Behavioral biases and representative agent (or the brain as a central planner with heterogeneous doers)

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Homo Economicus

Axiomatic approach

- Lotteries space X (for instance $R^m \times \Delta_m$)
- $x = (x^1, \pi^1; \cdots; x^n, \pi^n)$
- $\succeq$ preference relation on X

Theorem (VNM)

If $\succeq$ is complete, transitive, Archimedean and satisfies

$$x \succeq y \Rightarrow \alpha x + (1-\alpha) r \succeq \alpha y + (1-\alpha) r, \qquad \forall r, \forall \alpha \in [0,1] \quad \text{(Independence)}$$

then $x \succeq y \iff E[u(x)] \geq E[u(y)]$ with $u : R \to R$ non-decreasing and concave.

Subjective probabilities

- Random variables space (payoffs space) X
- $\succeq$ preference relation on X

Theorem (Savage)

If $\succeq$ satisfies the rationality axioms then $x \succeq y \Longleftrightarrow \sum_{j=1}^{m} \pi^j u(x^j) \geq \sum_{j=1}^{m} \pi^j u(y^j)$ for some $u : R \to R$ non-decreasing and concave and some $\pi \in \Delta_m$.

Pareto optimum

- n agents, $i = 1, \ldots, n$
- u_i, $i = 1, \ldots, n$
- $Q_i = \left(\pi_i^j\right)$, $i = 1, \ldots, n$
- $U_i(x) = E^{Q_i}\left[u_i(x)\right] = E\left[M_i u_i(x)\right]$
- total endowment x

Definition

An allocation $(x_1, \cdots, x_n)$ such that $\sum_{i=1}^n x_i = x$ is Pareto optimal if there is no $(y_1, \cdots, y_n)$ such that $\sum_{i=1}^n y_i = x$ and $U_i(y_i) \geq U_i(x_i), i = 1, \ldots, n$, and $U_j(y_j) > U_j(x_j)$ for some j.

Characterization of Pareto optima

Theorem

$(x_{1,}^*, \cdots, x_n^*)$ is a PO iff $\exists\, (\lambda_i) \in \Delta_n$ s.t.
$(x_{1,}^*, \cdots, x_n^*) = \arg\max_{\sum_{i=1}^n x_i = x} \sum_{i=1}^n \lambda_i U(x_i)$.

For given weights $(\lambda_i) \in \Delta_n$, the social welfare value function is defined by

$$U(x) = \max_{\sum_{i=1}^n x_i = x} \sum_{i=1}^n \lambda_i U_i(x_i) = \sum_{i=1}^n \lambda_i U_i(x_i^*)$$

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- $D_t^i = \exp(-\rho_i t)$
- $M_t^i = \left.\frac{dQ_i}{dP}\right|_{F_t}$
- $U_i(x(0), x(1), \cdots, x(T)) = \sum_{t=0}^{T} \exp(-\rho_i t) U_i(x(t)) = \sum_{t=0}^{T} E\left[M_t^i D_t^i u_i(x(t))\right]$

Behavioral features

- Hyperbolic discounting (Loewenstein and Prelec): ρ_i is not constant and decreases with time

$$\sum_{j=1}^{m} w^j(x)u(x^j)$$

with

$$w^j(x) = w(p_j + \cdots + p_n) - w(p_{j+1} + \cdots + p_n).$$

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- Cumulative Prospect Theory (Tversky and Kahneman), if $x_1 < x_2 < \cdots < x_n$

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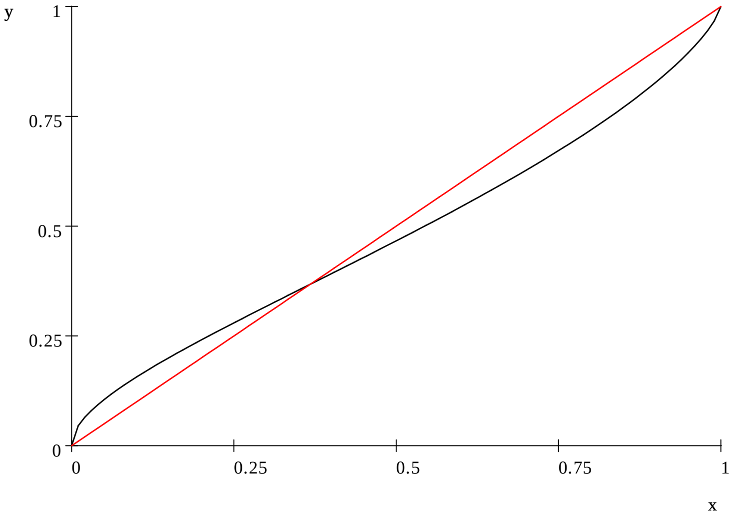
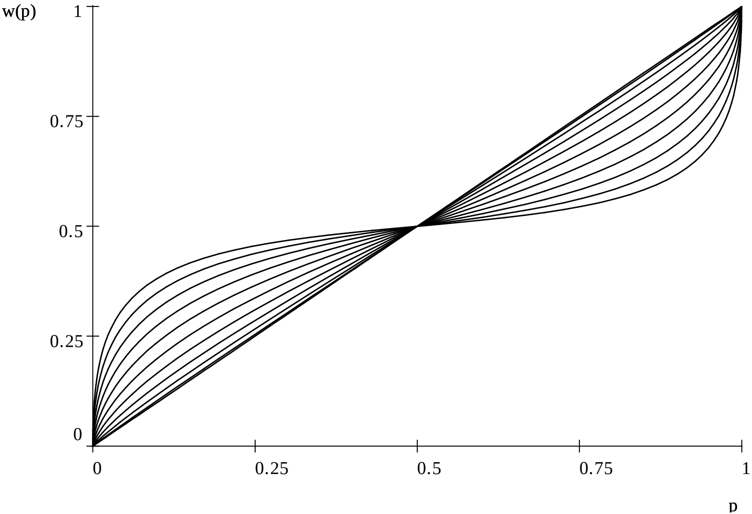
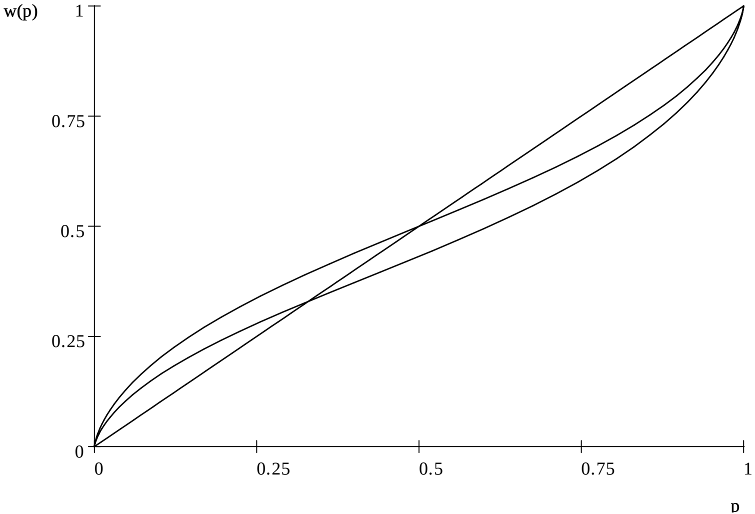
Parametric and non-parametric calibrations



Discriminability



Attractivity



Representative agent

- All agents have the same CRRA utility function $u(s) = \frac{s^{1-\frac{1}{\eta}}}{1-\frac{1}{\eta}}$.
- The number of agents can be finite or infinite

Theorem

If the characteristics $\left(M_t^i, D_t^i, \lambda_i\right)_{i\in I}$ *are independent, then*

$$U(x) = E\left[M_t D_t u(x)\right]$$

with

$$M_t = \left(\frac{1}{|I|}\sum_{i\in I}\left(M_t^i\right)^\eta\right)^{\frac{1}{\eta}} \text{ and } D_t = \left(\frac{1}{|I|}\sum_{i\in I}\left(D_t^i\right)^\eta\right)^{\frac{1}{\eta}}.$$

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- We analyze the behavioral properties of the representative agent
- It can be seen as a model of individual decision making if RA=individual and agents=processes

Distribution of x for the representative agent

We denote by Q the representative agent probability

Theorem

$$f_t = \left(\frac{1}{|I|}\sum_{i\in I}\left(f_t^i\right)^\eta\right)^{\frac{1}{\eta}}$$

- If $\eta = 1$, $E^Q[x] = \frac{1}{|I|}\sum_{i\in I} E^{Q_i}[x]$ and $E^Q[x] = E^P[x]$ if no bias

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- If $\eta = 1$, $Var^Q[x] = Var^P[x] + Var_i\left[E^{Q_i}[x]\right]$

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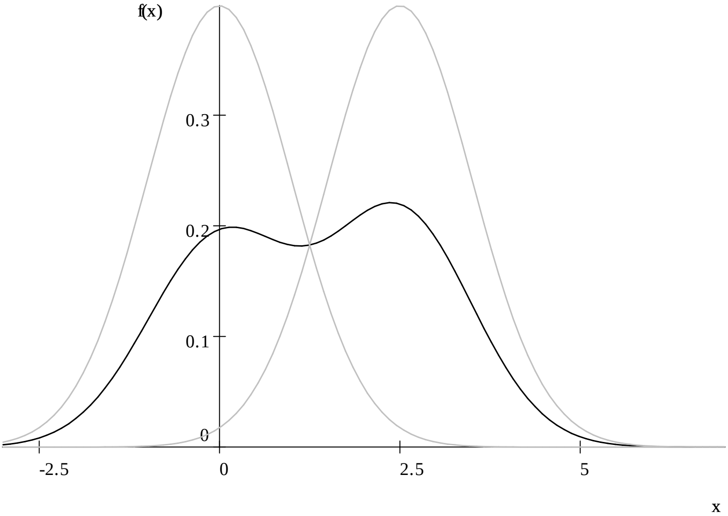
3. $Var^Q[\log x]$ *increases with the level or risk tolerance* η.

The bimodal representative agent distribution



Properties of the representative agent's belief

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If I is made of both optimistic and pessimistic agents.

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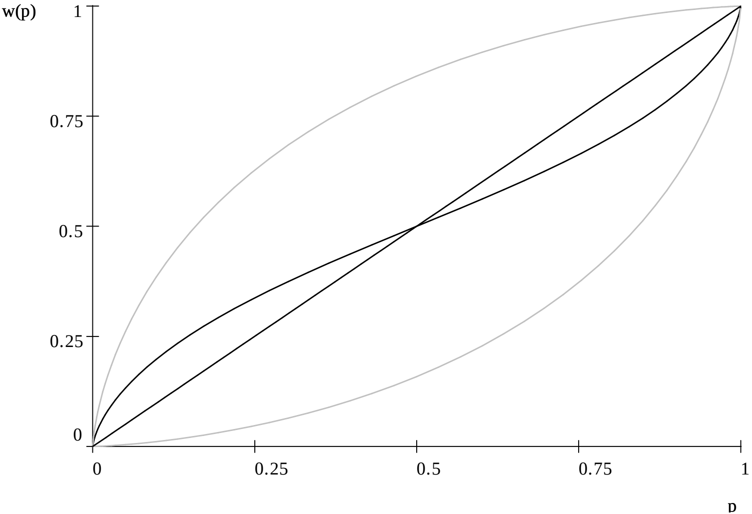
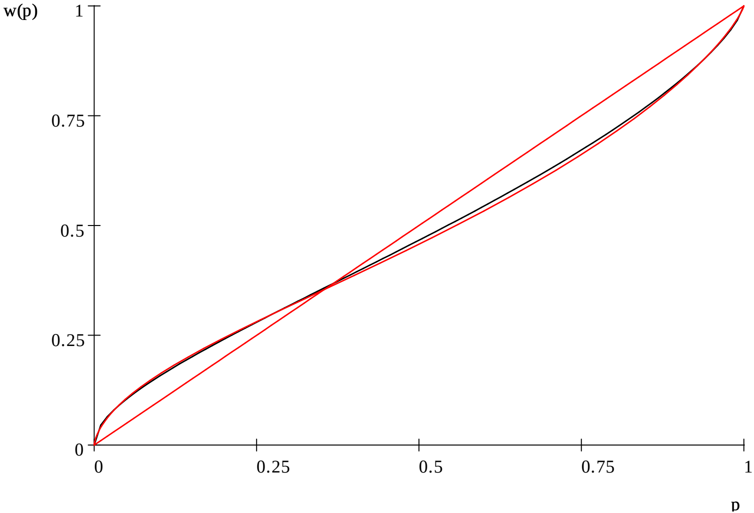
5. *The transformation w is inverse S-shaped (Abdellaoui, Gonzalez and Wu, Prelec)*

The transformation w



Prelec's function



Shifts

Theorem

If all the agents have log-utility functions then a left shift on the distribution of (μ_i) increases attractiveness and a symmetric spread on the distribution of (μ_i) decreases discriminability.

Time preference rates

- $D_t = \left(\sum_{i\in I} \frac{1}{|I|} \left(D_t^i\right)^\eta\right)^{\frac{1}{\eta}}$ where $D_t^i \equiv \exp\left(-\rho_i t\right)$.

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- Representative agent marginal and average time preference rates ρ_m and ρ_a

$$\begin{aligned} \rho_m^D(t) &= -\frac{D_t'}{D_t} \\ \rho_a^D(t) &= -\frac{1}{t}\log D_t \end{aligned}$$

Behavioral properties of the rep. agent's rates

Theorem

ρ_m and ρ_a are given by

$$\rho_a^D(t) = -\frac{1}{t}\log\left[\frac{1}{N}\sum_{i=1}^{N}\exp\left(-\eta\rho_i t\right)\right]^{1/\eta}$$

$$\rho_m^D(t) = \sum_{i=1}^{N}\frac{\exp\left(-\eta\rho_i t\right)}{\sum_{i=1}^{N}\exp\left(-\eta\rho_i t\right)}\rho_i$$

They are lower than the average of the time preference rates

$$\rho_m^D(t) < \frac{1}{N}\sum_{i=1}^{N}\rho_i \text{ and } \rho_a^D(t) < \frac{1}{N}\sum_{i=1}^{N}\rho_i$$

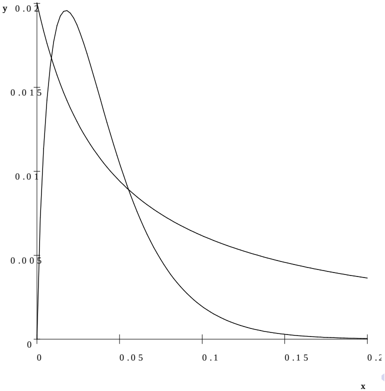
"Behavioral Properties" (Lengwiller, Gollier-Zeckhauser): ρ_m and ρ_a decrease with time and

$\lim \rho_m^D(t) = \lim \rho_a^D(t) = \inf(\rho_i)$

Specific Distributions

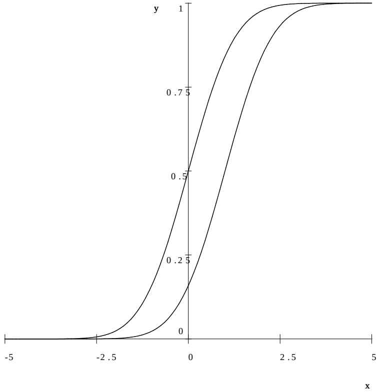
If we assume a Gamma distribution with mean m and variance v for the ρ_is we obtain (hyperbolic discounting)

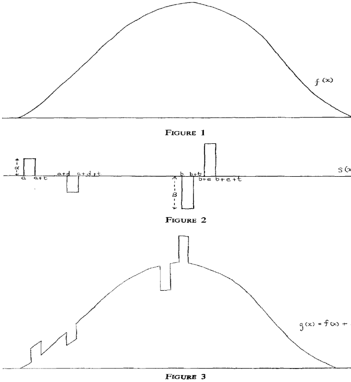
$$\rho_m^D(t) = \frac{m^2}{m + \eta v^2 t}$$



FSD and SSD shifts

A FSD (resp. SSD) dominated shift on the distribution f_ρ of individual marginal time preference rates decreases the representative agent average time preference rate ρ_a^D.





MLR and PD shifts

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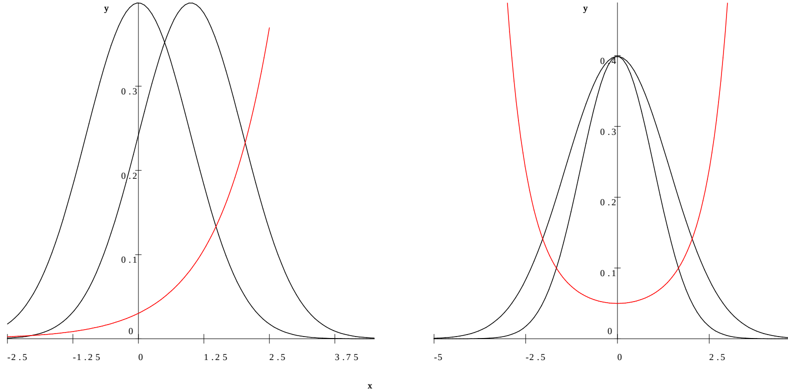

Optimism and pessimism

Definition

An agent is said to be optimistic (resp. pessimistic) if $\frac{f_i}{f}$ is nondecreasing . Agent i is said to be more optimistic than agent j and we denote by $f_i \succcurlyeq f_j$ if and only if $\frac{f_i}{f_j}$ is nondecreasing.

The probability weighting function g_i transforms the objective decumulative distribution function F into the agent's subjective decumulative distribution function F_i, i.e. $F_i = g_i \circ F$.

- $\frac{f_i}{f}$ is nondecreasing (resp. nonincreasing) if and only if g_i is convex (resp. concave). (Diecidue and Wakker, 2001) in a RDEU framework.

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- A MLR dominated shift for a given distribution reduces the mean.

Behavioral properties

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5. *The probability weighting function is concave for small probabilities, and convex for high probabilities.*

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2. *If the set $(f_i)_{i \in I}$ of agent's density functions is totally ordered with respect to the MLR order then a MLR dominated shift in agents' density functions distribution leads to a more pessimistic representative agent.*

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- Other models in the literature (Carillo-Brocas): asymmetric information