3rd BC Combinatorics Day

Saturday, April 25, 2015

University of Victoria

David Turpin Building, Room A110

Presented by PIMS and the University of Victoria, Department of Mathematics and Statistics

Schedule

9:00 **Coffee and Muffins**

9:30 **Amites Sarkar**, Western Washington University
Highly connected subgraphs of sparse graphs

10:00 **Ben Tremblay**, University of the Fraser Valley
Edge-coloured graphs and switching with the dihedral group

10:30 **Gara Pruesse**, Vancouver Island University
The most elegant bump-number algorithm

11:00 **Bill Bird**, University of Victoria
Non-isomorphic Colourings and Distinguishing Colourings

11:30 **Richard Anstee**, UBC-Vancouver
Multicoloured Matrices

12:00 **LUNCH BREAK**

2:00 **Donovan Hare**, UBC-Okanagan
On the Construction of Graphs With Critical Cograph Partitions

2:30 **Christopher van Bommel**, University of Victoria
Triangle Decompositions of Planar Graphs

3:00 **Sean McGuinness**, Thompson Rivers University
Hamilton Paths in the Cayley Graph of a Dihedral Group

3:30 **Eric Fusy**, UBC-Vancouver
Baxter permutations and meanders

4:00 **Ross Churchley**, SFU
Packing edge-disjoint odd (u, v)*-trails*

Abstracts

In Alphabetical Order

Richard Anstee, UBC-Vancouver

Multicoloured Matrices

One way to encode sets is by an element-set incidence (0,1)-matrix. A matrix is simple if it has no repeated columns. Let $\|A\|$ denote the number of columns of a matrix A. We say that F is a configuration in A or $F \prec A$ if there is a submatrix of A which is a row and column permutation of F. We generalize to r-matrices which are matrices with entries drawn from an r-set $\{0, 1, \ldots, r-1\}$. Our extremal problem considers a given r and a fixed family $\mathcal{F}$ of r-matrices. We tackle the extremal problem to determine, given m, the maximum of $\|A\|$ over all simple m-rowed r-matrices which contain no configuration in $\mathcal{F}$. We generalize a number of known results about forbidden configurations. Füredi and Sali have impressive generalizations of the ideas of VC-dimension. Lu and the speaker have generalized the Balogh-Bollobás result considering the finite families which yields a constant bound. A number of other results are considered in joint work with Sali.

Bill Bird, University of Victoria

Non-isomorphic Colourings and Distinguishing Colourings

A *colouring* of a graph G is an assignment of colours to the vertices of G with no inherent restrictions. In this context, the classical vertex colouring problem is often called 'proper colouring' for clarity. A colouring C is *distinguishing* if no non-trivial automorphism of the graph G fixes the colour of every vertex. The *distinguishing number* of G is the smallest number of colours in a distinguishing colouring of G.

This talk will cover bounds on the distinguishing number of arbitrary graphs and connections between distinguishability problems and classical isomorphism problems. An algorithmic technique to generate all unrestricted colourings up to isomorphism will also be discussed. This technique can be applied to computational searches for various graph theoretic structures, including independent sets, proper colourings and distinguishing colourings, to significantly reduce the search space on graphs with large automorphism groups.

Ross Churchley, SFU

Packing edge-disjoint odd (u, v)-trails

Menger's theorem gives a famous duality between packings and coverings of (u, v)-paths in a graph. But perfect duality may not exist if the paths are further restricted: for example, the maximum number of edge-disjoint odd (u, v)-paths may be less than the number of edges it takes to cover all such paths. In this talk, we explain an approximate duality for packings of odd trails: if a graph does not have k edge-disjoint odd (u, v)-trails, it has a set of fewer than $8k$ edges intersecting all such trails. This is joint work with Bojan Mohar and Hehui Wu.

Eric Fusy, UBC-Vancouver
Baxter permutations and meanders

I will explain on the link between the well studied class of Baxter permutations and a certain sub-class of meanders called monotone meanders, thereby reformulating results of Boyce in a bijective setting. The interplay between meanders and Baxter permutations is also fruitful as it provides a way to specify monotone meanders by local conditions, and then makes it possible to encode these meanders by non-intersecting triples of lattice paths.

Donovan Hare, UBC-Okanagan
On the Construction of Graphs With Critical Cograph Partitions

A *cograph* is a simple graph that does not contain an induced path on 4 vertices. A graph G is *c-colorable* in k colors if the vertices of G can be colored in k colors such that, for each color, the subgraph induced by the vertices assigned the color is a cograph. The smallest such k for a graph G is called the *c-chromatic* number of G and is denoted $c(G)$. In 2010, Gimbel and Nešetřil motivated the importance of this graphical parameter by showing how it fits in an axiomatization of coloring functions and by laying a foundation of results. In this talk, we describe some general constructions of c-chromatic graphs that are *critical* with respect of vertex deletions (i.e. $c(G) = k$, and for all vertices v, $c(G - v) < k$).

This is joint work with Peng Zhang.

Sean McGuinness, Thompson Rivers University
Hamilton Paths in the Cayley Graph of a Dihedral Group

A well-known conjecture states that every finite, connected Cayley graph has a Hamilton cycle. This has been shown to be true for several classes of groups. However, it is still open for the dihedral groups D_{2n}. What is known is that for n even the Cayley graph for D_{2n} is Hamilton connected. For n odd, the situation is much tougher. We will discuss how the solution for this case depends on finding Hamilton paths in specific cubic graphs which are call "Brick products?.

Gara Pruesse, Vancouver Island University
The most elegant bump-number algorithm

The bump number of a linear extension of a poset is the number of occurrences of adjacent elements in the ordering that are comparable in the poset; the bump number of the poset is the least bump number of any linear extension.

A linear-time algorithm to find the least-bump linear extension was identified in the 1980's, but has likely never been implemented due to its extreme complexity. In this talk, we present what we propose to be the "book" algorithm for bump number, easily implemented and fast in practice, which nevertheless has not yet been shown to be linear time. Either the book algorithm is not the most efficient, or the algorithm needs one more bit of cleverness to make it run in linear time, and it?s a puzzle we put to the audience to figure out which is the case.

Joint work with Derek Corneil and Lalla Mouatadid, Department of Computer Science University of Toronto.

Amites Sarkar, Western Washington University
Highly connected subgraphs of sparse graphs

Let G be a graph on n vertices with independence number α. How large a k-connected subgraph must G contain? It turns out that if n is sufficiently large ($n \geq \alpha^2 k + 1$ will do), then G always contains a k-connected subgraph on at least n/α vertices. This is sharp, since G might be the disjoint union of α equally-sized cliques. What if n is *not* sufficiently large? I'll present the (surprisingly complicated) answer when $\alpha = 2$ and $\alpha = 3$. Joint work with Shinya Fujita and Henry Liu.

Ben Tremblay, University of the Fraser Valley
Edge-coloured graphs and switching with the dihedral group

Let G be an m-edge-coloured graph, that is, a graph with m edge sets $E_1, E_2, \ldots, E_m$, where E_i is the set of edges of colour i. Given a group Γ_m of permutations of $\{1, 2, \ldots, m\}$, the operation of switching at a vertex x with respect to $\pi \in \Gamma_m$ permutes the colours of the edges incident with x according to π. We will consider switching when Γ_m is the entire symmetric group and also when it is the dihedral group. After characterizing the graphs which can be made monochromatic of some colour, we examine the complexity of a particular type of proper vertex colouring problem.

Christopher van Bommel, University of Victoria
Triangle Decompositions of Planar Graphs

The Nash-Williams conjecture states that all sufficiently dense triangle-divisible graphs are triangle decomposable. We explore triangle decompositions from the opposite perspective by looking at planar graphs. A triangle decomposition of a graph is a partition of its edge set into triangles. We also consider rational triangle decompositions, where each triangle of the graph is assigned a nonnegative rational weight and for every edge of the graph, the sum of the weights of the triangles containing that edge is 1. We demonstrate a necessary and sufficient condition for a planar multigraph to be triangle decomposable and develop a characterization of rationally triangle decomposable simple planar graphs. Joint work with Kieka Mynhardt.