A workshop on *Computational and Analytical Mathematics* in honour of Jonathan Borwein's 60th Birthday May 16–20, 2011

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June 9, 2011

1 Summary

Variational Analysis, Optimization Algorithms, High Performance Computing, Number Theory, and Experimental Mathematics are extremely active areas of mathematics that were explored at the workshop on *Computational and Analytical Mathematics*. The objectives of this workshop were to chart promising research directions in these areas, to survey the state-of-the-art of theory and practice, to identify emerging problems driven by applications, and to discuss new approaches for solving these problems. The workshop itself was dedicated to Jonathan Borwein in honor of his 60th birthday. Dr. Borwein is one

of the most productive Canadian mathematicians of his generation. He has had a profound impact on these areas of research with over a dozen books and 300 papers published, and with more than 2500 citations by more than 1250 authors.

The workshop attracted 93 participants from 14 countries. In addition, all the talks were streamed live over the internet, and will shortly be made available for download. At least three Australian universities had a live audience for the afternoon plenary presentations. Furthermore, there was also true remote participation as, three presentations were given from Australia!

The workshop provided a distinctive opportunity for the participants to meet as a group. In fact, many of the participants had not met before, and it is very unlikely that even a large cross-section of the attendees will meet again as a group at an ordinary conference. We expect this workshop to be the basis for new innovative research and collaborations due to its unique mix of experts with a broad range of applications.

2 Scientific Contributions to the Workshop

In this section, we summarize the scientific talks by the participants. All talks emphasized classical results, directions for future research, and promising applications. The topics are grouped into three somewhat distinct areas, but common themes that arose throughout the conference are Convex and Variational Analysis, Optimization Algorithms and High Performance Computing, and Number Theory and Experimental Mathematics.

2.1 Convex and Variational Analysis

Terry Rockafellar (University of Washington) showed how new methodology from variational analysis has fundamental impact in economics; in particular, the theory of economic equilibria [51]. Shawn Wang (University of British Columbia, Kelowna) gave explicit constructions for maximally monotone extensions of monotone linear relations [58]. Gerald Beer (California State University Los Angeles) characterized a class of metric spaces using an Ekeland-type principle [4]. The usefulness of variational methods in financial trading was brought to light by Qiji Zhu (Western Michigan University) [60]. The power of convex duality theory in the context of maximum likelihood estimation was the topic of the talk by James Burke (University of Washington) [12]. Rafal Goebel (Layola University) illustrated how the notion of (robustly) quasiconvex functions is useful in the study of viscosity solutions of second-order partial differential equations [28]. Heinz Bauschke (University of British Columbia, Kelowna) surveyed new results on fixed points and range properties of averages of firmly nonexpansive mappings [3]. Alexander Knecht[1] (La Sierra University) reported on the resolution of an open problem on convexity-preserving gradient mappings of Legendre functions [33]. Characterization of badly behaved conic linear systems were presented by Gabor Pataki (University of North Carolina at Chapel Hill) [49]. Lionel Thibault (Montpellier, France) reviewed several crucial properties of prox-regular sets [57]. New qualification conditions useful in the computation of normal cones were

[1] Undergraduate student

provided by Boris Mordukhovich (Wayne State) [45]. Marian Fabian (Czech Academy of Science) presented new sufficient conditions for attainment and subdifferentiability of a function with the square of the norm [25]. Alexander Ioffe (Israel Institute of Technology) reported on new results concerning regularity of set-valued mappings [32]. The characterization of Mazur's intersection property was the topic of the talk by Lixin Cheng (Xiamen University, China) [15]. Michel Théra (Limoges, France) presented new results on minimizing irregular convex functions [56]. Regina Burachik (University of South Australia) presented a new sufficient condition for the equality of two maximally monotone operators [11]. Marco Lopez (Alicante, Spain) studied the stability of feasible sets for a primal-dual pair of optimization problems [37]. Coercivity properties and their applications to convex risk measures were presented by Jose Orihuela (Murcia, Spain) [48]. Stephen Simons (UC Santa Barbara) showed how convex analysis can be combined with SSDB space theory to obtain and generalize various results on maximally monotone multifunctions [53]. Adrian Lewis (Cornell University) discussed a generic property of semi-algebraic sets and functions and the relationship with a proximal algorithm [35]. New results on bilevel optimal control problems were reported by Henry Bonnel (Université de la Nouvelle-Caledonie) [5]. John Giles (Newcastle, Australia) presented a continuity characterization of Asplund spaces related to a problem posed by Godefroy, Montesino and Zizler [27]. Recent results on subdifferential estimates were presented by Marc Lassonde (Université des Antilles et de la Guyane, France) [34]. Warren Moors (Auckland, New Zealand) reported on recent work on the fragmentability of groups and metric-valued function spaces [44].

2.2 Optimization Algorithms and High Performance Computing

Russell Luke (Universität Goettingen) provided a new algorithmic framework for sparse optimization [39]. Asen Dontchev (University of Michigan) demonstrated extensions of quasi-Newton methods for variational inequalities [22]. Patrick Combettes (Paris 6, France) introduced new primal-dual splitting algorithms for inclusions involving monotone operators [17]. Yves Lucet (University of British Columbia, Kelowna) reported on recent progress on computing classical convex analysis operators [38]. Pierre Maréchal (Toulouse, France) described a new reconstruction approach in thermoacoustic tomography [41]. David Bailey (Lawrence Berkeley Lab) reported on ground-breaking Indian algorithm for the computation of square roots dating back to 400 BC [1]. Tony Guttman (Melbourne, Australia) [2] described recent work relating the two-dimensional Ising model to solutions of the Painlevé equations. Dominikus Noll (Toulouse, France) showed how smooth and nonsmooth optimization techniques are key in the study of feedback control design [47]. David Bailey (Lawrence Berkeley Lab) showed how to evaluate integrals arising in quantum field theory and random walks [2]. Mason Macklem (Kelowna) presented implementation details on a parallel derivative-free optimization package, designed for the use on multi-user computing networks [40]. Richard Crandall (Reed College) provided a fascinating glimpse on the fractal distribution of brain synapses [19]. O-Yeat Chan (Newcastle, Australia) described a single-node traffic flow process on a graph and its asymptotic properties [14]. Warren Hare (University of British Columbia, Kelowna) examined the potential to apply

[2] Talk given remotely

proximal point methods in derivative-free optimization [31]. Vris Cheung (University of Waterloo) [3] reported on joint work with Henry Wolkowicz on facial reduction techniques to solve huge sensor network localization problems.

2.3 Number Theory and Experimental Mathematics

Bruno Salvy (INRIA, France) surveyed recent progress and challenges on the basic principles for automatic proofs of identities [52]. Frank Garvan (University of Florida) revealed some unexpected arithmetic properties of a class of partition functions [26]. Eva Curry (Acadia University) showcased new results on periodic orbits of the Euclidean algorithm and pseudodigits. Wadim Zudilin (Newcastle, Australia) [4] presented recent work on Boyd's conjectures on Mahler measures [61]. Richard Brent (Australian National University) [5] provided simple and convenient algorithms for computing Bernoulli and Tangent numbers [10]. Jonathan Borwein (Newcastle, Australia) surveyed his current projects in Australia [7]. A certain inverse central binomial series was evaluated and its asymptotic behaviour analyzed by Lawrence Glasser (Clarkson University) [24]. Kevin Hare (University of Waterloo) presented new results on the structure of the set of Garcia numbers [30]. The Gauss factorial was introduced and an extension of the Gauss binomial coefficient theorem was presented by Karl Dilcher (Dalhousie) [21]. Peter Borwein (Simon Fraser) reflected on the academic and life path of his brother [8]. Tilings of the Euclidean space by convex bodies were discussed by Sinai Robins (Nanyang Technological University) [50]. Marc Chamberland (Grinnell College) showed various new factorizations made possible by computer algebra systems [13]. Rob Corless (University of Western Ontario) reported on recent work with German Kalugin on integral representations of special functions [18]. Roland Girgensohn (Munich, Germany) showed the audience how to compute Bernoulli convolutions using Excel [23]. Generating functions and identities for generalized log-sine integrals were discussed by Armin Straub (Tulane University) [54]. Minimal polynomials of Kloosterman sums on finite fields, which have applications in cryptography and coding theory, were the topic of the talk by Petr Lisonek (Simon Fraser) [36]. Michael Monagan (Simon Fraser) considered the problem of interpolating a sparse multivariate polynomial over a ring [43]. Keith Taylor (Dalhousie) explored the analytical theory connected with crystals and crystallographic symmetries [55]. Karyn Mclellan (Dalhousie) reported on recent extensions on work on random Fibonacci sequences [42]. Rob Noble (Dalhousie) described on how to obtain divisibility properties for the asymptotic coefficients of Delannoy numbers lying in a number field [46]. Erick Wong (University of British Columbia, Vancouver) [6] determined probabilities of real and integer matrices to have real eigenvalues [59]. David Borwein (University of Western Ontario) reviewed work on cmputing integrals involving the sinc function and volumes of symmetric convex polyhedra [6]. A q-analog of Euler's evaluation of the double zeta function was provided by David Bradley (University of Maine) [9].

[3] Graduate student
[4] Talk given remotely
[5] Talk given remotely
[6] Graduate student

3 Further Outcomes of the Meeting

H. Bauschke, M. Théra and H. Wolkowicz are currently co-editing a special issue of the journal *Mathematical Programming*. This issue is projected to appear in print in 2012.

Furthermore, August 1, 2011 is the deadline for the volume *Computational and Analytical Mathematics*, to be part of the Springer series *Springer Proceedings in Mathematics* [7] and directly related to the mathematical research presented at this conference. This volume will be co-edited by D. Bailey, H. Bauschke, P. Borwein, F. Garvan, M. Théra, J. Vanderwerff, and H. Wolkowicz.

Acknowledgment

The organizers thank the following sponsors for their support in making this workshop possible: IRMACS, SFU, MITACS, AMSI, PIMS, CARMA, Fields.

References

[1] D.H. Bailey: "A quartically convergent square root algorithm", research talk presented at this workshop.

[2] D.H. Bailey: "Hand-to-hand combat with thousand-digit integrals", plenary research talk presented at this workshop.

[3] H.H. Bauschke: "On averages of firmly nonexpansive mappings", research talk presented at this workshop.

[4] G. Bauschke: "Notes on the Ekeland variational principle", research talk presented at this workshop.

[5] H. Bonnel: "Semivectorial bilevel convex optimal control problems", research talk presented at this workshop.

[6] D. Borwein: "Sinc integrals", research talk presented at this workshop.

[7] J. Borwein: "What I am doing in Australia", research talk presented at this workshop.

[8] P. Borwein: "The once and future Jon", research talk presented at this workshop.

[9] D. Bradley: "A q-analog of Euler's reduction formula for the double zeta function", research talk presented at this workshop.

[10] R. Brent: "Computing Bernoulli and tangent numbers", research talk presented at this workshop.

[11] R.S. Burachik: "On a sufficient condition for equality of two maximal monotone operators", research talk presented at this workshop.

[7] See `http://www.springer.com/series/8806` for details.

[12] J. Burke: “Maximum likelihood estimation of mixture densities”, research talk presented at this workshop.

[13] M. Chamberland: “A feast of factoring”, research talk presented at this workshop.

[14] O-Y. Chan: “Chipping away”, research talk presented at this workshop.

[15] L. Cheng: “Analytic characterizations of Mazur’s intersection property”, research talk presented at this workshop.

[16] V. Cheung: “Strong duality in semidefinite programming and facial reduction with applications to sensor network localization and molecular conformation”, research talk presented at this workshop.

[17] P.L. Combettes: “Monotone+Skew decomposition of inclusion problems: algorithms and applications”, plenary research talk presented at this workshop.

[18] R. Corless: “Stieltjes, Poisson and similar representations of functions”, research talk presented at this workshop.

[19] R. Crandall: “On the fractal distribution of brain synapses”, plenary research talk presented at this workshop.

[20] E. Curry: “Pseudodigits”, research talk presented at this workshop.

[21] K. Dilcher: “Gauss factorials: properties and applications”, research talk presented at this workshop.

[22] A.L. Dontchev: “Inexact Newton methods for variational inequalities”, research talk presented at this workshop.

[23] R. Girgensohn: “Computing Bernoulli convolutions with Excel”, research talk presented at this workshop.

[24] L. Glasser: “An inverse central binomial series”, research talk presented at this workshop.

[25] M. Fabian: “Attainment and (sub)differentiability of the infimal/supremal convolution of a function and of the square of the norm”, research talk presented at this workshop.

[26] F. Garvan: “The Andrews spt-function and higher-order generalizations”, research talk presented at this workshop.

[27] J.R. Giles: “A continuity characterization of Asplund spaces”, research talk presented at this workshop.

[28] R. Goebel: “Viscosity characterization and convex analysis of quasiconvex and robustly quasiconvex functions”, research talk presented at this workshop.

[29] T. Guttman: “The susceptibility of the two-dimensional Ising model”, plenary research talk presented at this workshop.

[30] K. Hare: “Garcia numbers”, research talk presented at this workshop.

[31] W. Hare: “Derivative-free optimization via proximal point methods”, research talk presented at this workshop.

[32] A. Ioffe: “On some problems of the regularity theory of variational analysis”, research talk presented at this workshop.

[33] A. Knecht: “Legendre functions whose gradients preserve convexity”, research talk presented at this workshop.

[34] M. Lassonde: “Subdifferential estimate of the directional derivative and optimality criterion for lower semicontinuous functions”, research talk presented at this workshop.

[35] A. Lewis: “Nonsmooth optimization and semi-algebraic sets”, plenary research talk presented at this workshop.

[36] P. Lisonek: “Minimal polynomials of Kloosterman zeros”, research talk presented at this workshop.

[37] M.A. Lopez: “Lipschitzian properties of the infinite-dimensional dual pair”, research talk presented at this workshop.

[38] Y. Lucet: “Computing the conjugate of convex piecewise linear-quadratic bivariate functions”, research talk presented at this workshop.

[39] D.R. Luke: “Variational analysis of sparsity optimization”, research talk presented at this workshop.

[40] M. Macklem: “Load-balancing for parallel derivate-free optimization methods on shared computing networks”, research talk presented at this workshop.

[41] P. Maréchal: “Aspects of thermoacoustic tomography”, research talk presented at this workshop.

[42] K. Mclellan: “Two approaches to the growth of random Fibonacci sequences”, research talk presented at this workshop.

[43] M. Monagan: “Sparse polynomial interpolation”, research talk presented at this workshop.

[44] W. Moors: “Fragmentability of groups and metric-valued function spaces”, research talk presented at this workshop.

[45] B. Mordukhovich: “Constraint qualifications and optimality conditions for nonconvex semi-infinite and infinite programs”, research talk presented at this workshop.

[46] R. Noble: “Asymptotics of the weighted Delannoy numbers”, research talk presented at this workshop.

[47] D. Noll: “Optimization for feedback control”, plenary research talk presented at this workshop.

[48] J. Orihuela: “Compactness, optimality and risk”, research talk presented at this workshop.

[49] G. Pataki: “Bad semidefinite programs: they all look the same”, research talk presented at this workshop.

[50] S. Robins: “Translational tilings of Euclidean space by a convex body, with multiplicity”, research talk presented at this workshop.

[51] R.T. Rockafellar: “Variational analysis and economic equilibrium”, plenary research talk presented at this workshop.

[52] B. Salvy: “Automatic proofs of identities”, plenary research talk presented at this workshop.

[53] S. Simons: “SSD spaces and maximal monotonicity,” research talk presented at this workshop.

[54] A. Straub: “Applications and evaluations of log-sine integrals”, research talk presented at this workshop.

[55] K.F. Taylor: “Analysis with crystals”, research talk presented at this workshop.

[56] M. Théra: “Minimizing irregular convex functions”, research talk presented at this workshop.

[57] L. Thibault: “Prox-regular sets”, research talk presented at this workshop.

[58] X. Wang: “Maximally monotone linear subspace extensions of monotone subspaces: explicit constructions and characterizations”, research talk presented at this workshop.

[59] E. Wong: “Eigenvalues of random integer matrices”, research talk presented at this workshop.

[60] Q. Zhu: “Variational methods in trading”, research talk presented at this workshop.

[61] W. Zudilin: “Mahler measures and modular functions”, research talk presented at this workshop.