Behavioral biases and representative agent (or the brain as a central planner with heterogeneous doers)

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Axiomatic approach

- Lotteries space $X$ (for instance $R^m \times \Delta_m$)
- $x = (x^1, \pi^1; \cdots; x^n, \pi^n)$
- $\succeq$ preference relation on $X$

Theorem (VNM)

If $\succeq$ is complete, transitive, Archimedean and satisfies

$$x \succeq y \implies \alpha x + (1 - \alpha) r \succeq \alpha y + (1 - \alpha) r, \quad \forall r, \forall \alpha \in [0, 1]$$

(Independence)

then $x \succeq y \iff E[u(x)] \geq E[u(y)]$ with $u : R \rightarrow R$ non-decreasing and concave.
Subjective probabilities

- Random variables space (payoffs space) $X$
- $\succeq$ preference relation on $X$

**Theorem (Savage)**

If $\succeq$ satisfies the rationality axioms then

$$x \succeq y \iff \sum_{j=1}^{m} \pi^j u(x^j) \geq \sum_{j=1}^{m} \pi^j u(y^j)$$

for some $u : R \rightarrow R$ non-decreasing and concave and some $\pi \in \Delta_m$. 

Subjective probabilities
Pareto optimum

- $n$ agents, $i = 1, \ldots, n$
- $u_i$, $i = 1, \ldots, n$
- $Q_i = \left( \pi_i^j \right)$, $i = 1, \ldots, n$
- $U_i(x) = E^{Q_i} [u_i(x)] = E [M_i u_i(x)]$
- total endowment $x$

**Definition**

An allocation $(x_1, \cdots, x_n)$ such that $\sum_{i=1}^{n} x_i = x$ is Pareto optimal if there is no $(y_1, \cdots, y_n)$ such that $\sum_{i=1}^{n} y_i = x$ and $U_i(y_i) \geq U_i(x_i), i = 1, \ldots, n$, and $U_i(y_j) > U_i(x_j)$ for some $j$. 
Characterization of Pareto optima

**Theorem**

\((x_1^*, \ldots, x_n^*)\) is a PO iff \(\exists (\lambda_i) \in \Delta_n \text{ s.t.} \)
\[(x_1^*, \ldots, x_n^*) = \arg \max \sum_{i=1}^n x_i = x \sum_{i=1}^n \lambda_i U(x_i).\]

For given weights \((\lambda_i) \in \Delta_n\), the social welfare value function is defined by

\[U(x) = \max_{\sum_{i=1}^n x_i = x} \sum_{i=1}^n \lambda_i U_i(x_i) = \sum_{i=1}^n \lambda_i U_i(x_i^*).\]
Time preferences

\[ t = 0, 1, \ldots, T \]
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- $\rho_i, i = 1, \ldots, n$
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- \( M_t^i = \left. \frac{dQ_i}{dP} \right|_{F_t} \)
- \( U_i(x(0), x(1), \ldots, x(T)) = \sum_{t=0}^{T} \exp(-\rho_i t) U_i(x(t)) = \sum_{t=0}^{T} E \left[ M_t^i D_t^i u_i(x(t)) \right] \)
Behavioral features

- Hyperbolic discounting (Loewenstein and Prelec): $\rho_i$ is not constant and decreases with time

\[
\sum_{j=1}^{m} w^j(x) u(x^j)
\]

with

\[
w^j(x) = w(p_j + \cdots + p_n) - w(p_{j+1} + \cdots + p_n).
\]
Behavioral features

- Hyperbolic discounting (Loewenstein and Prelec): $\rho_i$ is not constant and decreases with time.
- Prospect Theory (Kahneman and Tversky): Probabilities depend upon the prospect $\sum_{j=1}^{m} w(\pi^j)u(x^j)$. 

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- Cumulative Prospect Theory (Tversky and Kahneman), if $x_1 < x_2 < \cdots < x_n$

$$\sum_{j=1}^{m} w^j(x)u(x^j)$$

with

$$w^j(x) = w(p_j + \cdots + p_n) - w(p_{j+1} + \cdots + p_n).$$
Parametric and non-parametric calibrations
Discriminability

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Decision Theory
The log-normal case
Time preference rates
More general distributions
Interpretation

Figure:

\[ w(p) \]

[Graph showing discriminability with a logarithmic scale on the x-axis and a linear scale on the y-axis, with values ranging from 0 to 1.]
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Attractivity

Figure:
Representative agent

- All agents have the same CRRA utility function
  \[ u(s) = \frac{s^{1-\frac{1}{\eta}}}{1-\frac{1}{\eta}}. \]

- The number of agents can be finite or infinite

**Theorem**

*If the characteristics \((M^i_t, D^i_t, \lambda_i)_{i \in I}\) are independent, then*

\[ U(x) = E\left[ M_t D_t u(x) \right] \]

*with*

\[ M_t = \left( \frac{1}{|I|} \sum_{i \in I} (M^i_t)^\eta \right)^{\frac{1}{\eta}} \quad \text{and} \quad D_t = \left( \frac{1}{|I|} \sum_{i \in I} (D^i_t)^\eta \right)^{\frac{1}{\eta}}. \]
Our aim

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- We analyze the behavioral properties of the representative agent
- It can be seen as a model of individual decision making if RA=individual and agents=processes
We denote by $Q$ the representative agent probability

**Theorem**

$$f_t = \left( \frac{1}{|I|} \sum_{i \in I} (f_t^i)^\eta \right)^{\frac{1}{\eta}}$$

- If $\eta = 1$, $E^Q [x] = \frac{1}{|I|} \sum_{i \in I} E^{Q_i} [x]$ and $E^Q [x] = E^P [x]$ if no bias
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- If $\eta = 1$, $\text{Var}^Q [x] = \text{Var}^P [x] + \text{Var}_i [E^{Q_i} [x]]$
The log-normal case

- \( \log x \sim \mathcal{N}(\mu, \sigma^2) \)

**Theorem**

If agents' beliefs are heterogeneous enough, the distribution of \( \log x \) is bimodal.

1. More variance, excess kurtosis and same mean (if no bias),
2. \( \text{Var} \log x \) increases with the level of risk tolerance \( \eta \).
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The bimodal representative agent distribution
Properties of the representative agent’s belief

**Theorem**

*If I is made of both optimistic and pessimistic agents.*

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5. The transformation $w$ is inverse $S$-shaped (Abdellaoui, Gonzalez and Wu, Prelec)
The transformation $w$
Prelec’s function
Shifts

Theorem

If all the agents have log-utility functions then a left shift on the distribution of $\mu_i$ increases attractiveness and a symmetric spread on the distribution of $\mu_i$ decreases discriminability.
Time preference rates

\[ D_t = \left( \sum_{i \in I} \frac{1}{|I|} (D^i_t)^\eta \right)^{\frac{1}{\eta}} \] where \( D^i_t \equiv \exp(-\rho_i t) \).
Time preference rates

- \( D_t = \left( \sum_{i \in I} \frac{1}{|I|} \left( D_t^i \right)^\eta \right)^{1/\eta} \) where \( D_t^i \equiv \exp(-\rho_i t) \).
- Representative agent marginal and average time preference rates \( \rho_m \) and \( \rho_a 

\[ \rho_m^D (t) = -\frac{D_t'}{D_t} \]

\[ \rho_a^D (t) = -\frac{1}{t} \log D_t \]
Behavioral properties of the rep. agent’s rates

**Theorem**

\( \rho_m \) and \( \rho_a \) are given by

\[
\rho_a^D(t) = -\frac{1}{t} \log \left[ \frac{1}{N} \sum_{i=1}^{N} \exp(-\eta \rho_i t) \right]^{1/\eta}
\]

\[
\rho_m^D(t) = \sum_{i=1}^{N} \frac{\exp(-\eta \rho_i t)}{\sum_{i=1}^{N} \exp(-\eta \rho_i t)} \rho_i
\]

They are lower than the average of the time preference rates

\[
\rho_m^D(t) < \frac{1}{N} \sum_{i=1}^{N} \rho_i \quad \text{and} \quad \rho_a^D(t) < \frac{1}{N} \sum_{i=1}^{N} \rho_i
\]

“Behavioral Properties” (Lengwiller, Gollier-Zeckhauser): \( \rho_m \) and \( \rho_a \) decrease with time and

\[
\lim_{t \to +\infty} \rho_m^D(t) = \lim_{t \to +\infty} \rho_a^D(t) = \inf \rho_i
\]
Specific Distributions

If we assume a Gamma distribution with mean $m$ and variance $\nu$ for the $\rho_i$'s we obtain (hyperbolic discounting)

$$\rho^D_m(t) = \frac{m^2}{m + \eta \nu^2 t}$$
A FSD (resp. SSD) dominated shift on the distribution $f_\rho$ of individual marginal time preference rates decreases the representative agent average time preference rate $\rho^D_a$.
A MLR (resp. PD) dominated shift on the distribution $f_{\rho}$ of individual marginal time preference rates decreases the representative agent average time preference rate $\rho^D_m$. 
Optimism and pessimism

Definition

An agent is said to be optimistic (resp. pessimistic) if \( \frac{f_i}{f} \) is nondecreasing. Agent \( i \) is said to be more optimistic than agent \( j \) and we denote by \( f_i \succ f_j \) if and only if \( \frac{f_i}{f_j} \) is nondecreasing.

The probability weighting function \( g_i \) transforms the objective decumulative distribution function \( F \) into the agent’s subjective decumulative distribution function \( F_i \), i.e. \( F_i = g_i \circ F \).

\( \frac{f_i}{f} \) is nondecreasing (resp. nonincreasing) if and only if \( g_i \) is convex (resp. concave). (Diecidue and Wakker, 2001) in a RDEU framework.
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- A MLR dominated shift for a given distribution reduces the mean.
Behavioral properties

Theorem

*If there are at least one optimistic agent and one pessimistic agent*

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4. $f_M \sim_{\infty} f_{\text{opt}}^{\max}$ and $f_M \sim_{-\infty} f_{\text{pess}}^{\max}$. 

5. The probability weighting function is concave for small probabilities, and convex for high probabilities.
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Shifts

Theorem

1. If log-utilities and if agent’s density functions are totally ordered with respect to the FSD order then a FSD dominated shift in agents’ density functions distribution leads to a less attractive density function for the representative agent.
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Theorem

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2. If the set \( (f_i)_{i \in I} \) of agent’s density functions is totally ordered with respect to the MLR order then a MLR dominated shift in agents’ density functions distribution leads to a more pessimistic representative agent.
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Interpretation

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- The attribution is made in order to maximize the total utility (specialization of the processes).
- Other models in the literature (Carillo-Brocas): asymmetric information.