

**Submittee:** Nathan Ng

**Date Submitted:** 2015-11-30 14:27

**Title:** Alberta Number Theory Days 7

**Event Type:** Conference-Workshop

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**Location:**

Banff International Research Station

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**Dates:**

Friday, June 12, 2015- Sunday, June 14, 2015

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**Topic:**

Number Theory

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**Methodology:**

Lectures.

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**Objectives Achieved:**

See attached sheet.

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**Scientific Highlights:**

See attached sheet.

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**Organizers:**

Ng, Nathan, Mathematics and Computer Science, University of Lethbridge

Patnaik, Manish, Mathematics and Statistics, University of Alberta

Steele, Ander, Mathematics and Statistics, University of Calgary

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**Speakers:**

Speaker: Amir Akbary (Mathematics and Computer Science, University of Lethbridge)

Title: On the greatest prime factor of some divisibility sequences

Abstract:

Let  $P(m)$  denote the greatest prime factor of  $m$ . For integer  $a > 1$ , M. Ram Murty and S. Wong proved that, under the assumption of the ABC conjecture,  $P(a^n - 1) \ll_{\epsilon} a^{n^{2-\epsilon}}$  for any  $\epsilon > 0$ . We study analogous results for the corresponding divisibility sequence over the function field  $\mathbb{F}_q(t)$  and for some divisibility sequences associated to elliptic curves over the rational field  $\mathbb{Q}$ .

This is a joint work with Soroosh Yazdani.

Speaker: Mark Bauer, (Mathematics and Statistics, University of Calgary)

Title: Cubic Irrationalities and a Ramanujan-Nagell Analogue

Abstract: In this talk we consider a cubic analogue of the Ramanujan-Nagell equation. In particular, we show that by constructing explicit restricted irrationality measures for the cube root of two that are, in some very real sense, better than expected it is possible to derive meaningful and interesting bounds on the difference between the cube of an integer and powers of 2. These measures naturally yield results about certain Diophantine equations.

This is joint work with Michael Bennett.

Speaker: William Casselman, (Mathematics, University of British Columbia)

Title: Newton polygons and ramification

Abstract:

I shall explain the characterization of ramification groups in terms of Newton polygons, following an old suggestion of Tate

and a recent paper of Jonathan Lubin. If time permits, I'll also explain either Lubin's proof of Sen's theorem on p-adic dynamics

or the use of Newton polygons to compute local Galois groups (following Romano, Greve, and Pauli).

Speaker: Thomas Creutzig, (Mathematics and Statistics, University of Alberta)

Title: Logarithmic Hopf links and modular forms

Abstract:

Representations of a regular vertex operator algebra or conformal field theory form a modular semi-simple tensor category. The categorical  $SL(2, \mathbb{Z})$ -action is generated by the Hopf link and twist corresponding to the elements  $S$  and  $T$  of the modular group. This action coincides with the modular group action on the space of trace functions. Further the Hopf link defines representations of the fusion ring of the vertex operator algebra.

If the vertex operator algebra has indecomposable but reducible modules then nothing is known about a similar modular story. I will introduce what I call logarithmic Hopf links, explain how they relate to representations of the fusion ring and especially in an example how they relate to modular group action on the space of trace and pseudo trace functions.

Speaker: Clifton Cunningham, (Mathematics and Statistics, University of Calgary)

Title: Lifting Hilbert modular forms to spin modular forms

Abstract:

In this talk I will explain how Hilbert modular forms for totally real fields of degree  $n$  determine automorphic representations of forms of the group  $Spin(2n+1)$  over  $\mathbb{Q}$ . Using examples, I will show that the automorphic representations obtained in this way may or may not be holomorphic.

Joint work with Lassina Dembele.

Speaker: Stephan Ehlen, (Mathematics and Statistics, McGill University)

Title: On Two Arithmetic Theta Lifts

Abstract:

In his 1997 Annals paper 'Central Derivatives of Eisenstein Series and Height Pairings', Kudla constructs Green's functions for special cycles on Shimura curves. The construction generalizes to higher dimensional orthogonal and unitary Shimura varieties. We show how to obtain these Green's functions as regularized theta lifts of certain 'truncated' Poincare series. This construction allows us to apply the well-developed methods for such theta lifts. In particular, we are able to study their CM values and show a direct relation to Bruinier's automorphic Green functions (which are regularized

theta lifts of harmonic weak Maass forms). One of the interesting applications in arithmetic geometry is that our results imply the modularity of the difference of two arithmetic generating series for Kudla-Rapoport cycles on unitary Shimura varieties.

This is joint work with Siddarth Sankaran.

**Speaker:** Julia Gordon, (Mathematics, University of British Columbia)

**Title:** Product formulas for the size of an isogeny class of elliptic curves

**Abstract:**

Consider the question: how likely is a random elliptic curve over the finite field  $\mathbb{F}_p$  to have exactly  $N$  rational points, where  $N$  is a given integer in the appropriate range? In 2003, Gekeler gave an explicit answer based on a heuristic that was too strong to be literally true, thus the answer appeared somewhat mysterious. We provide an explanation for

this formula by making an explicit and very natural connection with a formula of Langlands and Kottwitz which expresses the size of an isogeny class of principally polarized abelian varieties in terms of an adelic orbital integral. Then we discuss a possible extension of Gekeler's computations from elliptic curves to abelian varieties.

This is joint work with Jeff Achter.

**Speaker:** James Parks (Mathematics and Computer Science, University of Lethbridge)

**Title:** The asymptotic constant for amicable pairs of elliptic curves

**Abstract:**

Let  $E$  be an elliptic curve defined over  $\mathbb{Q}$ . If  $p$  is a prime of good reduction then we define the group of points on the reduced elliptic curve over  $\mathbb{F}_p$  as  $E_p(\mathbb{F}_p)$ . Silverman and Stange defined a pair of distinct primes  $(p, q)$  such that  $|E_p(\mathbb{F}_p)| = q$  and  $|E_q(\mathbb{F}_q)| = p$  as amicable pairs. In this talk we consider the function that counts such pairs on average over a family of elliptic curves and discuss results related to the constant obtained in the asymptotic result.

**Speaker:** Anna Puskas, (Mathematics and Statistics, University of Alberta)

**Title:** Demazure-Lusztig operators, crystals and metaplectic Whittaker functions

**Abstract:**

We will discuss the study of metaplectic  $p$ -adic Whittaker functions in terms of Demazure-Lusztig operators. These operators appear in relevant results in the non-metaplectic setting: the work of Brubaker, Bump and Licata in describing Iwahori-Whittaker functions, Tokuyama's theorem and the Demazure character formula. Their metaplectic versions, introduced in joint work with Gautam Chinta and Paul E. Gunnells, satisfy similar formulae. They provide a combinatorial tool to link two metaplectic generalizations of the Casselman-Shalika formula. The two approaches represent the spherical Whittaker function either as a sum over a Weyl group (Chinta-Offen and McNamara) or as a sum over a highest weight crystal (Brubaker-Bump-Friedberg and McNamara). By generalizing Tokuyama's theorem in terms of metaplectic Demazure-Lusztig operators, we show that these approaches produce the same result. Additionally, we will report on recent joint work with Manish Patnaik to express metaplectic Iwahori-Whittaker functions in terms of Demazure-Lusztig operators.

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**Links:**

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**File Uploads:**

Additional Upload 1: [http://www.pims.math.ca/files/final\\_report/FINALREPORT-antd-2015.pdf](http://www.pims.math.ca/files/final_report/FINALREPORT-antd-2015.pdf)

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