Applied Mathematical Evaluation of Solar Tower Systems: Hybrid Renewable Energy Development

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Outline

• Motivation and Background
• Analytical Study
• Double-Inlet Collector
• Inflatable Tower
• Combined Solar Cycle
• Related Investigations
Motivation

• Fossil fuels are still the dominant source of energy.

• The share of solar energy is still very low (0.43% of the US energy consumption in 2015 and 0.58% in 2016)
Nuclear energy provided around 8% of the total primary energy consumption and all of it was used for electricity generation. The share of solar energy in 2015 US electricity generation was just 0.4% in 2015 and 0.6% in 2016.
Another Motivation/Bad News!

- The single **largest** consumers of fresh water in the United States are **thermal power plants** used to produce electricity.
- Approximately **half** of all fresh water consumed in the United States is used to absorb waste heat from thermal power plants.
- Typically, this water is either returned to the body of water from which it was extracted, or more commonly it is evaporated to the atmosphere in a **cooling tower**.
- Some nuclear power plants (Palo Verde in particular) are currently restricted from expansion by the lack of fresh water.
Similar Historical Approach

No Change In Energy Policy for 300,000 Years,
Throw a Little Carbon on the Fire
Solar Power Towers

• Concentrated Solar Power Plants

• Solar Chimney Power Plants
Solar Chimney Power Plant
Our Goal

• Having solar towers more efficient to be utilized in hybrid energy systems

• To address the water consumption issue in the cooling process
Manzanares Prototype

<table>
<thead>
<tr>
<th>Prototype component</th>
<th>Size (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean collector radius</td>
<td>122.00</td>
</tr>
<tr>
<td>Collector height</td>
<td>1.85</td>
</tr>
<tr>
<td>Chimney radius</td>
<td>5.08</td>
</tr>
<tr>
<td>Chimney height</td>
<td>196</td>
</tr>
<tr>
<td>Rotor blade length</td>
<td>5</td>
</tr>
</tbody>
</table>

- Isidoro Cabanyes, Spanish colonel proposed this idea (1903).
- The most famous prototype built in Manzanares at Spain in 1982 and rebuilt in 1989.
- China recently started to invest on this industry.
Analytical Study

\[ \frac{dA}{A} + \frac{d\rho}{\rho} + \frac{du}{u} = 0 \quad \text{(Continuity)} \]

\[ dp + \rho u du = 0 \quad \text{(Momentum)} \]

\[ c_p dT - dq + u du = 0 \quad \text{(Energy)} \]

\[ dp = d(\rho RT) \quad \text{(State)} \]

\[ \dot{W} \approx \frac{\dot{m}(p_{c,o} - p_{t,i})}{\rho_{turb}} \]
Analytical Study-Collector

\[ dp = \frac{\dot{m}^2}{\rho} \left( \frac{dA}{A^3} - \frac{q''(2\pi r)dr}{\dot{m}(2\pi rh_c)^2 T_c p} + \frac{udu}{A^2 c_p T} + \frac{dp}{A^2 p} \right) \]

\[ p_{c,o} - p_{c,i} \approx \left[ \frac{\dot{m}^2}{2\rho_{m,c}} \left( \frac{1}{A_{c,i}^2} - \frac{1}{A_{c,o}^2} \right) + \frac{q''\dot{m}}{2\pi h_c^2 c_p \rho_{m,c} T_{m,c}} ln \frac{r_{c,i}}{r_{c,o}} \right] \]
Analytical Study-Tower

\[ p_{t,o} = p_\infty \left(1 - \frac{gh_t}{c_p T_\infty}\right)^{\frac{c_p}{R}} \]

\[ \rho_{t,i} = \rho_{c,o} = \rho_\infty \left(1 + \frac{T_\infty - T_{c,o}}{T_\infty}\right) \]

\[ p_{t,o} - p_{t,i} \simeq -\rho_{m,t} gh_t - \frac{\dot{m}^2}{2 \rho_{m,t}} \left(\frac{1}{A_{t,o}^2} - \frac{1}{A_{t,i}^2}\right) \]
Analytical Study - Power Calc

\[ \dot{W} \approx \frac{\dot{m}(p_{c,o} - p_{t,i})}{\rho_{turb}} \]

\[ \dot{W} \approx \frac{\dot{m}}{(\rho_{c,o} + \rho_{t,i})/2} \left[ -\frac{\dot{m}^2}{2\rho_{m,c}} \left( \frac{b - 1}{bA_{c,o}^2} \right) + \frac{q''\dot{m}}{2\pi h_c c_p \rho_{m,c} T_{m,c}} \ln \frac{r_{c,i}}{r_{c,o}} + (\rho_{\infty} - \rho_{m,t})gh_t \right] \]
Analytical Study-Power Calc.
Computational Study

Turbulent Flow

Air

Natural Convection
Comparative Study - Evaluation
Analytical Power

- Solar Irradiation (W/m²)
  - 200
  - 400
  - 600
  - 800
  - 1000

- η (efficiency)
  - 1
  - 0.7
  - 0.5
  - 0.4
  - 0.3
  - 0.2
  - 0.1

- 9 m/s updraft velocity as the maximum reported value

- Output Power (kW)
  - Mass flowrate (kg/sec)

- Output Power (kW)
  - Mass flowrate (kg/sec)
Double-Inlet Collector
Analytical Approach

\[ m_{i2} = 2\pi r (h_{c,i2} - h_{c,i1}) u_{i2} \]

\[ u_{i2} = u_{ref} \left( \frac{h_{c,i2}}{h_{ref}} \right)^\alpha \]

\[ \alpha = \frac{1}{ln(h_{ref}/h)} \]

\( u_{ref} \) is the wind velocity measured at \( h_{ref} \) which is considered as the reference wind velocity value, and \( \alpha \) is the wind shear exponent.
Double-Inlet Collector
Numerical Analysis
Numerical Analysis

Graphs showing the radial direction along the collector (m) vs. density (ρ) and temperature (T) for different solar radiation intensities (400 Wm⁻² to 1000 Wm⁻²).
Numerical Analysis

![Graphs showing velocity and power vs. radial distance and heat flux for different heat flux densities.](image)
Experiments

“A computer lets you make more mistakes faster than any invention in human history— with the possible exceptions of handguns and tequila.” - Mitch Ratcliffe
Experimental Analysis

70°C HT- Average of 150 images
Experimental Results

- Mean:
  \[ u = 9.01 \text{ mm/s} \]
  \[ u \in [7.21, 10.7] \frac{\text{mm}}{\text{s}} \]
Examples of the Area Validation Metric

The modified form for CDF has a separate tracking

- Experiments larger than simulation, $d^+$
- Experiments smaller than simulation, $d^-$

\[ [S + F_s d^-, S + F_s d^+] \text{ where } F_s = 1.25 \]
Mismatch Representation

We can represent the model form uncertainty in an interval by having the mismatch.

- \( u_{model} = [S - F_s d^{-}, S - F_s d^{+}] \frac{mm}{s} = [8.15, 10.3] \frac{mm}{s} \)

- \( S \) is the simulation results
- \( d \) represents the mismatch

The validation metric is defined to be the area between the EDF/CDF from the simulation and the EDF/CDF from the experiment.
Model Verification

• Observed order of accuracy: \( \hat{p} = 1.043 \)

\[
\hat{p} = \frac{\ln\left(\frac{f_3 - f_2}{f_2 - f_1}\right)}{\ln(r)} \quad r = \frac{h_2}{h_1} = \frac{h_3}{h_2}
\]

• Estimated exact solution (RE): \( \bar{u} = 10.508 \text{ mm/s} \)

• Estimated DE: \( \varepsilon_h = 0.1154 \text{ mm/s} \)

\[
\varepsilon_h \approx u_h - \bar{u}
\]

• Grid Convergence Index: \( GCI = 0.3463 \text{ mm/s} \)
Inflatable Tower
Inflatable Tower

2 m/s

15 m/s
Combined Cycle
Thermal Cycle of a PWR
The Proposed Cycle
Combined Solar Cycle

- **Ambient Temperature vs. Thermal Efficiency**
  - Nuclear Only
  - Combined (night)
  - Combined (Day)

- **Solar Plant Design Power vs. Increase in Efficiency**
  - Combined (night)
  - Combined (Day)

- **Available Heat Flux vs. Turbine Output Power**
  - Experimental Data
  - CFD Results
  - Applying 600 W/m² waste heat
On going work

Optimal collector shape: roof and ground*

Heat waste recovery**

*Will be presented at the next PIMS Workshop on Mathematical Sciences and Clean Energy Applications
** Sponsored by Compo Energy
Heat Exchanger

Heat exchanger is a key component in any thermal power cycle. Thermal design and numerical model especially for high temperature and pressure applications

Computational and experimental inspection and testing per ASME codes

Propose enhancement in thermal-hydraulics performance of HX

Involving advanced manufacturing to fabricate
Excessive distortion of tubesheet due to temperature differential

Initial design
(T.S. 4 7/8") (150X magnification)

Final Design
(T.S. 8") (150X magnification)
Thermal Response

Important aspect in HX for inspection and testing

- Cladding
- Thermal fatigue
- Porous composite

Effective thermal material properties of the applied composite shall be evaluated
More Applications

• Porous media
  – Materials & structures containing voids of different phases

• Examples
  – Aerogels
  – 3D-printed structures
  – Nuclear fuel assemblies
  – Carbon composites
  – Pipelines
Computational Model

\[ k_b = \text{bulk thermal conductivity} \]
\[ k^* = \text{dimensionless effective thermal conductivity} \]
\[ L = \text{cell length \\& width} \]
\[ q = \text{heat flux} \]
\[ R = \text{void radius} \]
\[ T = \text{temperature} \]
\[ \alpha = \text{porosity} \]

\[ \alpha = \frac{\pi R^2}{L^2} \]

\[ k^* = \frac{q_3 L}{k_b (T_3 - T_1)} \]
Code Verification

Method of Manufactured Solutions (MMS)

Solution

\[ T_{MMS}(x,y) = \cos\left(\frac{x\pi}{L}\right) \sin\left(\frac{y\pi}{L} + 0.75\right) \]

Source Term

\[ Q_{MMS}(x,y) = (k_x) \cos\left(\frac{x\pi}{L}\right) \sin\left(\frac{y\pi}{L} + 0.75\right) \frac{\pi^2}{L^2} \]

Error

\[ \epsilon_h = \sqrt{\frac{\sum_{i=1}^{N_V} [T_i - T_{MMS}(x_i,y_i)]^2}{N_V}} \]
Some Results

- Predictions for $k^*$ and $\ln(\alpha)$
- Comparison with Landauer, Numerical, Smith 1, Smith 2, Edrisi 1, Edrisi 2
- Trends in $k^*$ and $\ln(\alpha)$
- Various percentages: 4.9%, 11.0%, 19.6%, 30.7%, 44.2%, 60.1%

Graphs showing the relationship between $\ln(k^*)$ and $\ln(\alpha)$ for different values of $h$.
Summary

- Hybrid clean energy systems can be a solution to provide a more stable baseline of energy
- Design, development and modeling advanced hybrid clean energy power cycle are presented
- Computational fluid dynamics and heat transfer evaluation of energy conversion components
- Experimental prototype analysis was performed to validate our numerical model
References


Thank you!
Questions?

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