Title / Abstract

Swarm dynamics and equilibria for a nonlocal aggregation model

Razvan Fetecau (SFU)

We consider the aggregation equation $\varrho_t - \nabla\cdot(\varrho\nabla K*\varrho)=0$ in R^n, where the interaction potential K models short-range repulsion and long-range attraction. We study a family of interaction potentials with repulsion given by a Newtonian potential and attraction in the form of a power law. We show global well-posedness of solutions and investigate analytically and numerically the equilibria and their global stability. The equilibria have biologically relevant features, such as _nite densities and compact support with sharp boundaries. This is joint work with Yanghong Huang and Theodore Kolokolnikov.

A paradox of state-dependent diffusion and how to resolve it

Paul Tupper (SFU)

Abstract: Consider a particle diffusing in a confined volume which is divided into two equal regions. In one region, the diffusion coefficient is twice the value of the diffusion coefficient in the other region. Will the particle spend equal proportions of time in the two regions in the long term? Statistical mechanics would suggest yes, since the number of accessible states in each region is presumably the same. However, another line of reasoning suggests that the particle should spend less time in the region with faster diffusion, since it will exit that region more quickly. We demonstrate with a simple microscopic model system that both predictions are consistent with the information given. Thus, specifying the diffusion rate as a function of position is not enough to characterize the behaviour of a system, even assuming the absence of external forces. We propose an alternative framework for modelling diffusive dynamics in which both the diffusion rate and equilibrium probability density for the position of the particle are specified by the modeller. We introduce a numerical method for simulating dynamics in our framework that samples from the equilibrium probability density exactly and is suitable for discontinuous diffusion coefficients. This is joint work with Xin Yang.

Global wellposedness for the Navier-Stokes-Maxwell system and its asymptotic

Slim Ibrahim (UVic)

Abstract: In this talk, I will first review the results about the existence of global small solutions of the Navier-Stokes-Maxwell system, and then I will show how these solutions converge to the solutions of the standard MHD problem, as the speed of light converges to infinity. The latter part is a joint work with D. Arsenio and N. Masmoudi.

Global Maxwellians over All Space
and Their Relation to Conserved Quantites of Classical Kinetic Equations

C. David Levermore (UMD)

Abstract: When a classical kinetic equation is posed over the entire D dimensional space, every solution with finite mass, energy, and second spatial moments formally has $4 + 2D + D(D-1)/2$ conserved quantities associated with it. The family of global Maxwellians with finite mass over the entire space has the same number of free parameters. We show that a unique global Maxwellian can be associated with each such initial data by matching the values of their conserved quantities. Moreover, the set of all such values is characterized by an inequality on the trace norm of the angular momentum matrix.

Kaniel-Shinbrot Iteration and Global Solutions
of the Cauchy Problem for the Boltzmann Equation

C. David Levermore (UMD)
(joint work with Claude Bardos, Irene Gamba, and Francois Golse)

Abstract: Kaniel-Shinbrot iteration can be used to establish the existence of solutions to the Boltzmann equation over all space for some initial data that is pointwise bounded above and below by a class of global Maxwellians larger than previously considered. When these solutions are global in time, we obtain results on their large-time asymptotics.

Marketing on Random Graphs

Reinhard Illner (UVic)

Regularized 13 Moment Equations: Derivation and boundary value problems

Henning Struchtrup (UVic)

Abstract: The regularized 13 moment equations of rarefied gas dynamics provide a stable set of macroscopic equations to approximate to the Boltzmann equation within the super-Burnett order. The transport coefficients in the equations depend on the molecular interaction model. To give insight into the origin of the equations, I will outline the derivation of the R13 equations for a monatomic hard sphere gas in the linear regime. The equations are based on an extended Grad-type moment system, which is systematically reduced by means of the Order of Magnitude Method . The derivation of the appropriate boundary conditions from the Grad distribution function will be outlined. The resulting R13 equations include the linear Burnett and super-Burnett equations, which are obtained by Chapman-Enskog expansion. While the Burnett coefficients agree with literature values, this is the first time that super-Burnett coefficients are computed for a hard sphere gas. Some comments on the extension to the non-linear regime will be provided as well.

Boundary conditions for the R13 equations must be derived from the boundary conditions for the Boltzmann equation. I'll briefly discuss generalization of Maxwell boundary conditions to include isotropic scattering, and then use these to set up boundary conditions for R13. Different ways to realize adiabatic walls are discussed. Results for 2-D cavity flows for the non-linear Maxwell gas show intricate flow and heat flux patterns that are also observed in solutions of the Boltzmann equation, but unattainable with the laws of classical hydrodynamics (Navier-Stokes and Fourier laws). An interesting observation is a heat flux entering the gas at a fully adiabatic, but rough, wall in DSMC and R13 simulations. The R13 boundary conditions reveal that this heat is produced in the slip, hence we speak of viscous slip heating.

If time permits, I'll also show an extension of Maxwell boundary conditions to include velocity dependent accommodation coefficients, and some pertinent simulations.

Logarithmic Expansions and the Stability of Periodic Patterns of Localized Spots for Reaction-Diffusion Systems in R^2

Michael Ward (UBC)
(joint work with Juncheng Wei (UBC), David Iron (Dalhousie) and John Rumsey (Dalhousie))

The linear stability of steady-state periodic patterns of localized spots in R^2 for the two-component Gierer-Meinhardt (GM) and Schnakenburg reaction-diffusion models is analyzed in the semi-strong interaction limit corresponding to an asymptotically small diffusion coefficient ε^2 of the activator concentration. In the limit $\varepsilon \to 0$, localized spots in the activator are centered at the lattice points of a Bravais lattice with constant area $|\Omega|$. To leading order in $\nu = -1/\log\varepsilon$, the linearization of the steady-state periodic spot pattern has a zero eigenvalue when the inhibitor diffusivity satisfies $D = D_0/\nu$, for some D_0 independent of the lattice and the Bloch wavevector $\kb$. From a combination of the method of matched asymptotic expansions, Floquet-Bloch theory, and the rigorous study of certain nonlocal eigenvalue problems, an explicit analytical formula for the continuous band of spectrum that lies within an $O(\nu)$ neighborhood of the origin in the spectral plane is derived when $D = D_0/\nu + D_1$, where $D_1 = O(1)$ is a de-tuning parameter. The periodic pattern is linearly stable when D_1 is chosen small enough so that this continuous band is in the stable left-half plane $\mathrm{Re}(\lambda) < 0$ for all $\kb$. Moreover, for both the Schnakenburg and GM models, our analysis identifies a model-dependent objective function, involving the regular part of the Bloch Green's function, that must be maximized in order to determine the specific periodic arrangement of localized spots that constitutes a linearly stable steady-state pattern for the largest value of D. From a numerical computation, based on an Ewald-type algorithm, of the regular part of the Bloch Green's function that defines the objective function, it is shown within the class of oblique Bravais lattices that a regular hexagonal lattice arrangement of spots is optimal for maximizing the stability threshold in D.