Long term discounting and heterogeneous beliefs

Elyès Jouini
Université Paris-Dauphine

PIMS, 2008

Introduction

- Consensus consumer and intertemp. asset pricing with heterog. beliefs, J. & Napp, Restud, 2007

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- Is there a pessimistic bias in individual beliefs? Evidence from a simple survey, BenMansour, J. & Napp, Th & Decision, 2006

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- Are risk averse agents more optimistic? A Bayesian estimation approach, Ben Mansour, J., Marin, Napp & Robert, J. of Fin. Econometrics, 2008

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- A metaphore for the brain?

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- Historically (on average on the last century)
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  - Risky assets had a 4% (in France and 7% in the US) rate of return

The use of the NPV in risky environments

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- ρ is the pure time preference rate

Project evaluation

- Project with cash-flows $(X_t)_t = (X_0, \cdots, X_t, \cdots)$

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- Known conditional (to today information) distribution
- Known joint-distribution with $(c_t)_t$
- Each agent has a proportion ε of the project
- $V_1 = \sum_{t=0}^{\infty} \exp(-\rho t) E\left[u(c_t + \varepsilon X_t)\right]$

Project evaluation

- $$V_1 > V_0 \Longleftrightarrow \frac{\partial}{\partial \varepsilon} \sum_{t=0}^{\infty} \exp(-\rho t) E\left[u(c_t + \varepsilon X_t)\right] > 0$$
  $$V_1 > V_0 \Longleftrightarrow \sum_{t=0}^{\infty} \exp(-\rho t) E\left[X_t u'(c_t)\right] > 0$$

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- The project is desirable iff

$$NPV = \sum_{t=0}^{\infty} \exp(-r(t)t) B_t > 0$$
$$\exp(-r(t)t) = \exp(-\rho t) \frac{E\left[u'(c_t)\right]}{E\left[u'(c_0)\right]}$$

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\end{aligned}
$$
- Specific case: constant, deterministic growth g and

Project evaluation

Theorem

In presence of risk and uncertainty about (c_t) *and* (X_t)*, the project is desirable iff its NPV is positive. NPV is computed through 2 steps*

1. *Each payoff is replaced by its certainty equivalent*
2. *The payoffs are actualized at a discount rate that is independent of the project*
3. *The NPV is then equal to the sum of the present values of the certainty equivalent values*

Choice of the discount rate

- $$r(t) = \rho - \frac{1}{t} \ln \frac{E\left[u'(c_t)\right]}{E\left[u'(c_0)\right]}$$

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- If $u'(c) = c^{-\gamma}$ (γ is the RRA parameter) and if $\ln \frac{c_t}{c_0} \sim \mathcal{N}(\mu t, \sigma^2 t)$, then

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- Three terms
  - Pure time preference rate
  - Wealth effect: a high growth rate increases future wealth and reduces the marginal value of 1€ in the future ($\nearrow$ with γ)
  - Precautionary effect: a high level of risk increases the value of 1€ in the future ($\Longrightarrow$ precautionary saving)

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	$\alpha = 10\%$	$\alpha = 30\%$
$\gamma = 0.5$	$\pi = 0.3\%$	$\pi = 2.3\%$
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$\gamma = 40$	$\pi = 8.4\%$	$\pi = 28.7\%$

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- Large debate (Stern Review)

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- The level of uncertainty is so small that it does not impact the discount rate

Decreasing discount rates

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- For large horizons, larger wealth ($r(t) \nearrow$) and larger risk ($r(t) \searrow$)
- In the standard model, log-linear risk and log-linear growth $\Longrightarrow$ no impact

The isosyncratic risk

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- In general

$$\begin{aligned} B_t &= \frac{E\left[X_t u'(c_t)\right]}{E\left[u'(c_t)\right]} = E\left[X_t\right] + \frac{E\left[X_t u'(c_t)\right] - E\left[X_t\right] E\left[u'(\right.}{E\left[u'(c_t)\right]} \\ &= E\left[X_t\right] + cov\left(X_t, \frac{u'(c_t)}{E\left[u'(c_t)\right]}\right) \end{aligned}$$

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- B_t is lower than $E\left[X_t\right]$ when X_t is positively correlated with c_t ($u' \searrow$): risk premium

The CAPM approach

- $u'(c_t) \simeq u'(E(c_t)) + (c_t - E(c_t))u''(E(c_t)), \qquad E[u'(c_t)] \simeq u'(E(c_t))$

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- The risk premium $\nearrow$ with γ and with the covariance between the project and the total wealth

Project-specific discount rate and CAPM formula

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- $$\begin{aligned} E\left[R_X\right] &= E\left[\frac{X}{V}\right] = (1+r)E\left[\frac{X}{B}\right] \\ &= (1+r)(1+\theta cov\left(R_X, \lambda c\right) \\ E\left[R_M\right] &= (1+r)\left(1+\theta cov\left(R_M, \lambda c\right)\right) \\ r_X &= r + (r_M - r)\frac{cov(R_X, R_M)}{var(R_M)} \end{aligned}$$

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-
$$V = \frac{1}{1+r_X}E\left[X\right]$$

Project-specific discount rate and the CCAPM formula

Elyès Jouini
Université Paris-Dauphine

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- $dV_t = r_V V_t dt + \sigma_V V_t dW_t^V$

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- $dM_t = r_M M_t dt + \sigma_M M_t dW_t^M$
- $dV_t = r_V V_t dt + \sigma_V V_t dW_t^V$
- $\langle dW_t, dW_t^M \rangle = dt$ and $\langle dW_t, dW_t^V \rangle = \rho dt$

$$\begin{aligned}
r_V &= \frac{1}{t} \ln E\left[\frac{V_t}{V_0}\right] = r_f + \frac{1}{t} \ln E\left[\frac{V_t}{B_t}\right] \\
&= r_f + \frac{1}{t} \ln E \frac{E\left[V_t\right] E\left[u'(c_t)\right]}{E\left[V_t u'(c_t)\right]} \\
&= r_f + \rho \sigma_c \sigma_V \\
r_M &= r_f + \sigma_c \sigma_M \\
r_V - r_f &= \beta \left(r_M - r_f\right), \qquad \beta = \frac{\sigma_M \cdot \sigma_V}{\left\|\sigma_M\right\|^2}
\end{aligned}$$

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- $$r^V - r_t^f = \sum \beta_i \left(r_t^i - r_t^f \right); \qquad \beta = \frac{\sigma_i \cdot \sigma_V}{\left\| \sigma_i \right\|^2}$$

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- APT: generalization to n factors
- $$r^V - r_t^f = \sum \beta_i \left(r_t^i - r_t^f \right); \qquad \beta = \frac{\sigma_i \cdot \sigma_V}{\|\sigma_i\|^2}$$
- r^V is then the excess return that is required by the market for a project whose risk characteristics are described by $(\beta_1, \cdots, \beta_n)$

To sum up

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  - Common to all projects

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- Project specific discount rate: factor models

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- Hyperbolic discounting, Loewenstein and Thaler, 1989, temporal inconsistencies
- Sustainable welfare function, Chichilnisky (1996) and Li and Löfgren (2000)

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- Divergence of opinion among agents or among experts (Jouini-Napp, 2007)

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- Example: $\mu_1 = 1\%$, $p_1 = 50\%$ and $\mu_2 = 3\%$, $p_2 = 50\%$ $\to r_0 = 3.92\%$ and $r_\infty = 1.92\%$.