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$\mathbf{R}^{2n}$ is a universal symplectic manifold for reduction.

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The authors show that if a manifold Q is of finite type, that is, $H^k(Q,\mathbf{Z})$ is finite-dimensional, then the noncanonical cotangent bundle $(T^*Q, d\theta_Q+\tau_Q^*\Omega)$ can be obtained by a Marsden-Weinstein reduction of $T^*\mathbf{R}^n=\mathbf{R}^{2n}$ relative to a torus action. Here Ω is a 2-form on Q, $\tau_Q\colon T^*Q\to Q$ is the bundle map and θ_Q is the canonical 1-form. Since any symplectic manifold (Q,Ω) is a reduction of $(T^*Q, d\theta_Q+\tau_Q^*\Omega)$ relative to the zero section, it follows that any symplectic manifold can be obtained by a reduction of $\mathbf{R}^{2n}$ with the standard symplectic structure. The authors point out that the problem of determining those symplectic manifolds that arise from Marsden-Weinstein reduction is a hard open question: The torus action on $\mathbf{R}^{2n}$ which when reduced gives $(T^*Q, d\theta_Q+\tau_Q^*\Omega)$ is obtained in two steps. First, since Q is of finite type, Ω can be expressed as a sum of integral 2-forms, each of which can be realized as the curvature of an S^1-bundle over Q. The fiber product P is a principal torus bundle over Q. P is then equivariantly embedded in $\mathbf{R}^n$, where the torus action is given by the orthogonal group. The lift of the torus action to $T^*\mathbf{R}^n$ commutes with the vertical action on $T^*\mathbf{R}^n$ determined by the lift of the normal bundle of P in $\mathbf{R}^n$ to $T^*\mathbf{R}^n$. The product action is an abelian Hamiltonian action on $T^*\mathbf{R}^n$, which by Kummer's construction reduces $T^*\mathbf{R}^n$ to $(T^*Q, d\theta_Q+\tau_Q^*\Omega)$.

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