Submittee: Michael Ward
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Title: AARMS-PIMS Summer School in Differential Equations and Scientific Computing
Event Type: Summer-School

Location:
Dalhousie University, Halifax, Nova Scotia

Dates:
July 6th-31st, 2015

Topic:
The topics for the four courses comprising the workshop were the following:

1) pattern formation problems in applied partial differential equations modeling by reaction-diffusion theory: spectral theory for such problems.

2) the mathematics of nonlinear waves, swarming behavior, and collective dynamics.

3) The analysis of structure-preserving discretizations and geometric integration for the numerical analysis of differential equations.

4) The numerical analysis of singularly perturbed ODEs and PDES with applications to advection dominated flows.

Methodology:
This summer school is intended for graduate and senior undergraduate students, as well as young researchers. Each participant is expected to register for two of the four courses. Each course runs for four weeks and consists of five ninety-minute lectures per week. These are graduate courses approved by Dalhousie and we will facilitate transfer credit to the extent possible.

Course descriptions

Course 1: Waves and patterns in nonlinear systems Instructors: Andrea Bertozzi and Ricardo Carrettero This topics course on waves and pattern formation covers a variety of settings. Topics include: Linear waves (dissipation, dispersion) Kortweg de Vries (KdV) equation: derivation from fluids, cnoidal wave solutions, solitons, conserved quantities. Nonlinear Schrodinger (NLS) equation: dark and bright solitons, conserved quantities, Bose Einstein condensates, variational approximation, soliton-soliton and soliton-trap interactions, vortices. Well posedness, finite time blowup, and pattern formation in nonlocal transport equations. Fluid dynamics: from incompressible Euler to Aggregation equations.

Course 2: Topics in Reaction-Diffusion Systems: Theory and Applications Instructors: Michael Ward and Juncheng Wei This course is intended to expose students to analytical aspects for the study of spatial-temporal patterns in reaction-diffusion (RD) systems. This course consists of two parts: In Part I, we briefly discuss Turing stability analysis, weakly nonlinear theory, and bifurcations of spatially homogeneous solutions to RD systems. For various specific systems with an extreme diffusivity ratio of the diffusing species, we then illustrate and analyze asymptotically different classes of localized patterns, including transition layers and spot patterns. The goal is to develop a facility in the use of systematic asymptotic and singular perturbation methodologies for analyzing various types of localized patterns. Part II is devoted to a rigorous treatment of localized solutions. First we will introduce the method of finite dimensional Liapunov-Schmidt method for scalar equations and then we will apply it to construct various localized patterns for reaction-diffusion systems (spikes, layers, etc). Next we study the stability of these localized patterns. For large eigenvalues we introduce the method of Nonlocal Eigenvalue Problem (NLEP). For small eigenvalues, we return to the method of finite dimensional reduction. Finally if time permits we will introduce the infinite dimensional reduction method.

Course 3: Structure-preserving discretization of differential equations Instructors: Elena Celledoni and Brynjulf Owren Since the early 1990s the study of structure-preserving discretization and geometric integration have emerged as important new fields in the numerical approximation of ordinary and (time-dependent) partial differential equations. Here, concepts like Lie group integrators play a key role in the design of time integrators for PDEs that preserve, e.g., first integrals. This course, presented by two of the leading contributors to the field, will provide a thorough introduction to the basic concepts underlying structure-preserving discretization and then lead the students to the present state of the art. (A representative reference: S. Christiansen, H.Z. Munthe-Kaas and B. Owren, Topics in structure-preserving discretization, Acta Numerica 20 (2011), 1-119.)

Course 4: Numerical analysis of singularly perturbed ODEs and PDES Instructor: Martin Stynes Boundary-value problems for singularly perturbed ordinary differential equations and partial differential equations of convection-diffusion type, and their time-dependent versions arise in many physical applications. Since their solutions usually possess sharp boundary (or interior) layers, one is interested in designing robust numerical schemes whose convergence properties are independent of the singular perturbation parameter 0 epsilon
1. Here, layer-adapted (spatial or temporal) meshes (for example Shishkin meshes) usually form a key part of such methods. This course
is presented by one of the world leaders in this field; it will provide a thorough introduction to the numerical analysis of singularly perturbed differential equations and to its current state of the art. (A representative reference: G.-G. Roos, M. Stynes and L. Tobiska, Robust Numerical Methods for Singularly perturbed Differential Equations (2nd edition), Springer Series in Computational

Mathematics, Springer-Verlag, Berlin, 2008.)

These courses were supplemented by various weekend workshops in the Halifax area over the month of July and early August which the students were strongly encouraged to attend to hear latest advances in disciplines closely related to their summer courses. Many of these students did attend these workshops. See the link below for details on this.

Objectives Achieved:
This was a summer school with the aim of providing advanced training for graduate students and the required background to work in four distinct and active areas of applied mathematics.

Organizers:
Carretero, Ricardo, Dept. of Mathematics, San Diego State University

Kolokolnikov, Theodore, Dept. of Mathematics, Dalhousie University

Ward, Michael, Dept. of Mathematics, Univ. of British Columbia

Brunner, Hermann, Dept. of Mathematics, Memorial Univ. and the Hong Kong Baptist Univ, Hong Kong.

Speakers:
There were four courses for the workshop: Three of these courses were team-taught:

Course 1 (Nonlinear Waves); cotaught by
Andrea Bertozzi, Dept. of Mathematics, UCLA
Ricardo Carretero, Dept. of Mathematics, San Diego State Univ.

Course 2: (Reaction Diffusion Systems); cotaught by
Michael Ward, Juncheng Wei, Dept. of Mathematics, UBC

Course 3: (Structure-preserving discretization of differential
equations), coataught by
Elena Celledoni and Brynjulf Owren, Dept. of Matthematics, Norwegian
Univ. of Science and Technology

Course 4: (Numerical analysis of singularly perturbed ODEs and PDES)
Martin Stynes, Prof. Mathematics, Beijing Computational Science Research Centre, Beijing

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Links:

https://aarms.math.ca/summer/2015/index.html

http://mathstat.dal.ca/~tkolokol/summer/

Comments / Miscellaneous:

The second link above is the link describing the four courses, and the biographies of the instructors. It also indicates that these courses were supplemented by various weekend workshops in the Halifax area, which the students were strongly encouraged to attend to hear latest advances in disciplines closely related to their summer courses. Many of these students did attend these workshops. One of these workshops was partially supported by PIMS, and a separate report on this activity has been filed. The Bluenose Workshop at St. Mary's was another related workshop (not supported by PIMS), that had high-level talks in the field of numerical analysis (closely related to the topics of course 3 and 4). These synergistic activities were very beneficial (we believe) for enriching the summer school experience and for the students to see currently active research topics presented.
