Differential Schemes and Differential Cohomology
Titles and Abstracts

The meeting will be held 25-26 June 2012 at the University of Calgary. The following speakers have been confirmed, although the times are TBA. There will be additional talks, but several of these will be determined at a BIRS workshop in Banff to be held the preceding weekend.

1. Speaker: Robin Cockett, University of Calgary

   Title: Tangent structure and differentials categorically

   Abstract:

   In 1984 Jiri Rosicky proposed in a short paper an abstract definition for a tangent bundle functor which generalized the usual tangent bundle functor of differential geometry. From this description Rosicky was able to show that one can, quite abstractly, define the Lie bracket on vector fields. Furthermore, when the tangent structure is "representable", he exhibited a connection to synthetic differential geometry.

   More recently, in 2009, Rick Blute, Robert Seely, and the speaker introduced the notion of a (Cartesian) differential category to provide an abstract description of differentiation. This was motivated not only by the desire to capture the essential algebraic manipulation underlying ordinary "multivariable" differential calculus but also by structures arising from Computer Science, Linear Logic, and combinatorics.

   A natural question to ask concerns the relationship between these notions. Slightly generalizing Rosicky's ideas to "tangent structure" allows one to link these notions by an adjoint. Every (Cartesian) differential category has tangent structure. Furthermore, within a category with (Cartesian) tangent structure there is always a full subcategory of "differential objects" – they are the tangent spaces – which, as their name suggests, form a differential category. Sometimes, as is the case with differential manifolds, one can actually "generate" the larger category from its differential objects.

   Of interest are the constructions to which categories with tangent structure are closed. Two constructions of interest are forming presheave and scheme categories. The former has been well-investigated and it is known that the differential structure does not (quite) transfer. The latter case has not been as well investigated, however, once again, the structure does not in general transfer – although there are special cases in which it does.

2. Speakers: Dingkang Wang and Yan Sun - University of Manitoba (Wang) and Key Laboratory of Mathematics Mechanization, Academy of Mathematics and Systems Science (Sun) (joint work with Yang Zhang)

   Title: A Signature-Based Algorithm for Computing Bases in Solvable Polynomial Algebras

   Abstract:

   Signature-based algorithms, including F5, F5C, G2V and GVW, are efficient algorithms for computing Groebner bases in commutative polynomial rings. In this paper we present a signature-based algorithm to compute Groebner bases in solvable polynomial algebras which include the usual polynomial rings and some non-commutative polynomial rings such as the Weyl algebra. The generalized rewritten criterion (proposed in Sun and Wang 2011) is used to construct this new algorithm. When this new algorithm uses the order implied by GVW, its termination is proved without special assumptions on the computing order of critical pairs. Data structures similar to F5 can be used to speed up this new algorithm, and Groebner bases for corresponding syzygy modules can be obtained from the outputs in polynomial time. Experimental data shows that most redundant computations can be avoided in this new algorithm.

3. Speaker: Carlos Arreche, Graduate Center, City University of New York

   Title: Solving Linear Differential Equations with Parameters: An Algorithmic Approach with Applications to Arithmetic Geometry

   Abstract:

   Towards the end of the last century J. Kovacic constructed an algorithm which: (i) determines the differential Galois group of a second-order linear homogeneous ordinary differential equation with coefficients in a field of rational functions over an algebraically closed field of characteristic 0; and (ii) determines from that group if the equation admits "elementary" solutions. We have been successful in extending Kovacic's methods to cover the case of one-parameter families of second-order linear homogeneous equations, and the talk will outline the basic ideas.

   The Galois theory of differential equations with parameters is originally due to Peter Landesman, and the special case of linear differential equations with parameters (Parametrized Picard-Vessiot Theory) has been recast quite recently in close analogy with the classical Picard-Vessiot theory by P. Cassidy and M. Singer. In recent work of Gorchinskiy and Ovchinnikov the work of Cassidy-Singer

has been related to Gauss-Manin connections. We will derive the Picard-Fuchs equation as a simple application of our algorithm in order to illustrate our methods, and discuss other potential applications to arithmetic and algebraic geometry if time permits.