MR1896340 (2003d:53144) 53D17 (17B63 37J05)
Gotay, Mark J. (1-HI); **Grabowski, Janusz** (PL-WASW-IM); **Kaneshige, Bryon** (1-HI)
Polynomial algebras on coadjoint orbits of semisimple Lie groups. (English summary)
J. Pure Appl. Algebra **170** (2002), *no. 1,* 29–34.

It is known that if $\mathfrak{g}$ is a real (finite-dimensional) semisimple Lie algebra then the dual space $\mathfrak{g}^*$ is a linear Poisson manifold under the Lie-Poisson bracket; the symplectic leaves are the orbits of the coadjoint representation of the group G on $\mathfrak{g}^*$.

The authors study the structure of the Poisson algebra $P(\mathcal{O})$ of polynomials on a coadjoint orbit $\mathcal{O}$. They first consider the symmetric algebra $S(\mathfrak{g})$, which can be identified with the algebra of polynomial functions on $\mathfrak{g}^*$, and prove the decomposition $S(\mathfrak{g}) = C(\mathfrak{g}) \oplus S(\mathfrak{g})'$ where $C(\mathfrak{g})$ denotes the Lie center of the Poisson algebra and $S(\mathfrak{g})' = \{S(\mathfrak{g}), S(\mathfrak{g})\}$ is the derived Lie ideal. Then they show that $P(\mathcal{O})$ admits a similar decomposition as a direct sum of its Lie center $Z(P(\mathcal{O})) = \mathbf{R}$ and its derived ideal $P(\mathcal{O})'$.

In the second part the authors prove that the Lie algebra $P(\mathcal{O})$ is essentially simple if and only if the orbit $\mathcal{O}$ is semisimple.

Reviewed by *Manuel F. Rañada*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.