

Title: Values of the zeta function at negative integers, from Euler to the trace formula

Abstract: Although the zeta function $\zeta(s)$ is often named after Riemann, it was Euler who discovered many of its remarkable properties. After making his name on the evaluation of $\zeta(2)$, Euler was able to obtain similar formulas at all positive even integers, and defined putative values at negative integers, where the series does not converge. Euler showed that these values at negative integers were all rational numbers. A comparison with the values at positive integers led him to guess the functional equation relating $\zeta(s)$ to $\zeta(1-s)$ (which was proved about one hundred years later by Riemann). I will begin by exposing some of this work, then show how the values at negative integers can be used to compute the dimension of certain spaces of automorphic forms. In a special case the dimension turns out to be 1, and this leads to a construction of local systems with exceptional Galois groups on the projective line (minus two points) over a finite field.