

92c:58023 58E30 49N99 58A20 58F05 70G99

Gotay, Mark J. (1-USNA)

A multisymplectic framework for classical field theory and the calculus of variations. I. Covariant Hamiltonian formalism.

Mechanics, analysis and geometry: 200 years after Lagrange, 203–235, North-Holland Delta Ser., North-Holland, Amsterdam, 1991.

Let Y be a fibred manifold over an $(n+1)$ -dimensional base manifold X , and let Θ be the canonical $(n+1)$ -form on $\Lambda^n T^*(J^{k-1}Y)$. Also, let us consider the subbundle Z^{k-1} of the points $z \in \Lambda^{n+1} T_{\varphi(x)}^*(J^{k-1}Y)$ such that $i_\xi i_\eta z = 0$ for all tangent vectors ξ, η in $T_{\varphi(x)}(J^{k-1}Y)$ vertical over X , where φ stands for a local section of $J^{k-1}Y \rightarrow X$. The author proposes the pair $(Z^{k-1}, -d\Theta)$ as a covariant Hamiltonian counterpart of the Lagrangian system $(J^{2k-1}Y, -d\Theta_{\mathcal{L}})$, where \mathcal{L} is a k th order Lagrangian density on Y and $\Theta_{\mathcal{L}}$ stands for a Lepagean equivalent of \mathcal{L} in the sense of Krupka. To justify this, he explains that $J^{k-1}Y$ -bundle maps $\sigma_{\mathcal{L}}: J^{2k-1}Y \rightarrow Z^{k-1}$ exist (Legendre transformations) such that $\Theta_{\mathcal{L}} = \sigma_{\mathcal{L}}^* \Theta$. Let $J^{1*}(J^{k-1}Y)$ be the bundle over $J^{k-1}Y$ whose fibre over $\varphi(x)$ consists of the affine maps $J_{\varphi(x)}^1(J^{k-1}Y) \rightarrow \Lambda^{n+1} T_x^*(X)$. It is proved as the main result that Z^{k-1} and $J^{1*}(J^{k-1}Y)$ are canonically isomorphic. Section 5 is devoted to reviewing the notion of regularity in higher-order variational calculus. Also, an interesting section of prospects is included. It should be noted that the Poincare-Cartan form associated to a linear connection was introduced for the first time in a paper by P. L. Garcia and the reviewer [Atti Accad. Sci. Torino Cl. Sci. Fis. Mat. Natur. 117 (1983), suppl. 1, 127–147; MR 86i:58042]. The references on this matter quoted in the paper under review are from a later date.

{For the entire collection see MR 91k:58003}.

Jaime Muñoz Masqué (E-CSIC-MS)