

One can prove a generalization of Landau's theorem where we restrict our attention to integers whose prime divisors all fall in a fixed arithmetic progression. In this talk, I will discuss the methods of Nicolas as they relate to the classical result, and also provide evidence that his methods could be generalized in the same vein to provide answers to similar questions related to the generalization of Landau's theorem.

Speaker: Habiba Kadiri (University of Lethbridge)
Title: Explicit results in prime number theory
Abstract: In 1962, Rosser and Schoenfeld gave a method to estimate the error term in the approximation of the prime counting function \psi(x).
Since then, progress on the numerical verification of the Riemann Hypothesis and widening the zero-free region of the Riemann zeta function have allowed numerical improvements of these bounds.
It is only recently that explicit zero density estimates have been used in this context.
We will present some of these results as well as consequences to the distribution of primes.

Speaker: Manish Patnaik (University of Alberta)

Title: Whittaker functions and Metaplectic Kac-Moody groups
Abstract: Metaplectic groups have had a rich interplay with number theory, generally via the theory of theta functions and their Fourier-Whittaker coefficients. We describe a recent construction of metaplectic covers of infinite-dimensional groups (joint with Anna Puskas) and explain its conjectural link to some concrete questions in analytic number theory.

Speaker: Wolfgang Riedler (University of Alberta)
Title: Self-Dual Vertex Operator Superalgebras and Superconformal Field Theories
Abstract: Recent work has related the equivariant elliptic genera of sigma models with K3 surface target space to a vertex operator superalgebra that realizes moonshine for Conwayâ€™s group. Motivated by this we consider conditions under which a self-dual vertex operator superalgebra may be identified with the bulk Hilbert space of a superconformal field theory. After presenting a classification result for self-dual vertex operator superalgebras with central charge up to 12, several examples of close relationships with bulk superconformal field theories are described, including those arising from sigma models for tori and K3 surfaces.

Speaker: Majid Shahabi (University of Calgary)
Title: Modular Forms for Abelian Varieties
Abstract: As the modularity theorem shows, classical modular forms are connected to Tate modules of elliptic curves over \mathbb{Q} through their L-functions. This connection is built through the automorphic representations of GL(2) and its subgroups. This talk concerns a generalization of this story to abelian varieties. The Langlands program predicts that for abelian varieties A over \mathbb{Q}, there should be an automorphic representation of GSpin over \mathbb{Q} such that the L-function of the automorphic representation coincides with the L-function coming from the Tate module of the abelian variety A. Recently, Gross has refined this prediction for certain abelian varieties A, showing exactly how to describe the weight and level of a type-B modular form f_A whose L-function matches the L-function of the Tate module of A. In this talk, I will review some of this story and will describe my own work on the group scheme of the level of the GSpin modular forms that arise in Gross' conjecture.

Speaker: Sahar Siavashi (University of Lethbridge)
Title: Wieferich primes and Wieferich numbers
Abstract: An odd prime p is called a \emph{Wieferich prime} (in base 2), if $2^{p-1} \equiv 1 \pmod{p^2}$. These primes first were considered by A. Wieferich in 1909, while he was working on a proof of Fermat's last theorem. This notion can be generalized to any integer base a>1. In this talk, we discuss the work that has been done regarding the size of the set of non-Wieferich primes and show that, under certain conjectures, there are infinitely many non-Wieferich primes in certain arithmetic progressions. Also we consider the congruence $a^{\varphi(m)} \equiv 1 \pmod{m^2}$, for an integer m with (a,m)=1, where \varphi is Euler's totient function. The solutions of this congruence lead to Wieferich numbers in base a. In this talk we present a way to find the largest known Wieferich number for a given base. In another direction, we explain the extensions of these concepts to other number fields such as quadratic fields of class number one.

Speaker: Katherine Stange (University of Colorado - Boulder)
Title: Circle packings, thin orbits and the arithmetic of imaginary quadratic fields
Abstract: Integral Apollonian circle packings are certain fractal packings of the plane with circles of disjoint interior, and integer curvatures. The set of curvatures which appears has been of recent interest as a challenging problem in the study of orbits of thin groups. Work of Bourgain, Fuchs and Kontorovich culminated in the demonstration that density one of the integers appear as curvatures,

up to a congruence restriction. In this talk, we'll rediscover Apollonian circle packings as part of the essential nature of the Gaussian integers and their Diophantine approximation, generalize to other quadratic fields to discover new circle packings, and discuss the extension of results on curvatures to these and other Kleinian packings.

Speaker: Ha Tran (University of Calgary)
Title: The Size Function For a Number Field
Abstract: The size function h^0 for a number field is an analogue of the dimension of the Riemann-Roch spaces of divisors on an algebraic curve. In this talk, we introduce this function and discuss the conjecture of Schoof and Van der Geer on the maximality of h^0 at the trivial divisor.

Speaker: Ursula Whitcher (American Mathematical Society)
Title: Zeta functions of alternate mirror Calabi-Yau pencils
Abstract: We prove that if two Calabi-Yau invertible pencils in projective space have the same dual weights, then they share a common polynomial factor in their zeta functions related to a hypergeometric Picard-Fuchs differential equation. The polynomial factor is defined over the rational numbers and has degree greater than or equal to the order of the Picard-Fuchs equation. This talk describes joint work with Charles Doran, Tyler Kelly, Adriana Salerno, Steven Sperber, and John Voight.

Links:

File Uploads:
Additional Upload 1: http://www.pims.math.ca/files/final_report/PIMS_Report_1.pdf
Additional Upload 2: http://www.pims.math.ca/files/final_report/SurveyResponsesANTD2017.pdf

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