

Topics in Kinetic Theory

Book of Abstracts - Principal Lecturers

■ Speaker: Pierre Degond

Title: Kinetic and Fluid Modeling of Complex Systems

Abstract: In this series of lectures we will discuss various issues related with the modeling of systems which exhibit self-organization capabilities such as traffic networks (vehicular traffic on highways, traffic of goods on supply chains) or socially interacting biological entities (insect swarms, fish schools, sheep herds, pedestrians). These systems exhibit complex self-organization behaviour at the macroscopic scale, while, at the microscopic scale, each individual only retains limited information about its environment and performs simple tasks and interactions with its neighbors.

Kinetic and fluid models are powerful tools to study these questions. We will discuss a selection of topics related to the questions discussed above.

Possible plan of the series of lectures:

(1) Introduction and examples: issues, problems and models (2) A critical review of the chaos assumption (3) Hydrodynamic equations for a 'universal' swarming model: the Vicsek model (4) Fokker-Planck and diffusion models for displacements and foraging behaviour.

■ Speaker: Yan Guo

Title: A L^p - L^∞ Approach in the Boltzmann Study

Abstract: After a short review of some basic background in the Boltzmann theory, we will introduce a recent L^p - L^∞ approach leading to new results in the field. We will focus on several applications of such a new method which include (1) boundary value problems (2) Euler and acoustic limit (3) phase transition in a binary kinetic fluid.

■ Speaker: Axel Klar

Title: Mathematical models for fibre dynamics and fibres lay down in non-woven production

Abstract: The understanding of the forms generated by the lay-down of flexible fibres onto a conveyor belt is of great interest in the production process of non-woven materials that find their applications for example in composite materials like filters or textile and hygiene industry. In the meltspinning process of non-woven materials hundreds of individual fibres being obtained by the continuous extrusion of a melted polymer are stretched and entangled by highly turbulent air flows to, finally, form a web on the conveyor belt. The quality of this web and the resulting non-woven material -- in terms of homogeneity and load capacity -- depends essentially on the dynamics and the deposition of the fibres.

Here, deterministic and stochastic models for turbulent fibre dynamics and fibre lay-down based on stochastic differential equations and Fokker-Planck equations are presented. They are investigated using a variety of methods from numerical, asymptotic and stochastic analysis. Finally, these models are used to assess the quality of non-woven materials in industrial applications.

2009 Topics in Kinetic Theory

Book of Abstracts – Invited Lecturers

- Speaker: Håkan Andreasson

Title: The Einstein-Vlasov-Maxwell system and sharp bounds on the mass-radius ratio of spherically symmetric charged objects

Abstract: In general relativity an important question is how compact a spherically symmetric static object possibly can be, i.e., how large can the ratio M/R be? Here M is the total ADM mass and R the area radius of the static object. A bound on M/R limits the gravitational red shift that can be observed. In 1959 Buchdahl proved, under the assumptions that the pressure is isotropic and that the energy density is non-increasing outwards, the bound $M/R \leq 4/9$. I have shown that this bound holds generally, independently of the assumptions made by Buchdahl. If the object admits charge the analogous problem is to find an upper bound on M for a given area radius R and charge Q . This problem has resulted in a number of papers in recent years but neither a transparent nor a general inequality similar to the Buchdahl inequality has been found. Using insights from my result on the non-charged case I have derived the surprisingly transparent inequality $\sqrt{M} \leq \sqrt{\frac{R}{9}} + \sqrt{\frac{R}{9} + \frac{Q^2}{3R}}$.

I will discuss the proofs of these results which hold for any matter model. However, since several ideas originate from my study of static solutions of the Einstein-Vlasov (-Maxwell) system, special attention will be given to the features of these solutions.

- Speaker: Kazuo Aoki (*Joint work with T. Tsuji (Kyoto Univ.) and G. Cavallaro (Univ. Rome 1)*)

Title: Approach to steady motion of a plate moving in a collision less gas under a constant external force

Abstract: We consider a thin plate accelerated or decelerated in a collision less (or free-molecular) gas at rest by a constant external force. The force is in the direction perpendicular to the plate. In this situation, the plate velocity approaches its final constant velocity as time goes on. It is shown numerically that, under the diffuse-reflection boundary condition, the difference between the plate velocity and its final value decreases in proportion to an inverse power of time. This agrees with the previous theoretical result obtained under the assumption that the initial plate velocity is sufficiently close to the final one.

- Speaker: Anton Arnold

Title: Quantum Fokker-Planck models: kinetic and operator theory approaches

Abstract: Dissipative open quantum systems like quantum-Fokker-Planck (QFP) models play an important role for quantum Brownian motion, quantum optics, and the numerical simulation of nano-semiconductor devices. Their time evolution can either be described by quantum kinetic Wigner function models or in the density matrix formalism.

In this talk we mostly focus on the QFP equation in density matrix formalism. We discuss existence and uniqueness of a trace-preserving solution, existence of a unique (normalized) steady state, and large-time convergence (in trace class norm) to it. This involves tools from semi-group theory, operator theory, and quantum probability. The proofs cover quadratic confinement potentials along with sub-quadratic perturbations.

References: A. Arnold, F. Fagnola, L. Neumann: Quantum Fokker-Planck models: the Lindblad and Wigner approaches, "Quantum Probability and related Topics - Proceedings of the 28th Conference" (Series: QP-PQ: Quantum Probability and White Noise Analysis - Vol. 23), J.C. Garcia R. Quezada, S.B. Sontz (Eds.), World Scientific 2008.

A. Arnold, C. Sparber: Quantum dynamical semi-groups for diffusion models with Hartree interaction, *Comm. Math. Phys.* 251, no.1 (2004) 179-207.

Speaker: Jose Antonio Carrillo

Title: Some kinetic models for swarming

Abstract: I will discuss several features of kinetic models for collective behavior of individuals. Issues about modelling, particular solutions, stability, well-posedness and long time asymptotics will be addressed. Particular attention will be put on milling and flocking solutions for some models.

- Speaker: Stephane Cordier

Title: Mesoscopic description of the behavior of a simple financial market

Abstract: In this talk, we present a mesoscopic description of the behavior of a simple financial market where the agents can create their own portfolio between two investment alternatives: a stock and a bond. The model is derived starting from the Levy-Levy-Solomon microscopic model using the methods of kinetic theory and consists of a linear Boltzmann equation for the wealth distribution of the agents coupled with an equation for the price of the stock. From this model, under a suitable scaling, we derive a Fokker-Planck equation and show that the equation admits a self-similar lognormal behavior. (Numerical examples will be shown).

- Speaker: Jean Dolbeault (*Joint work with C. Mouhot and C. Schmeiser*)

Title: Hypocoercivity for kinetic equations with linear relaxation terms

Abstract: This talk will be devoted to a simple method for proving the hypocoercivity associated to a kinetic equation involving a linear time relaxation operator. It is based on the construction of an adapted Lyapunov functional satisfying a Gronwall-type inequality. The method clearly distinguishes the coercivity at microscopic level, which directly arises from the properties of the relaxation operator, and a spectral gap inequality at the macroscopic level for the spatial density, which is connected to the diffusion limit. The case of a linear BGK model and of a relaxation operator which corresponds at macroscopic level to the linearized fast diffusion will be considered.

- Speaker: Klemens Fellner (*Joint work with Jose A. Carrillo, Laurent Desvillettes, Jose A. Canizo*)

Title: Coagulation-Fragmentation Models with Diffusion

Abstract: We consider existence, large time behaviour, and fast-reaction limits of coagulation-fragmentation models with spatial diffusion. As continuous in size model we study the Aizenman-Bak model of coagulation/fragmentation. Discrete in size models are addressed with more general coagulation/fragmentation coefficients. The diffusion coefficients are allowed to degenerate in size. The main applied techniques include a-priori estimates based on the dissipation of an entropy functional, entropy entropy-dissipation approaches, moment bounds, and duality methods.

- Speaker: Hyung Ju Hwang

Title: Boundary value problems in the Vlasov-Poisson system

Abstract: We are concerned with boundary value problems arising in the Vlasov-Poisson system. The Vlasov-Poisson system models a collisionless plasma. When a boundary condition is included in the problem it is known that singularities can occur but that weak solutions exist globally in time. However, existence of a strong solution and uniqueness of a weak solution for such a BVP have been open so far.

Firstly, we study global existence of strong solutions for the Vlasov-Poisson system in convex bounded domains with specular boundary conditions and with a prescribed outward electrical field on the boundary. Secondly, we discuss uniqueness of weak solutions for the BVP problem in one dimension with specular reflection on the boundary.

- Speaker: Ansgar Juengel

Title: Electron transport and heating in semiconductor devices and circuits

Abstract: Thermal effects in semiconductor devices and electric circuits are modeled and numerically simulated. The particle temperatures are computed from the energy-transport equations, which are derived from the semiconductor Boltzmann equation in the diffusion limit. The complete model also includes a heat equation for the lattice temperature, the electric network equations, and the thermal network describing the heat evolution in the circuit elements.

The final model is a system of nonlinear partial-differential- algebraic equations, which are discretized in time by BDF methods and in space by mixed finite elements. The heating effects are illustrated by several numerical examples.

- Speaker: Tai-Ping Liu

Title: Invariant Manifolds for Stationary Boltzmann Equation

Abstract: We will present the recent work by Shih-Hsien Yu and the author on the stable, un- stable and center manifolds for the stationary Boltzmann equation. The main technique is the time-asymptotic analysis based on the Green's function. Among the applications, we offer analytical understanding of the Sone's diagram of bifurcation phenomena for complete condensation boundary problem. We also show that the Boltzmann shock profiles are monotone.

- Speaker: Simona Mancini

Title: A Fokker-Planck model from neuroscience for two interacting populations

Abstract: We will consider the decision making modelling of two interacting populations, in particular the firing rates of two families of neurons. Their behaviour in time is described by a system of stochastic partial differential equations (see [1]).

We are interested in the study of the statistical properties of a large set of interacting neurons. Hence we deduce, as classically done, a kinetic model (Fokker-Planck equation) describing the evolution in time of the distribution function of the two firing rates.

The flux term in our kinetic model doesn't derive from a potential, and so we don't have an exponential explicit formula for the solution of the associated stationary equation. Nevertheless, we can prove the existence and uniqueness of a positive solution to the stationary problem, also, applying relative entropy methods, the convergence in time of the solution towards this equilibrium state.

Finally, we perform and discuss numerical simulation of the Fokker-Planck equation. Our results are in agreement with those find in [1] by means of moments analysis on the stochastic system. Moreover, we remark the slow-fast behaviour for the evolution of the distribution function, and we will then briefly investigate the reduction of the stochastic system to one stochastic differential equation, yielding to one-dimensional Fokker-Planck model.

[1] G. Deco, D. Marti, Biological Cybernetics, (2007).

Speaker: Daniel Matthes

Title: Distribution of wealth in market economies -- a kinetic approach

Abstract: Kinetic equations model the redistribution of energy and momentum in an ensemble of particles due to binary collisions. The Boltzmann equation, where the collisions are taken literally, i.e., particles interact according to the laws of classical mechanics, provides one possible interpretation. Recently, various alternative interpretations have become popular. In this talk, we are concerned with the econophysics approach, where particles are thought of as interacting agents in a closed economy, the particle energy is identified with an agent's wealth, and the binary "collisions" are trades, in which wealth is exchanged according to certain laws, which obviously differ from those of classical mechanics. In particular, some randomness is involved, which corresponds to gains or losses due to risky investments.

I intend to provide an overview on the recently obtained analytical results: existence and uniqueness of transient and stationary solutions, the creation of Pareto tails and the phenomenon of wealth condensation, the regularity of solutions and their equilibration behaviour.

- Speaker: Florian Mehats

Title: Stable steady states and self-similar blow up solutions for the relativistic gravitational Vlasov-Poisson system

Abstract: *(Joint work with Mohammed Lemou and Pierre Raphael)*

We consider the three dimensional gravitational Vlasov-Poisson system in the relativistic case. This system displays finite time blow up solutions under a critical size condition, according to the associated interpolation inequality. Two results will be presented. The first one concerns the subcritical case. Using standard concentration compactness techniques, we show that the breaking of the scaling symmetry allows the existence of stable relativistic ground states. The second case concerns the critical case. We exhibit a family of finite time blow up self-similar solutions and prove that their blow up dynamic is stable with respect to radially symmetric perturbations.

- Speaker: Luc Mieussens

Title: Analysis of an asymptotic preserving scheme for linear kinetic equations in the diffusion limit

Abstract: I will present a new numerical scheme for linear kinetic equations that preserves the diffusion limit (proposed with Mohammed Lemou). We use the micro-macro decomposition of the distribution function into a local equilibrium state and a deviation: this decomposition allows us to reformulate the kinetic equation into a coupled system of a macroscopic equation for the density and a kinetic equation for the deviation. This system is discretized by using various techniques (staggered grids, upwind and central finite differences, semi-implicit time discretization) so as to obtain an asymptotic preserving scheme: when the scale factor goes to zero, the scheme obtained in the limit is consistent with the limit of the equation itself (the so-called diffusion limit). In this talk, I will present a rigorous analysis of this scheme (work in collaboration with Jian-Guo Liu and Mohammed Lemou): by using simple energy estimates, we can prove that the scheme is indeed uniformly stable and accurate, that is to say stability and accuracy are obtained for time and grid steps that can be independent of the scale factor.

- Speaker: Phil Morrison

Title: On Landau Damping

Abstract: Since Landau's paper on the damping the bears his name, and Van Kampen's early work, mathematically rigorous work has been done by several authors (Degond, Maslov, Gedoryuk, Glassey, Schaeffer).

In this talk I will give my thoughts on Landau damping, in light of our recent numerical results for Vlasov-Poisson (with Heath, Gamba and Mischler) that uses a discontinuous Galerkin approach.

- Speaker: Reinel Sospedra

Title: Global Classical Solutions to the 3D Relativistic Vlasov-Maxwell System with Bounded Spatial Density

Abstract: The relativistic Vlasov-Maxwell system (RVM in short) describes the time evolution of a collisionless plasma whose particles interact through the self-induced electromagnetic field. The plasma is assumed to be at high temperatures, thus the particles may travel at speeds comparable to the speed of light. The main open problem concerning this system is to prove whether classical solutions develop singularities in finite time. Glassey and Strauss showed that local classical solutions can be continued globally in time if the moment of the particles is controlled. They proved that such control is achieved if the kinetic energy density (i.e., the moment of order $\ell = 1$ of the distribution function in the momentum variable) remains bounded for bounded times. Later, C. Pallard relaxed this assumption to $0 < \ell < 1$. In this talk, we show that the limit case $\ell = 0$ also holds true, and so we improve the previous results to the boundedness of the spatial density function.

- Speaker: Robert Mills Strain III

Title: Global Newtonian limit for the Relativistic Boltzmann Equation near Vacuum

Abstract: We discuss recent work to prove global existence of solutions to the Cauchy Problem for the Relativistic Boltzmann equation with near vacuum data whose smallness condition can be shown by scaling to be independent of the speed of light. We further show for the first time that the Newtonian limit, $c \rightarrow \infty$, is valid globally in time.

- Speaker: Henning Struchtrup (*Joint work with Peyman Taheri, Victoria*)

Title: Analytical and numerical solutions of the regularized 13 moment equations for rarefied gases

Abstract: The regularized 13 moment equations describe rarefied gas flows in the transition regime, for Knudsen numbers up to unity. The equations are of super-Burnett order in the sense of the Chapman-Enskog method. For boundary value problems in simple geometries the equations can be analytically solved. The analytical solution explicitly show rarefaction effects such as Knudsen layers, and non-linear bulk effects such as the characteristic dip in the temperature profile in Poiseuille flow. Explicit solutions are discussed for steady and unsteady flows in plane and cylindrical geometries. Comparison with numerical solutions of the Boltzmann equation, give proof of the accuracy of the equations within their range of validity. The question of extending the numbers of variables will be discussed as well.

- Speaker: Vitali Vougalter

Title: Threshold eigenvalues and resonances for the linearized NLS equation

Abstract: We prove the instability of threshold resonances and eigenvalues of the linearized NLS operator. We compute the asymptotic approximations of the eigenvalues appearing from the endpoint singularities in terms of the perturbations applied to the original NLS equation. Our method involves such techniques as the Birman-Schwinger principle and the Feshbach map.