Pacific Institute for the Mathematical Sciences

PIMS Workshop on New Trends in Localized Patterns in PDES

May 10- 13, 2021

8:30AM Pacific

Schedule of Talks

***All times Pacific**

Monday (May 10)

8:30 am - 9:00 am	Opening ceremony and chat
	Zoom
9:00 am - 9:45 am	**Arjen Doelman**, Leiden University, Netherlands
	Localized slow patterns in singularly perturbed 2-component reaction-diffusion equations
10:00 am -10:45 am	**Alan Champneys**, University of Bristol
	Localised patterns and semi-strong interaction, a unifying framework for reaction-diffusion systems
11:00 am -11:45 am	**Andrew Krause**, University of Bristol
	Recent Progress and Open Frontiers in Turing-Type Morphogenesis
12:00 pm -12:45 pm	**Rustum Choksi**, McGill University
	The Phase-Field-Crystal Model at Large and Small Scales
12:45 pm - 1:15 pm	**Celebrating Michael** (Michele Titcombe)
1:30 pm - 3:00pm	**Break**
3:00 pm - 3:45 pm	**Theodore Kolokolnikov**, Dalhousie University
	A ring of spikes for the Schnakenberg model
4:00 pm - 4:45 pm	**David Iron**, Dalhousie University
	Noise and Spike Dynamics for the Gierer-Meinhardt Equations
5:00 pm - 5:30 pm	**Daniel Gomez**, University of British Columbia
	Pattern formations in the presence of inhomogeneous boundary conditions
5:30 pm - 6:00 pm	**Chunyi Gai**,
	Resource-mediated competition between two plant species with different rates of water intake

Tuesday (May 11)

8:30 am - 9:15 am	**Philip Maini,** University of Oxford
	Modelling collective cell movement in biology and medicine
9:30 am - 10:15 am	**Matthias Winter**, Brunel University
	Stable spikes for a reaction-diffusion system with two activators and one inhibitor
10:30 am - 11:15 am	**Sean Lawley**, University of Utah
	Extreme first passage times
11:30 am - 12:15 pm	**Iain Moyles**, York University
	Regularization of the Ostwald supersaturation model for Liesegang bands
12:15 pm - 12:45 pm	Celebrating Michael
12:30pm - 3:00pm	**Break**
3:00 pm - 3:45 pm	**Justin Tzou**, Macquarie University
	A brief overview of methods and results for localized patterns and narrow escape problems
4:00 pm - 4:45 pm	**Jia Gou**, UC Riverside
	Growth Control in the Drosophila Wing Imaginal Disc

5:00 pm - 5:45 pm **Yana Nec**, Thompson River University

Spike patterns as a window into non-injective transient diffusive processes

6:00 pm - 6:30 pm **Jason Gilbert**, University of Saskatchewan

Global Optimization of the Mean First Passage Time for Narrow Capture Problems in Elliptic Domains:

Wednesday (May 12)

8:30 am - 9:15 am **Yasumasa Nishiura**, Hokkaido University

Traveling pulses with oscillatory tails, figure-eight stack of isolas, and dynamics inheterogeneous media

9:30 am - 10:15 am **Edgar Knobloch**, UC Berkeley

The conserved Swift-Hohenberg equation and crystallization

10:30 am -11:15 am **Xiaofeng Ren**, George Washington University

Non-hexagonal lattices from a two species interacting system

11:30 am - 12:15 pm **Alan Lindsay**, Univ. Notre Dame

Mathematics of diffusive signaling and the role of receptor clustering in chemoreception

12:15 pm - 12:45 pm Michael Ward's Contributions (Theodore, David, Alan)

Thursday (May 13)

8:30 am - 9:15am **Leah Edelstein-Keshet and Andreas Buttenschoen,** University of British Columbia

Patterns, waves, and bifurcations in cell migration

9:30 am - 10:15 am **Thomas Hillen**, University of Alberta

Symmetries and Bifurcations in Non-Local Cell Adhesion Models

10:20 am - 11:05 am **Alexei Cheviakov**, University of Saskatchewan

Narrow Escape and Narrow Capture Problems in Three Dimensions: Solutions and the Optimization of Trap Configurations

11:10 am - 11:55 am **Jay Newby**, University of Alberta

Stochastic models of rare events in cell biology

12:00 pm - 12:45 pm **P.C. Bressloff**, University of Utah

Asymptotic analysis of target fluxes in the three-dimensional narrow capture problem

12:45 pm - 1:15 pm "Zoom" Birthday Celebration

Speaker Abstracts

P.C. Bressloff, University of Utah

Asymptotic analysis of target fluxes in the three-dimensional narrow capture problem

We develop an asymptotic analysis of target fluxes in the three-dimensional (3D) narrow capture problem. The latter concerns a diffusive search process in which the targets are much smaller than the size of the search domain. The small target assumption allows us to use matched asymptotic expansions and Green's functions to solve the diffusion equation in Laplace space. In particular, we derive an asymptotic expansion of the Laplace transformed flux into each target in powers of the non-dimensionalized target size ϵ. One major advantage of working directly with fluxes is that one can generate statistical quantities such as splitting probabilities and conditional first passage time moments without having to solve a separate boundary value problem in each case. However, in order to derive asymptotic expansions of these quantities, it is necessary to eliminate Green's function singularities that arise in the limit $s \longrightarrow 0$, where s is the Laplace variable. We achieve this by considering a triple expansion in ϵ, s and $\Lambda \sim \epsilon/s$. This allows us to perform partial summations over infinite power series in Λ, which leads to multiplicative factors of the form $\Lambda^n/(1+\Lambda)^n$. Since $\Lambda^n/(1+\Lambda)^n \longrightarrow 1$ as $s \longrightarrow 0$, the singularities in s are eliminated. We then show how corresponding asymptotic expansions of the splitting probabilities and conditional MFPTs can be derived in the small-s limit. The resulting expressions agree with previous asymptotic expansions derived by solving separate boundary values problems for each statistical quantity, although the latter were only carried out to second order in the expansions. Here we also determine the third order contributions, which are $O(\epsilon^2)$ and $O(\epsilon)$ in the case of the splitting probabilities and conditional MFPTs, respectively. Finally, we briefly indicate how to extend the analysis to partially absorbing targets.

Alan Champneys, University of Bristol

Localised patterns and semi-strong interaction, a unifying framework for reaction-diffusion systems

Systems of activator-inhibitor reaction-diffusion equations posed on an infinite line are studied using a variety of analytical and numerical methods. A canonical form is considered that contains all known models with simple cubic autocatalytic nonlinearity and arbitrary constant and linear kinetics. Restricting attention at first to models that have a unique homogeneous equilibrium, this class includes the classical Schnakenberg and Brusselator models, as well as other systems proposed in the literature to model morphogenesis. Such models are known to feature Turing instability, when activator diffuses more slowly than inhibitor, leading to stable spatially periodic patterns. Conversely in the limit of small feed rates, semi-strong interaction asymptotic analysis as introduced by Michael Ward and his collaborators shows existence of isolated spike-like patterns.

Connecting these two regions, a certain universal two-parameter state diagram is revealed in which the Turing bifurcation becomes sub-critical, leading to the onset of homoclinic snaking. This regime then morphs into the spike regime, with the outer-fold being predicted by the semi-strong asymptotics. A rescaling of parameters and field concentrations shows how this state diagram can be studied independently of the diffusion rates. Temporal dynamics is found to strongly depend on the diffusion ratio though. A Hopf bifurcation occurs along the branch of stable spikes, which is subcritical for small diffusion ratio, leading to collapse to the homogeneous state. As the diffusion ratio increases, this bifurcation typically becomes supercritical, interacts with the homoclinic snaking and also with a supercritical homogeneous Hopf bifurcation, leading to complex spatio-temporal dynamics. The details are worked out for a number of different models that fit the theory using a mixture of weakly nonlinear analysis, semi-strong asymptotics and different numerical continuation algorithms.

The theory is extended include models, such as Gray-Scott, with bistability of homogeneous equilibria. A homotopy is studied that takes a Schnakenberg-like glycolysis model for r = 0 to the Gray-Scott model for r = 1. Numerical continuation is used to understand the complete sequence of transitions to two-parameter bifurcation diagrams within the localised pattern parameter regime as r varies. Several distinct codimension-two bifurcations are dis-covered including cusp and quadruple zero points for homogeneous steady states, a degenerate heteroclinic connection and a change in connectedness of the homoclinic snaking structure. The analysis is repeated for the Gierer-Meinhardt system, which lies outside the canonical framework. Similar transitions are found under homotopy between bifurcation diagrams for the case where there is a constant feed in the active field, to it being in the inactive field. Wider implications of the results are discussed for other kinds of pattern-formation systems as well as to distinguishing between different kinds of observed behaviour in the natural world.

Alexei Cheviakov, University of Saskatchewan

Narrow Escape and Narrow Capture Problems in Three Dimensions: Solutions and the Optimization of Trap Configurations

Narrow escape (NE) problems are concerned with the calculation of the mean first passage time (MFPT) for a Brownian particle to escape a domain whose boundary contains N small windows (traps). NE problems arise in escape kinetics modeling in chemistry and cell biology, including receptor trafficking in synaptic membranes and RNA transport through nuclear pores. The related Narrow capture (NC) problems are characterized by the presence of small traps within the domain volume; such traps may be fully absorbing, or have absorbing and reflecting boundary parts. The MFPT of Brownian particles traveling in domains with traps is commonly modeled using a linear Poisson problem with Dirichlet-Neumann boundary conditions. We provide an overview of recent analytical and numerical work pertaining to the understanding and solution of different variants of NE and NC problems in three dimensions. The discussion includes asymptotic MFPT expressions in in the limit of small trap sizes, the cases of spherical and non-spherical domains, same and different trap sizes, the dilute trap fraction limit and MFPT scaling laws for N 1 traps, and the global optimization of trap positions to seek globally and locally optimal MFPT-minimizing trap arrangements. We also present recent comparisons of asymptotic and numerical solutions of NE problems to results of full numerical Brownian motion simulations, in the usual case of constant diffusivity, as well as considering more realistic anisotropic diffusion near the domain boundary.

Rustum Choksi, McGill University

The Phase-Field-Crystal Model at Large and Small Scales

The Phase-Field-Crystal (PFC) model is a simple yet surprisingly useful model for successfully capturing the phenomenology of grain growth in polycrystalline materials. PFC models are variational with a free energy functional which is very similar (in some cases, identical) to the well-known Swift-Hohenberg free energy. In this talk, we will discuss the simplest PFC functional and its gradient flow.

The first part of the talk will focus on large scales and address the model's uncanny ability to o capture certain features of grain growth. We introduce a novel atom-based grain extraction and measurement method, and then use it to provide a comparison of multiple statistical grain metrics between (i) PFC simulations, (ii) experimental thin film data for aluminum, and (iii) simulations from the Mullins model.

In the second part of the talk, we investigate the PFC energy landscape at small scales (the local arrangement of atoms). We address patterns which are numerically observed as steady states via the framework of the modern theory of rigorous computations. In doing so, we make rigorous conclusions on the existence of similar states. In particular, we show that localized patterns and grain boundaries are critical and not simply metastable states. Finally, we present preliminary work on connections and parameter continuation in the PFC system. This talk consists of work from the PhD thesis of Gabriel Martine La Boissoniere at McGill. Parts of the talk also involve joint work with S. Esedoglu (Michigan), K. Barmak (Columbia) and J.-P. Lessard (McGill).

Arjen Doelman, Leiden University, Netherlands

Localized slow patterns in singularly perturbed 2-component reaction-diffusion equations

Localized patterns in singularly perturbed reaction-diffusion equations typically consist of slow parts – in which the associated solution follows an orbit on a slow manifold in a reduced spatial dynamical system – alternated by fast excursions – in which the solution jumps from one slow manifold to another, or back to the original slow manifold. In this talk we consider the existence and stability of localized slow patterns that do not exhibit such jumps, i.e. that are completely embedded in a slow manifold of the singularly perturbed spatial dynamical system. These patterns have rarely been considered in the literature, for two reasons: (i) in the classical Gray-Scott/Gierer-Meinhardt type models that dominate the literature, the flow on the slow manifold is linear and thus cannot exhibit homoclinic pulse or heteroclinic front solutions; (ii) the slow manifolds occurring in the literature are typically trivial, or 'vertical' – i.e. given by $u \equiv u_0$, where u is the fast variable – so that the stability problem is determined by a simple (decoupled) scalar equation. The present talk is motivated by several explicit ecosystem models (of singularly perturbed reaction-diffusion type) that do give rise to nontrivial normally hyperbolic slow manifolds on which the flow may exhibit both homoclinic and heteroclinic orbits – that correspond to either stationary or traveling localized patterns. The associated spectral stability problems are at leading order given by a nonlinear, but scalar, eigenvalue problem with Sturm-Liouville type characteristics and we establish that homoclinic pulse patterns are typically unstable, while heteroclinic fronts can either be stable or unstable. However, we also show that homoclinic pulse patterns that are asymptotically close to a heteroclinic cycle may be stable. This result is obtained by explicitly determining the leading order approximations of 4 critical asymptotically small eigenvalues. Through this somewhat subtle analysis – that involves several orders of magnitude in the small parameter – we also obtain full control over the nature of the bifurcations – saddle-node, Hopf, global, etc. – that determine the existence and stability of the heteroclinic fronts and/or homoclinic pulses. Finally, we show that heteroclinic orbits may correspond to stable (slow) interfaces (in 2-dimensional space), while the homoclinic pulses must be unstable as localized stripes –even when they are stable in 1 space dimension.

Chunyi Gai, Dalhousie University

Resource-mediated competition between two plant species with different rates of water intake

We propose an extension of the well-known Klausmeier model of vegetation to two plant species that consume water at different rates. Rather than competing directly, the plants compete through their intake of water, which is a shared resource between them. In semi-arid regions, the Klausmeier model produces vegetation spot patterns. We are interested in how the competition for water affects co-existence and stability of patches of different plant species. We consider two plant types: a thirsty species and a frugal species, that only differ by the amount of water they consume, while being identical in all other aspects. We find that there is a finite range of precipitation rate for which two species can co-exist. Outside of that range, (when the rate is either sufficiently low or high), the frugal species outcompetes the thirsty species. As the precipitation rate is decreased, there is sequence of stability thresholds such that thirsty plant patches are the first to die off, while the frugal spots remain resilient for longer. The pattern consisting of only frugal spots is the most resilient. The next-most-resilient pattern consists of all-thirsty patches, with the mixed pattern being less resilient than either of the homogeneous patterns. We also examine numerically what happens for very large precipitation rate. We find that for sufficiently high rate, the frugal plant takes over the entire range, outcompeting the thirsty plant.

Jason Gilbert, University of Saskatchewan

Global Optimization of the Mean First Passage Time for Narrow Capture Problems in Elliptic Domains

Narrow escape and narrow capture problems which describe the average times required to stop the motion of a randomly travelling particle within a domain have applications in various areas of science. While for general domains, it is known how the escape time decreases with the increase of the trap sizes, for some specific 2D and 3D domains, higher-order asymptotic formulas have been established, providing the dependence of the escape time on the sizes and locations of the traps. Such results allow the use of global optimization to seek trap arrangements that minimize average escape times. In a recent paper, the escape time expansion for a 2D elliptic domain was derived, providing the dependence of the average MFPT on sizes and locations of small internal traps. The goal of this work is to systematically seek global minima of MFPT for $1 \leq N \leq 50$ traps, and compare the corresponding putative optimal trap arrangements for different values of the domain eccentricity.

Daniel Gomez, University of British Columbia

Boundary Layer Solutions in the Gierer-Meinhardt System

Abstract: The singularly perturbed Gierer-Meinhardt system has been a prototypical reaction diffusion system for the analysis of localized multi spike solutions. Motivated by recent interest in bulk-surface coupled systems, in this talk we address the structure and linear stability of multi spike solutions in the presence of inhomogeneous boundary conditions. Such inhomogeneities are shown to lead to the formation of both stable symmetric and asymmetric boundary bound spike solutions in one-dimensional domains and analogous solutions in higher dimensions.

Jia Gou, UC Riverside

Growth Control in the Drosophila Wing Imaginal Disc.

How organ size is controlled during development has been a subject of scientific study for centuries, but the growth control mechanisms are still poorly understood. The Drosophila wing imaginal disc has widely been used as a model system to study the regulation of growth. Growth control in the Drosophila wing disc involves various local signals, including signaling pathways, mechanical signals, etc. We developed a model of the Hippo pathway, which is the core regulatory pathway that mediates cell proliferation and apoptosis in Drosophila and mammalian cells, and contains a core kinase mechanism that affects control of the cell cycle and growth. We investigated the regulatory role of two upstream components Fat and Ds on the downstream mediator Yki of the pathway, and provide explanations to some of the seemingly contradictory experimental results. We found that a number of non-intuitive experimental results can be explained by subtle changes in the balances between inputs to the Hippo pathway. Since signal transduction and growth control pathways are highly con-served across species and directly involved in tumor growth, much of what is learned about Drosophila will have relevance to growth control in mammalian systems. Our recent work on morphogen transport in the wing disc will also be discussed.

Thomas Hillen, University of Alberta

Symmetries and Bifurcations in Non-Local Cell Adhesion Models

Cellular adhesion is one of the most important interaction forces between cells and other tissue components. In 2006, Armstrong, Painter and Sherratt introduced a non-local PDE model for cellular adhesion, which was able to describe known experimental results on cell sorting and pattern formation. The pattern formation arises through non-local attractive interactions of the cells. In this talk I will analyse the underlying symmetries and bifurcations that lead to the observed patterns. (joint work with A. Buttenschoen).

David Iron, Dalhousie University

Noise and Spike Dynamics for the Gierer-Meinhardt Equations

The stability and dynamic properties of spike-type solutions to the Gierer- Meinhart equations are well understood. We examine the effect of adding noise to the equations on the spike-dynamics. We derive a stochastic ordinary differential equation for the motion of a single spike as well as the distribution of spike location from the associated Fokker-Plank equation. With sufficiently large amplitude noise, it is possible for the spike to reach the boundary of the domain and become effectively trapped for some time. In this case, we calculate the expected time to capture.

Leah Edelstein-Keshet and Andreas Buttenschoen, UBC

Patterns, waves, and bifurcations in cell migration

Cell migration plays a central roles in embryonic development, wound healing and immune surveillance. In 2008, Yoichiro Mori, Alexandra Jilkine and LEK published a reaction-diffusion model for the initial step of cell migration, the front-back chemical polarization that sets a cell's directionality. (More detailed mathematical properties of this model were described by the same group in 2011.) Since then, progress has been made in investigating how that simple "wave-pinning" mechanism is shaped and tuned by feedback from other proteins, from the cell's environment (extracellular matrix), from interplay with larger signaling networks, and from cell-cell interactions. In this talk, we will describe some of this progress and mathematical questions that arise. In particular, AB will demonstrate how his numerical PDE bifurcation analysis has helped us to understand how cells repolarize to reverse their direction of motion.

Theodore Kolokolnikov, Dalhousie University

A ring of spikes for the Schnakenberg model

Consider N spikes on located along a ring inside a unit disk. This highly symmetric configuration corresponds to an equilibrium of a two-dimensional Schnakenberg model; the ring radius can be characterized in terms of the modified Green's function. We study the stability of such a ring with respect to both small and large eigenvalues (corresponding to spike position and spike height perturbations, respectively), and characterize the instability thresholds. For sufficiently large feed rate, we find that a ring of 8 or less spikes is stable with respect to both small and large eigenvalues, whereas a ring of 9 spikes is unstable with respect to small eigenvalues. For 8 spikes or less, as the feed rate is decreased, a small eigen-value instability is triggered first, followed by large eigenvalue instability. For 8 spikes, this instability appears to be supercritical, and deforms a ring into a square-type configuration. The main tool we use is circulant matrices and an analogue of the floquet theory.

Edgar Knobloch, UC Berkeley

The conserved Swift-Hohenberg equation and crystallization

The phase-field model, also known as the conserved Swift-Hohenberg equation, provides a useful model of crystallization that is derivable from the more accurate dynamical density functional theory. I will survey the properties of this model focusing on spatially localized structures and their organization in parameter space. I will highlight the role played by conserved mass and discuss the role played by these structures in the thermodynamic limit in both one and two spatial dimensions. I will then discuss dynamic crystallization via a propagating crystallization front. Two types of fronts can be distinguished: pulled and pushed fronts, with different properties. I will demonstrate, via direct numerical simulation, that the crystalline structures deposited by a rapidly moving front are not in thermodynamic equilibrium and so become disordered as they age. I will conclude with a discussion of a two-wavelength generalization of the model that exhibits quasicrystalline order in both two and three dimensions and of the associated spatially localized structures with different quasicrystalline motifs. The possible role of metastable spatially localized structures in nucleating crystallization will be highlighted.

Andrew Krause, Oxford University

Recent Progress and Open Frontiers in Turing-Type Morphogenesis

Motivated by recent work with biologists, I will showcase some mathematical results on Turing instabilities in complex domains. This is scientifically related to understanding developmental tuning in a variety of settings such as mouse whiskers, human fingerprints, bat teeth, and more generally pattern formation on multiple scales and evolving domains. Such phenomena are typically modelled using reaction-diffusion systems of morphogens, and one is often interested in emergent spatial and spatiotemporal patterns resulting from instabilities of a homogeneous equilibrium, which have been well-studied. In comparison to the well-known effects of how advection or manifold structure impacts unstable modes in such systems, I will present results on instabilities in heterogeneous systems, as well as those arising in the set-ting of evolving manifolds. These contexts require novel formulations of classical dispersion relations, and may have applications beyond developmental biology, such as in population dynamics (e.g. understanding colony or niche formation of populations in heterogeneous environments). These approaches also help close the vast gap between the simple theory of diffusion-driven pattern formation, and the messy reality of biological development, though there is still much work to be done in validating even complex theories against the rich dynamics observed in nature. I will close by discussing a range of open questions, many of which fall well beyond the extensions I will discuss.

Alan Lindsay, Univ. Notre Dame

Mathematics of diffusive signaling and the role of receptor clustering in chemoreception.

Cells receive chemical signals at localized surface receptors, process the data and make decisions on where to move or what to do. Receptors occupy only a small fraction of the cell surface area, yet they exhibit exquisite sensory capacity. In this talk I will give an overview of the mathematics of this phenomenon and discuss recent results focusing on receptor organization. In many cell types, receptors have very particular spatial organization or clustering - the biophysical role of which is not fully understood. In this talk I will explore how the number and configuration of receptors allows cells to deduce directional information on the source of diffusing particles. This involves a wide array of mathematical techniques from asymptotic analysis, homogenization theory, computational PDEs and Bayesian statistical methodologies. Our results show that receptor organization plays a large role in how cells decode their environmental situation and infer the location of distant sources.

Sean Lawley, University of Utah

Extreme first passage times

The first passage time (FPT) of a diffusive searcher to a target determines the timescale of many physical, chemical, and biological processes. While most studies focus on the FPT of a given single searcher, another important quantity in some scenarios is the FPT of the first searcher to find a target from a large group of searchers. This fastest FPT is called an extreme FPT and can be orders of magnitude faster than the FPT of a given single searcher. In this talk, we will explain recent results in extreme FPT theory and give special attention to the case of extreme FPTs to small targets. Time permitting, we will also explain results on extreme FPTs of subdiffusion modeled by a fractional time derivative and superdiffusion modeled by a fractional Laplacian."

Phillip Maini, University of Oxford

Modelling collective cell movement in biology and medicine

Collective cell movement occurs throughout biology and medicine and there are many common features shared across different areas. I will review work we have carried out over the past few years on (i) systematically deriving a PDE model for tumour angiogenesis from a discrete formulation and comparing this model with the classical, phenomenological snail-trail model; (ii) agent-based models for cranial neural crest cell migration in a collabo-ration with experimental biologists that has revealed a number of new biological insights.

Iain Moyles, York University

Regularization of the Ostwald supersaturation model for Liesegang bands

Many chemical systems exhibit a regular pattern of precipitate bands known as Liesegang rings. We review results from a prominent model of Liesegang rings which was ill-posed and prevented the termination and formation of multi-band patterns due to discontinuity in precipitate growth. Introducing a regularization to this process, we demonstrate numerically and asymptotically how a multi-band pattern can emerge and analyze its structure.

Yana Nec, Thompson River University

Spike patterns as a window into non-injective transient diffusive processes

Complex natural systems at times manifest transitions between disparate diffusive regimes. Efforts to devise measurement techniques capable of identifying the cross-over moments have recently borne fruit, however interpretation of findings remains contentious when the bigger picture is considered. This study generalises the 1D Gierer-Meinhardt reaction – diffusion model to a system that permits transitions between regular diffusive regimes with distinct diffusivities as well as sub-diffusion of a variable order. This is a sufficiently general, yet tractable description for the dynamics of a pattern qualitatively redolent of molecular clusters subject to transient anomalous diffusion mechanisms. The resulting system of equations substantiates the difficulties encountered when attempting to distinguish between various diffusive regimes in experimental settings: a non-monotonic dependence of the pat-tern's evolution on parameters defining the diffusion mechanism is a common occurrence, as is a non-injective mapping between a given sequence of diffusion regimes and ensuing drift behaviour.

Jay Newby, University of Alberta

Stochastic models of rare events in cell biology

I will discuss asymptotic and numerical tools to study metastable dynamics in stochastic processes that exhibit dynamics over multiple time scales and those that have both discrete and continuous states. I will illustrate my analysis with the application to epigenetic switching in gene circuits. I will also discuss spontaneous action potential initiation by stochastic ion channels. Finally, I will briefly motivate future directions for spatial rare events in a model of cell-cell contact formation.

Yasumasa Nishimura

Traveling pulses with oscillatory tails, figure-eight stack of isolas, and dynamics in heterogeneous media

The interplay between 1D traveling pulses with oscillatory tails (TPO) and heterogeneities of bump type is studied for a generalized three-component FitzHugh-Nagumo equation. We first present that stationary pulses with oscillatory tails (SPO) forms a "snaky" structure in homogeneous space, then TPO branches take a form of "figure-eight-like stack of isolas" located close to the snaky structure of SPO. Here we adopt voltage-difference as a bifurcation parameter. A drift bifurcation from SPO to TPO can be found by introducing another parameter at which these two solution sheets merge. As for the heterogeneous problem, in contrast to monotone tail case, there appears a nonlocal interaction between the TPO and the heterogeneity that creates infinitely many saddle solutions. The response of TPO shows a variety of dynamics including pinning and depinning processes in addition to penetration and rebound. Stable/unstable manifolds of these saddles interact with TPO in a complex way, which causes a subtle dependence on the initial condition and a difficulty to predict the behavior after collision even in one-dimensional space. Nevertheless, for 1D case, a systematic global exploration of solution branches (HIOP) induced by heterogeneities, and the reduction method to finite-dimensional ODEs allow us to clarify such a subtle dependence of initial condition and detailed mechanism of the transitions from penetration to pinning and pinning to rebound from dynamical system view point. It turns out that the basin boundary between two different outputs against the heterogeneities forms an infinitely many successive reconnections of heteroclinic orbits among those saddles as the height of the bump is changed, which causes the subtle dependence of initial condition. This is a joint work with Takeshi Watanabe.

Xiaofeng Ren, George Washington University

Non-hexagonal lattices from a two species interacting system

A two species interacting system motivated by the density functional theory for triblock copolymers contains long range interaction that affects the two species differently. In a two species periodic assembly of discs, the two species appear alternately on a lattice. A minimal two species periodic assembly is one with the least energy per lattice cell area. There is a parameter b in [0,1] and the type of the lattice associated with a minimal assembly varies depending on b. There are several thresholds defined by a number B=0.1867... If b is in [0, B), a minimal assembly is associated with a rectangular lattice; if b is in [B, 1-B], a minimal assembly is associated with a square lattice; if b is in (1-B, 1], a minimal assembly is associated with a rhombic lattice. Only when b=1, this rhombic lattice is a hexagonal lattice. None of the other values of b yields a hexagonal lattice, a sharp contrast to the situation for one species interacting systems, where hexagonal lattices are ubiquitously observed.

Matthias Winter, Brunel University

Stable spikes for a reaction-diffusion system with two activators and one inhibitor

We consider a reaction-diffusion system with two activators and one inhibitor. We prove rigorous results on the existence and stability of spiky patterns. We show that for certain conditions on the parameters these solutions can be stable. The approach is based on analytical methods such as elliptic estimates, Liapunov-Schmidt reduction and nonlocal eigenvalue problems. This is joint work with Weiwei Ao and Juncheng Wei.

Justin Tzou, Macquarie University

A brief overview of methods and results for localized patterns and narrow escape problems

Abstract: We will give a brief overview of results in localized pattern formation and narrow escape problems that have been achieved through hybrid asymptotic-numerical methods. We will then briefly discuss how we have used these methods to extend results to surfaces with variable curvature.