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A universal reduction procedure for Hamiltonian group actions.

The geometry of Hamiltonian systems (Berkeley, CA, 1989), 33–51,
Math. Sci. Res. Inst. Publ., 22, Springer, New York, 1991.

D. Gorenstein [Bull. Amer. Math. Soc. (N.S.) 14 (1986), no. 1, 1–98; MR 87k:20001] has said, “I for one feel it is the existence of singular solutions that gives mathematics its rich texture, and their suppression, were that possible, would make mathematics too homogenized for my taste.” The authors of the paper under review can perhaps be said to consider the reduction of Poisson structures in a similar light, though it must also be said that the thrust of their paper is in a sense to regularize the singularities that arise. They address the problem of inducing Poisson structures on the reduced phase spaces generated by a Hamiltonian action of a Lie group G on a symplectic manifold M with equivariant momentum mapping $J: M \rightarrow \mathfrak{g}^*$. The standard construction [J. Marsden and A. Weinstein, Rep. Math. Phys. 5 (1974), no. 1, 121–130; MR 53 #6633] breaks down in singular cases, for which a number of competing techniques exist: these techniques apply in different contexts and can disagree in their predictions [see Arms, Gotay and G. Jennings, Adv. Math. 79 (1990), no. 1, 43–103, MR 91a:58051]. The present paper offers a new natural reduction procedure which always assigns a Poisson structure to each reduced phase space. Using slices [see R. S. Palais, Ann. of Math. (2) 73 (1961), 295–323; MR 23, A3802; Arms, Marsden and V. E. Moncrief, Comm. Math. Phys. 78 (1980/81), no. 4, 455–478; MR 82m:58028], the authors show that if the action of G on M is proper then J has quadratic singularities and the reduced Poisson structures are nondegenerate. They also show that if, additionally, the preimages of coadjoint orbits under J are closed and the Marsden-Weinstein procedure applies, then its reduced structures agree with theirs. Finally, they discuss the special case of compact group actions and illustrate their results in the case of the spherical pendulum.

{For the entire collection see MR 92c:58002}.

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