

## Graph Complexes, Configuration Spaces and Manifold Calculus

Vancouver, May 22 – 25, 2018

4th Floor, Earth Sciences Building,

2207 Main Mall

	Monday 21	Tuesday 22	Wednesday 23	Thursday 24	Friday 25
8:45 AM	Arrivals and check into accommodation	Registration & Breakfast* (ESB4133)			
9:30 AM		Paul Arnaud SONGHAFUO T.	Ryan BUDNEY	Ricardo CAMPOS	Victor TURCHIN
10:30 AM		Coffee Break	Coffee Break	Coffee Break	Coffee Break
11:00 AM		Brenda JOHNSON	Pedro BOAVIDA	Najib IDRISSI	Robin KOYTCHEFF
12:00 PM		Lunch, Self	Lunch, <b>Hosted*</b> (ESB4133)	Lunch, Self	Lunch, Self
2:00 PM		Kristine BAUER	Eduardo FERNANDEZ	Keely GROSSNICKLE	
3:00 PM		Coffee Break	Coffee Break	Coffee Break	
3:30 PM - 4:30 PM		Franjo SARCEVIC	Dev SINHA	Pascal LAMBRECHTS	
6:30 PM				Conference Dinner	

Meeting room: ESB 4192

\*Breakfast and Lunch Room: ESB 4133

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## Abstracts

1. **Kristine BAUER, University of Calgary, Canada**

**Title:** Abelian functor calculus and differential categories

**Abstract:** Functor calculus, invented by T. Goodwillie in the 1990's, is a useful tool for computing a variety of kinds of invariants. There are a number of different “flavours” of calculus, including homotopy calculus, manifold calculus, orthogonal calculus and the abelian functor calculus of Johnson and McCarthy.

In joint work with B. Johnson, C. Osborne, E. Riehl and A. Tebbe, we showed that abelian functor calculus is a cartesian differential category in the sense of R. Blute, R. Cockett and R. Seely. In this talk, I will explain how this shows that the relationship between abelian functor calculus and classical differential calculus of function is a consequence of this structure, and not just an analogy. I will discuss some of the consequences of this structure and parallels to other types of functor calculus.

2. **Pedro BOAVIDA de BRITO, IST, University of Lisbon, Portugal**

**Title:** Presentations of configuration categories

**Abstract:** Configuration spaces of points in a manifold naturally assemble into a topological category, called the configuration category. These categories are deeply related to spaces of embeddings, via manifold calculus. In this talk, I will discuss an attempt (work in progress with M. Weiss) at giving presentations (as in generators and relations) for configuration categories. When the cardinality of the configurations is bounded, and under mild finiteness conditions on the manifold, the presentations we obtain are finite.

3. **Ryan BUDNEY, University of Victoria, Canada**

**Title:** Some observations on the high-dimensional splicing operad

**Abstract:** There is an operad called “the splicing operad” that acts on spaces of embeddings of one disc in another. In the case of the interval in the 3-disc, this operad can be described completely (up to some hard-to-resolve problems in hyperbolic 3-manifolds), moreover this operad provides a rather concise description of the homotopy type of the space of knots in the 3-disc. In the case of more general discs, little is known about this operad, while often much more is known about the corresponding space of knots. I will discuss some of the more accessible features of the splicing operad in high dimensions.

4. **Ricardo CAMPOS, University of Paris 13, France**

**Title:** A graphical model for (framed) configuration spaces of points

**Abstract:** We study the real homotopy type of configuration spaces on smooth closed manifolds. In this talk, I will give an introduction to the problem of understanding configuration spaces and present an algebraic model of these spaces using graph complexes similar to Kontsevich's. We will study the natural action from the (framed) little discs operad on the space of (framed) configuration of points, with applications to Manifold Calculus in mind and we will see that under some conditions the same type of graphs are able to capture the real homotopy type of this operadic module. (Joint work with Julien Ducoulombier, Najib Idrissi and Thomas Willwacher)

5. **Eduardo FERNANDEZ, ICMAT-UCM, Spain**

**Title:** Loops of Legendrians in contact 3-manifolds

**Abstract:** The theory of Legendrian submanifolds plays a central role in Contact Topology. In this talk we focus our attention in the 3-dimensional case, more specifically in the contact manifolds  $(\mathbb{R}^3, \xi_{\text{std}})$  and  $(\mathbb{S}^3, \xi_{\text{std}})$  (or any contact 3-fold  $(M, \xi)$  in which the Euler class of the contact distribution  $\xi$  vanishes). The starting point is the introduction of the so called classical invariants of Legendrian submanifolds. It turns out that these invariants are actually formal invariants. Following the formal viewpoint we are able to introduce new invariants for loops of Legendrian submanifolds. As an application we show that the natural action of the group  $\text{Cont}(\mathbb{S}^3, \xi_{\text{std}})$  in the space of Legendrians of  $(\mathbb{S}^3, \xi_{\text{std}})$  induces a homotopy injection on certain connected components of the space of Legendrian knots. Moreover, we find other families of examples that do not come from restriction of global contact isotopies. This reproves, in an elementary way, the classical Kálmán's examples of smoothly trivial loops of Legendrian knots that are non trivial loops in the space of Legendrian embeddings. This is a joint work with Francisco Javier Martínez-Aguinaga (ICMAT-UCM) and Francisco Presas (ICMAT).

6. **Keely GROSSNICKLE, Kansas State University, USA**

**Title:** Cycle Index Sum for Non- $k$ -Equal Configurations

**Abstract:** I will present the cycle index sum of the symmetric group action on the homology of the configuration spaces of points in a Euclidean space with the condition that no  $k$  of them are equal. Configuration spaces form a bimodule structure over the little  $d$ -disc operad. This connection will be explained in my talk as well as the induced structure on the homology. (Joint work with Victor Turchin)

7. **Najib IDRISSE, ETH Zurich, Switzerland**

**Title:** Configuration Spaces of Manifolds with Boundary

**Abstract:** We study the real homotopy type of configuration spaces of smooth compact manifolds with boundary. We built combinatorial model based on graph complexes for these configuration spaces. We have three different approaches:

- (a) the Swiss-Cheese operad naturally acts on colored configurations in the manifold, and we build models using Willwacher's graphical model for this operad;
- (b) the collection of configurations in a collar around the boundary of the manifold is naturally endowed with a homotopy associative algebra structure, by gluing, which naturally acts on the collection of configurations of the whole manifold, and we build models for this action;

(c) under dimensionality and connectivity assumptions, we provide a small model inspired by the Lambrechts–Stanley model for configuration spaces of closed manifolds.

(Joint work with Ricardo Campos, Pascal Lambrechts, and Thomas Willwacher)

8. **Brenda JOHNSON, Union College, USA**

**Title:** Functor Precalculus

**Abstract:** Functor calculi have been developed in a variety of forms and contexts. Each of these calculi comes equipped with its own definition of polynomial or degree  $n$  functor. Such definitions are often formulated in terms of the behavior of the functor on certain types of cubical diagrams. Using the discrete calculus developed with Kristine Bauer and Randy McCarthy as a starting point, we identify a category-theoretic framework, which we call a precalculus, that provides a means by which notions of degree for functors can be defined via cubical diagrams. We show how such precalculi might be used to produce functor calculi. This is work in progress with Kathryn Hess.

9. **Robin KOYTCHEFF, University of Louisiana at Lafayette, USA**

**Title:** Graph complexes, formality, and configuration space integrals for braids

**Abstract:** In joint work with Rafal Komendarczyk and Ismar Volic, we study the space of braids, that is, the loop space of the configuration space of points in a Euclidean space. We relate two different integration-based approaches to its cohomology, both encoded by complexes of graphs. On the one hand, we can restrict configuration space integrals for spaces of long links to the subspace of braids. On the other hand, there are integrals for configuration spaces themselves, used in Kontsevich’s proof of the formality of the little disks operad. Combining the latter integrals with the bar construction and Chen’s iterated integrals yields classes in the space of braids. We show that these two integration constructions are compatible by relating their respective graph complexes. As one consequence, we get that the cohomology of the space of long links surjects onto the cohomology of the space of braids.

10. **Pascal LAMBRECHTS, Catholic University of Louvain, Belgium**

**Title:** TBA

**Abstract:** TBA

11. **Franjo SARCEVIC, University of Sarajevo, Bosnia**

**Title:** Calculus of functors for  $r$ -immersion

**Abstract:** An  $r$ -immersion of a smooth manifold  $M$  to a smooth manifold  $N$  is an immersion that has no  $r$ -fold self-intersections, i.e. at most  $r - 1$  points of  $M$  are mapped to the same point in  $N$ . I’ll talk about these immersions from the perspective of manifold calculus of functors. For a given space of  $r$ -immersion, we construct the Taylor tower whose purpose is to approximate this space in a suitable way and our goal is to show that this tower converges.

12. **Dev SINHA, University of Oregon, USA**

**Title:** Hopf invariants and the calculus tower for knots

**Abstract:** Hopf invariants measure homotopy classes of maps by computing generalized linking numbers. They completely detect homotopy groups up to torsion. We developed these with Ben Walter with the application to the calculus tower for knots in mind, in particular the conjecture that the tower is a universal Vassiliev invariant. We have recently published spectral sequence calculations which show this should be fully possible. Such an application would require partial models for the integer-valued cochains of Euclidean configuration spaces. In this talk, we review these developments and formulate questions which would resolve the tower conjecture as well as have use more broadly in topology of configuration spaces and its applications to embedding calculus.

13. **Paul Arnaud SONGHAFUO TSOPMENE, University of Regina, Canada**

**Title:** Classification of homogeneous functors in manifold calculus

**Abstract:** Manifold calculus, due to Goodwillie and Weiss in the 1999's, is a calculus of functors suitable for studying contravariant functors from the poset  $\mathcal{O}(M)$  of open subsets of a manifold  $M$  to spaces. Being a calculus of functors, manifold calculus has a notion of polynomial functor, and therefore a notion of homogeneous functor. A deep result of Weiss states that any homogeneous functor of degree  $k$  can be constructed out of a fibration over the unordered configuration space  $F_k(M)$  of  $k$  points in  $M$ .

Recently, D. Stanley and myself have launched a project that consists of studying functors from  $\mathcal{O}(M)$  into general model categories. One of our new results is concerned with the classification of homogeneous functors. Let  $\mathcal{T}^{F_k(M)}$  be a triangulation of  $F_k(M)$ , and let  $\mathcal{T}_\bullet^{F_k(M)}$  denote the canonical associated simplicial set. We prove that every homogeneous functor of degree  $k$  is classified by a simplicial map from  $\mathcal{T}_\bullet^{F_k(M)}$  to a certain simplicial set  $\widehat{A}_\bullet$ . Furthermore, we prove that two functors are weakly equivalent if and only if their classifying simplicial maps are homotopic. We believe that our classification result is somehow related to that of Weiss.

14. **Victor TURCHIN, Kansas State University, USA**

**Title:** Graph-complexes in the study of embedding spaces

**Abstract:** I will briefly explain how graph-complexes appear in the study of the finite type invariants of classical knots and links in  $\mathbb{R}^3$  (after Bar Natan and Vassiliev). I will then review the known results due to Arone, Fresse, Lambrechts, Songhafou Tsopméné, Willwacher and myself that describe the rational homotopy of the spaces of (higher-dimensional) long knots and string links in terms of graph-homology. The remaining open problems will be discussed.