

**Alejandro Adem**, University of British Columbia

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Title: *Commuting elements and group cohomology*

Abstract:

In this talk we discuss simplicial spaces derived from the commuting elements in a group. We will describe some of their properties for finite groups as well as compact Lie groups, in particular their cohomology.

**Luchezar Avramov**, University of Nebraska - Lincoln

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Title: *Reverse homological algebra over local rings*

Abstract:

Classical techniques and results from algebraic topology, based on the notion of twisting cochain, are applied to a problem in reverse homological algebra: Given a ring  $R$  and left  $R$ -module  $k$  determine which left graded modules over  $\text{Ext}_R^*(k, k)$  have the form  $\text{Ext}_R^*(M, k)$  for some  $R$ -module  $M$ . The case when  $R$  is a commutative noetherian local ring and  $k$  its residue field has applications to group representations.

**Ragnar-Olaf Buchweitz**, University of Toronto Scarborough

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Title: *Supports for the Singularity Category of a Gorenstein Algebra*

Abstract:

It is known that for a commutative Gorenstein ring, the Hom-spaces in its singularity category, the stable category of maximal Cohen-Macaulay modules, are supported on the singular locus.

What is the exact annihilator, that is, the universal ideal that annihilates all these Hom-spaces? In joint work with H.Flenner we give various answers invoking Hochschild (co-)homology, Noether normalizations, and variants of conductor ideals. We also interpret the annihilators of self-extension groups in terms of covariant derivatives and deformation theory.

**Jon Carlson**, University of Georgia

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Title: *Generic Kernels and other constructions*

Abstract:

We will discuss some extensions and generalizations of notions that grew out of joint work with Friedlander, Pevtsova and Suslin. These involve generic constructions of submodules of a given module. In many cases the constructions are functorial.

**Christopher Drupieski**, University of Georgia

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Title: *Second cohomology for finite groups of Lie type*

Abstract:

Let  $G$  be a simple, simply-connected algebraic group over a field of characteristic  $p > 0$ , let  $q$  be a power of  $p$ , and let  $G(q)$  be the finite subgroup of  $F_q$ -rational points in  $G$ . In this talk I will describe conditions under which the first and second cohomology groups for  $G(q)$  with coefficients in an irreducible  $G(q)$ -module are isomorphic, via the restriction map, to the corresponding cohomology groups for the ambient algebraic group  $G$ . In this way, we are able to obtain new proofs of many first and second cohomology calculations for finite groups of Lie type. Even in cases where the restriction map is not an isomorphism, our methods are able to provide calculations of certain nonzero second cohomology groups. This work is in collaboration with the University of Georgia VIGRE Algebra Group.

**Karin Erdmann**, Oxford University

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Title: *Which selfinjective algebras have a support variety theory?*

Abstract:

Let  $A$  be a finite-dimensional selfinjective algebra over a field  $K$  (which is usually not a Hopf algebra), and let  $HH^*(A)$  be the Hochschild cohomology algebra of  $A$ , this acts on  $\text{Ext}_A^*(M, M)$  for any  $A$ -module

$M$ . If the Hochschild cohomology satisfies suitable finite generation properties then  $A$ -modules have supports defined via this action, and these supports share many properties of supports defined via group cohomology.

We present results of [ES] which give a reduction when  $A$  is Koszul, and we answer precisely when a weakly symmetric selfinjective algebra with radical cube zero has such supports.

We also discuss recent results for special biserial selfinjective algebras on the additive structure of Hochschild cohomology. As well, we show that many such algebras do not have such supports. These include the 4-dimensional algebra studied in [BGMS] whose Hochschild cohomology is finite-dimensional. Special biserial algebras occur amongst blocks of group algebras or Hecke algebras, and deformations.

[BGMS] R. Buchweitz, E.L. Green, D. Madsen, O. Solberg, *Finite Hochschild cohomology without finite global dimension*. Math. Res. Letters **12** (2005) 805-816.

[ES] K. Erdmann, O. Solberg, *Radical-cube zero weakly symmetric algebras and support varieties*. J. Pure Applied Algebra **216**(2011), 185-200.

**Rolf Farnsteiner**, Christian-Albrechts-Universität zu Kiel

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Title: *Categories of modules defined by nilpotent operators*

Abstract:

Let  $\mathcal{G}$  be a finite group scheme, defined over an algebraically closed field of characteristic  $p > 0$ . Given a  $\mathcal{G}$ -module  $M$ , every  $p$ -point  $\alpha : k[T]/(T^p) \rightarrow k\mathcal{G}$  with values in the algebra of measures on  $\mathcal{G}$  gives rise to a  $p$ -nilpotent operator

$$\alpha(t)_M : M \longrightarrow M \quad ; \quad m \mapsto \alpha(t)m \quad (t := T + (T^p)).$$

In a series of papers, Carlson, Friedlander, Pevtsova and Suslin have studied full subcategories of the category  $\text{mod } \mathcal{G}$  of finite-dimensional  $\mathcal{G}$ -modules that are defined via properties of the above operators. In our talk, we shall be mainly concerned with the subcategories  $\text{EIP}(\mathcal{G}) \subseteq \text{CR}(\mathcal{G})$ , whose objects satisfy the equal images property  $\text{im } \alpha(t)_M = V$  and the constant rank property  $\text{rk}(\alpha(t)_M) = d$  for all  $p$ -points  $\alpha$ , respectively.

The purpose of this talk is to draw attention to geometric and combinatorial methods that may yield a better understanding of these categories. If  $\mathcal{G}$  is an infinitesimal group scheme of height  $r$ , then every  $M \in \text{CR}(\mathcal{G})$  defines a morphism

$$\text{im}_M : \mathbb{P}(\mathcal{G}) \longrightarrow \text{Gr}_d(M) \quad ; \quad [\alpha] \mapsto \text{im } \alpha(u_{r-1})_M$$

from the projectivized variety of infinitesimal one-parameter subgroups of  $\mathcal{G}$  to the Grassmannian of  $M$ . Maps of this type turn out to provide information on the dimensions of indecomposable objects of  $\text{CR}(\mathcal{G})$  and  $\text{EIP}(\mathcal{G})$  for arbitrary finite group schemes.

The above categories can also be studied from the vantage point of Auslander-Reiten theory. While the indecomposable objects of  $\text{CR}(\mathcal{G})$  are a union of Auslander-Reiten components, equal images modules usually only appear at their “ends”. For a  $p$ -elementary abelian group  $E_r$  of rank  $r$ , modules of Loewy length  $\leq 2$  correspond to representations of the  $r$ -Kronecker quiver  $K_r$ . In the tame case  $r = 2$ , these modules are well understood. If  $r \geq 3$  methods from the theory of wild hereditary algebras provide information on the distribution of indecomposable equal images modules of Loewy length 2.

**Eric Friedlander**, University of Southern California

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Title: *Elementary subalgebras of restricted Lie algebras*

Abstract:

Jon Carlson, Julia Pevtsova, and I initiate the investigation of the variety  $\mathbb{E}(r, \mathfrak{g})$  of elementary subalgebras of dimension  $r$  of a ( $p$ -restricted) Lie algebra  $\mathfrak{g}$  for some  $r \geq 1$ . For various choices of  $\mathfrak{g}$  and  $r$ , we identify the geometric structure of  $\mathbb{E}(r, \mathfrak{g})$ . This projective algebraic variety encodes considerable information about the representations of  $\mathfrak{g}$ . Special classes of (restricted) representations of  $\mathfrak{g}$  lead to algebraic vector bundles on  $\mathbb{E}(r, \mathfrak{g})$ . For  $\mathfrak{g} = \text{Lie}(G)$  the Lie algebra of an algebraic group, rational representations of  $G$  enable us to realize familiar algebraic vector bundles on  $G$ -orbits of  $\mathbb{E}(r, \mathfrak{g})$ .

**David Hemmer**, SUNY Buffalo

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Title: *Cohomology from modules for symmetric groups and modules for symmetric groups from cohomology of braid groups*

Abstract:

I will speak about two topics related to the cohomology of modules for the symmetric group. The first is a series of results obtained by attempting to compute cohomology for natural choices of symmetric group modules, for example Specht modules and Young modules. These results include specific computations of cohomology, as well as various isomorphisms reminiscent of the generic cohomology theory of CPSvdK.

The second topic is some interesting character theory problems that arise from braid group cohomology via recent work of Church-Farb and others on representation stability.

**Radha Kessar**, University of Aberdeen

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Title: *A parametrisation of blocks of finite simple groups*

Abstract:

The past three decades have seen a lot of progress in our understanding of the modular representation theory of finite groups of Lie type in non-describing characteristic through a melding of Brauer and Lusztig theories. I will talk about joint work with Gunter Malle which fills in the final pieces of the problem of obtaining a parametrization of  $p$ -blocks of finite simple groups. As a consequence, we obtain one direction of Brauer's height zero conjecture from 1955.

**Alexander Kleshchev**, University of Oregon/Stuttgart

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Title: *Some new results on representations of KLR algebras*

Abstract:

We will discuss the affine cellular structure of KLR algebras of finite type: the standard and proper standard modules, homological properties, and a construction of simple modules.

**Markus Linckelmann**, University of Aberdeen

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Title: *On the Hochschild cohomology of block algebras*

Abstract:

One of the questions in the modular representation theory of finite groups is the extent to which the representation theoretic possibilities of a block of a finite group algebra are bounded by its local structure. Donovan's conjecture, predicting that for a fixed defect group there should be only finitely many Morita equivalence classes of block algebras, would imply that there are only finitely many Hochschild cohomology algebras arising from blocks with a fixed defect group. We will show that in each degree there is a bound on the dimension of the Hochschild cohomology in terms of the defect, which may be seen as a higher-dimensional analogue of a classical result of Brauer and Feit. Using Symonds' proof of a conjecture of Benson on the Castelnuovo-Mumford regularity of group cohomology, we show further that there are only finitely many Hilbert series of Hochschild cohomology algebras of blocks with a fixed defect group. This is joint work with Radha Kessar.

**Daniel Nakano**, University of Georgia

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Title: *Complexity for modules over the classical Lie superalgebra  $\mathfrak{gl}(m|n)$*

Abstract:

Let  $\mathfrak{g}$  be a classical Lie superalgebra and  $F$  be the category of finite dimensional  $\mathfrak{g}$ -supermodules which are completely reducible over the (reductive) zero graded component of  $\mathfrak{g}$ . In earlier work, Boe, Kujawa and Nakano [BKN] demonstrated that for any module  $M$  in  $F$ , the rate of growth of the minimal projective resolution (i.e., the complexity of  $M$ ) is bounded by the dimension of  $\mathfrak{g}_1$  (odd component). In this

talk I will show how to compute the complexity of the simple modules and the Kac modules for the Lie superalgebra  $\mathfrak{gl}(m|n)$ . In both cases the complexity is related to the atypicality of the block containing the module. A more intriguing connection that arises involves the combination of the support variety theory developed earlier by BKN, and the associated varieties introduced by Dufflo and Serganova.

My talk represents joint work with Brian Boe and Jonathan Kujawa.

**Brian Parshall**, University of Virginia

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Title: *p-filtrations of Weyl modules for semisimple groups*

Abstract:

Let  $G$  be a semisimple, simply connected algebraic group over an algebraically closed field of positive characteristic  $p$ . For a dominant weight  $\lambda$ , let  $\Delta(\lambda)$  be the Weyl module of highest weight  $\lambda$ . If  $\lambda = \lambda_0 + p\lambda_1$ , with  $\lambda_0$  restricted and  $\lambda_1$  dominant, put  $\Delta^p(\lambda) := L(\lambda_0) \otimes \Delta(\lambda_1)^{[1]}$ , where  $\Delta(\lambda_1)^{[1]}$  is the twist of  $\Delta(\lambda_1)$  through the Frobenius morphism. An old question asks if any Weyl module has a filtration with sections of the form  $\Delta^p(\mu)$ . In this talk, I discuss recent progress on this problem, and indicate some applications. (This is recent joint work with Leonard Scott.)

**Alexander Premet**, University of Manchester

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Title: *Humphreys' conjecture on small modular representations and finite W-algebras*

Abstract:

Let  $G$  be a reductive group over an algebraically closed field of characteristic  $p > 0$  and  $\mathfrak{g} = \text{Lie}(G)$ . In my talk, I'm going to discuss the current status of Humphreys' conjecture on the existence of  $\mathfrak{g}$ -modules with  $p$ -character  $\chi$  of dimension  $p^{d(\chi)}$ .

**Jeremy Rickard**, University of Bristol

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Title: *Block decompositions of thick subcategories*

Abstract:

There are many cases where there is a satisfactory classification of tensor ideal thick subcategories of a tensor triangulated subcategory in geometric terms (for example, the classification of tensor ideal thick subcategories of the stable module category of a finite group in terms of the spectrum of the group cohomology algebra). But for thick subcategories that are not tensor ideal, there seems to be no prospect of a classification. I will consider the question of classifying the subcategories that occur in block decompositions of tensor ideal subcategories, where there may be a more satisfactory theory. This is joint work, or based on joint work, with Jon Carlson.

**Vera Serganova**, University of California - Berkeley

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Title: *Associated varieties for algebraic supergroups and fiber functor*

Abstract:

The category of representations of a semi-simple algebraic supergroup over a field of characteristic zero is not semi-simple. In many ways the situation is similar to the classical case in positive characteristic.

Some time ago M. Duflo and I constructed a functor from this category to the category of equivariant quasicoherent sheaves on the cone of self-commuting odd elements of the corresponding Lie superalgebra. We call the support of the sheaf corresponding to a given representation the associated variety of the representation. This notion is analogous to the notion of rank variety in positive characteristic and associated variety for Harish-Chandra modules.

The geometric fiber of the corresponding sheaf at a generic point of the associated variety defines a functor from a block of the original category to a semi-simple block in the category of representations of a supergroup of the same type of smaller rank. I discuss applications of this fiber functor in representation theory: proof of Kac-Wakimoto conjecture and connection with modified dimension defined by N. Geer,



J. Kujawa and B. Patureau-Mirand. I also formulate open questions and conjectures about the fiber functor.

**Mark Walker**, University of Nebraska-Lincoln

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Title: *Support for complete intersections via higher matrix factorizations*

Abstract:

There is a well-known isomorphism joining the stable category of modules over a hypersurface and the homotopy category of matrix factorizations. I will talk about joint work with Jesse Burke in which we generalize this isomorphism to complete intersections, with the notion of a matrix factorization replaced by "twisted" matrix factorizations over schemes. Among other things, our approach allows for a more geometric description of the support varieties of modules over complete intersections.

**Peter Webb**, University of Minnesota

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Title: *Perfect complexes of group representations*

Abstract:

I construct a certain bilinear form on a Grothendieck group constructed from perfect complexes for a self-injective algebra and show that this form is non-degenerate. As a consequence it turns out that perfect complexes are determined up to homotopy by knowing only the dimensions of the spaces of homotopy classes of homomorphisms into all perfect complexes.

**Sarah Witherspoon**, Texas A&M University

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Title: *Varieties for modules of Hopf algebras*

Abstract:

The theory of support varieties for modules of finite groups may be generalized to finite dimensional Hopf algebras under some finiteness assumptions. Such Hopf algebras include group algebras, restricted Lie algebras, and small quantum groups, as well as many others of current interest. In this talk, we will give an overview of the general theory of varieties for modules of Hopf algebras, some applications, and recent developments.

**Fei Xu**, Shantou University

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Title: *Cohomology and support varieties of finite transporter category algebras*

Abstract:

Let  $G$  be a finite group and  $\mathcal{P}$  a finite  $G$ -poset. Suppose  $k$  is a field of characteristic  $p > 0$  dividing the order of  $G$ . The  $G$ -action on  $\mathcal{P}$  naturally gives rise to a  $G$ -action on  $k\mathcal{P}$ , the  $k$ -category algebra of  $\mathcal{P}$  (or the incidence algebra). Thus we may form a skew group algebra  $k\mathcal{P}[G]$ . When  $\mathcal{P}$  is a trivial  $G$ -poset, this is exact the group algebra  $kG$ . We introduce support varieties for finitely generated  $k\mathcal{P}[G]$ -modules. To do so, we need a finitely generated cohomology ring. It follows from an alternative construction of  $k\mathcal{P}[G]$  that there is a well-behaved cohomology theory. The  $G$ -poset  $\mathcal{P}$  can be used to build a category  $G \times \mathcal{P}$ , called the Grothendieck construction on  $\mathcal{P}$ . This is a finite category whose category algebra  $k(G \times \mathcal{P})$  is isomorphic to  $k\mathcal{P}[G]$  and whose geometric realization, the classifying space  $B(G \times \mathcal{P})$ , is the Borel construction on the  $G$ -space  $B\mathcal{P}$  (the classifying space, or the order complex, of  $\mathcal{P}$ ). We call  $H^*(B(G \times \mathcal{P}), k)$  the ordinary cohomology ring of  $k\mathcal{P}[G] = k(G \times \mathcal{P})$ . It is Noetherian, and moreover can be defined algebraically as  $\text{Ext}_{k(G \times \mathcal{P})}^*(\underline{k}, \underline{k})$  where  $\underline{k}$  is called the trivial module of  $k\mathcal{P}[G] = k(G \times \mathcal{P})$ . Let  $M$  and  $N$  be two finitely generated  $k\mathcal{P}[G] = k(G \times \mathcal{P})$ -modules. It has been shown that  $\text{Ext}_{k(G \times \mathcal{P})}^*(M, N)$  is finitely generated over  $\text{Ext}_{k(G \times \mathcal{P})}^*(\underline{k}, \underline{k})$ . Based on these we extend Carlson's support variety theory for group algebras to  $k(G \times \mathcal{P})$ , which we call a transporter category algebra.