Free Cash-Flow, Issuance Costs and Stock Volatility

by

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Introduction (1)

Research questions: Optimal level of cash holdings for a corporation? Implications in terms of security issuance and payout policy? When to issue new securities? Design of securities? Dynamics of prices?

guidance for a simple theoretical model

Why cash holding? Use cash to finance activities and investment when other sources of funding are costly.

- Cost of external finance: Hennessy and Whited 2007, JOF; Lee et al 1996, JFR; (Average cost of SEO: 7.1% of the proceeds of the issuing; SEO infrequent and lumpy) Bazdresh, 2005.

Why is it costly? High levels of cash induce managers to engage in wasteful activities.

- Easterbrook, 1984, Jensen, 1986
- Dittmar and Mahrt-Smith, 2007, JFE; Kalcheva and Lins, 2007, RFS
Main Results

- issuance and payout policies that maximize the value of the firm.
  - firms have target cash levels (cash in excess of certain threshold is returned to shareholders) (Opler et al, 1999, DeAngelo, DeAngelo and Stulz, 2006, JFE).
  - firms optimally issue equity. Equity adjustments take place in lumpy and infrequent issues.

- asset pricing implications of financing costs and agency
  - stock prices exhibit heteroskedasticity
  - dollar volatility of stock prices increases after a negative shock on stock prices. (Black, 1976, “When things go badly for the firm, its stock price will fall, and the volatility of the stock will go up.”)

Contribute to complement the CTCF literature initiated by Black and Cox, 1976, Leland, 1994.

Relation to the math. Fin. literature on optimal dividend and liquidity management policies: Jeanblanc and Shiryaev 1995; Sethi and Taksar, 2002; Lokka and Zervos, 2005; Cadenillas and Clark 2007.
The Model (1)

- Cumulative cash-flow process $R_t$:
  \[ R_0 = 0 \quad dR_t = \mu dt + \sigma dW_t. \]

- Frictions
  - Fixed and proportional issuance costs
    \[ m, \quad i, \quad m + \frac{i}{p} - f \]
  - Managerial inefficiencies

- Issuance policy
  - Dates at which new security is issued: $(\tau_n)_{n \geq 1}$
  - Issuance proceed: $(i_n)_{n \geq 1}$
  - Total issuance proceed:
    \[ I_t = \sum_{n \geq 1} i_n \mathbb{1}_{\tau_n \leq t} \]
  - Total fixed issuance costs:
    \[ F_t = \sum_{n \geq 1} f \mathbb{1}_{\tau_n \leq t} \]

- Cash reserves process
  - $M = \{M_t; t \geq 0\}$
    \[ M_0^- = m, \quad dM_t = (r - \lambda) M_t dt + dR_t + \frac{1}{p} dI_t - dF_t - dL_t \]
  - Bankruptcy time
    \[ \tau_B = \{ t \geq 0 \mid M_t < 0 \} \]
The Model (2)

▷ Value of the firm for a given policy

\[ v(m; (\tau_n)_{n \geq 1}, (i_n)_{n \geq 1}, L) = \mathbb{E}^m \left[ \int_0^T e^{-rt} (dL_t - dI_t) \right], \]

▷ Value function

\[ V^*(m) = \sup_{(\tau_n)_{n \geq 1}, (i_n)_{n \geq 1}, L} \{ v(m; (\tau_n)_{n \geq 1}, (i_n)_{n \geq 1}, L) \} \]

▷ Questions

- value function,
- optimal issuance and payout policies,
- optimal security,
- dynamics of security prices,
- testable asset pricing implications.

▷ First-best environment

\[ V(m) = m + \mathbb{E} \left[ \int_0^\infty e^{-rt} (\mu dt + \sigma dW_t) \right] = m + \frac{\mu}{r}. \]
Benchmark: \( p = 1, \ f = 0, \ \lambda > 0 \)

\( \triangleright \) distribute all initial cash reserve \( m \) as a special payment at date 0, hold no cash beyond that date.

\( \triangleright \) The pair \( (L, I) \)

\[
L_t = m 1_{\{t=0\}} + lt; \quad I_t = (l - \mu)t - \sigma W_t
\]

\[
V(m) = \mathbb{E}^m \left[ \int_0^\infty e^{-rt} (dL_t - dI_t) \right]
\]

\[
m + \mathbb{E} \left[ \int_0^\infty e^{-rt} (\mu dt + \sigma dW_t) \right] = m + \frac{\mu}{r}
\]

\( \triangleright \) Dynamics of security prices.

\( S = \{S_t; t \geq 0\} \) ex-payment price of a share of the security issued by the firm

\( N = \{N_t; t \geq 0\} \) number of outstanding shares

\[
V(M_t) = N_t S_t
\]

\[
dI_t = d(N_t S_t) - N_t dS_t = -N_t dS_t = -\frac{\mu}{r} \frac{dS_t}{S_t}
\]

\[
\frac{dS_t + dD_t}{S_t} = r dt + \frac{\sigma r}{\mu} dW_t
\]

where \( D_t \) is the cumulative payment per share process:

\[
dD_t = \frac{r}{\mu} S_t dt = \frac{l}{N_t} dt.
\]
Benchmark: \( p = 1, \ f = 0, \ \lambda > 0 \)

\[
\frac{dS_t}{S_t} = r \left(1 - \frac{l}{\mu}\right) dt + \frac{\sigma r}{\mu} dW_t
\]

\[
S_t = \mathbb{E} \left[ \int_t^\infty e^{-r(s-t)} \frac{lr S_s}{\mu} ds \mid \mathcal{F}_t \right]
\]

\[
= \mathbb{E} \left[ \int_t^\infty e^{-r(s-t)} \frac{l}{N_s} ds \mid \mathcal{F}_t \right].
\]
\[ p > 1, \ f > 0, \ \lambda > 0 \]

\[ V^*(m) = \sup_{I_t,L_t} \mathbb{E}^m \left[ \int_0^{\tau_B} e^{-rt} \left( dL_t - dI_t \right) \right] \]

▷ Cash reserve process \( M \) at the optimum.

- If issuance costs are “large”: diffusion process that is reflected back each time it hits \( \hat{m}_1 \), and that is absorbed at 0.
- If issuance costs are “not too large”: diffusion process that is reflected back each time it hits \( m_1^* \), and jumps to \( m_0^* \) each time it hits 0.

▷ Optimal issuance policy

- Firm value jumps from \( V^*(0) \) to \( V^*(m_0^*) \)
- Each time \( M \) hits zero, the amount \( V^*(m_0^*) - V^*(0) \) of new security is issued.
Stock price dynamics (1)

\[ S = \{S_t; t \geq 0\} \] ex-dividend price of a share in the firm
\[ N = \{N_t; t \geq 0\} \] number of shares issued by the firm

- Stock price does not jump at optimal issuance dates: \( S_{\tau_n} = S_{\tau_n^-} \)
- \( V^*(M_t) = N_t S_t \)

\[ dI_t = d(N_t S_t) - N_t dS_t = S_t dN_t \]
- \( V^*(m_0^*) - V^*(0) = S_{\tau_n}(N_{\tau_n} - N_{\tau_n^-}) \)

Proposition. The process \( N \) modelling the number of outstanding shares is given by:

\[
N_t = \begin{cases} 
1 & 0 \leq t < \tau_1, \\
\left[ \frac{V^*(m_0^*)}{V^*(0)} \right]^n & \tau_n \leq t < \tau_{n+1}.
\end{cases}
\]
Stock price dynamics (continuity)

\[ S_t = \mathbb{E}\left[ \int_t^\infty e^{-r(s-t)} \frac{dL_s}{N_s} \mid \mathcal{F}_t \right] \]

\[ e^{-rt}S_t = \mathbb{E}\left[ \int_0^\infty e^{-rs} \frac{dL_s}{N_s} \mid \mathcal{F}_t \right] - \int_0^t e^{-rs} \frac{dL_s}{N_s}. \]
Stock price dynamics (2)

- $V^*(M_t) = N_t S_t$
- $dS_t = d[V^*(M_t)]/N_{\tau_n} \quad \forall t \in [\tau_n, \tau_{n+1})$.

Proposition. Between two consecutive issuance dates $\tau_n$ and $\tau_{n+1}$, the instantaneous return on stock satisfies:

$$\frac{dS_t + dD_t}{S_t} = rdt + \sigma(N_{\tau_n} S_t) dW_t,$$

where

$$\sigma(v) \equiv \frac{V^*[1(V^*)^{-1}(v)]}{v}$$

$D_t$ denotes the cumulative dividend per share process:

$$dD_t = \frac{dL^m_t}{N_{\tau_n}}.$$

Consequences:

- Changes in the volatility of stock returns are negatively correlated with stock price movements.
- Changes in the volatility of stock prices are negatively correlated with stock price movements.
- Stock price cannot take arbitrarily large values.
- A reduction in issuance costs should lead to a fall in the volatility of stock returns.
Conclusion

Introducing growth opportunities...

● Interaction between dividend policy and decision to invest in a growth opportunity

● Role of issuance costs? Does a decrease in issuance costs encourage firms to invest in more risky projects? Consequences on the dynamics of stock prices?

● Non predictable growth opportunity
Conclusion

Taking into account issuance costs in corporate models allows to derive several implications on asset pricing.

Issuance costs provide a natural explanation for heteroscedasticity of stock prices.
Comparative statics

Proposition

• The elasticity of the value of the firm with respect to its cash reserves,

\[ \epsilon^*(m) = \frac{mV^*(m)}{V^*(m)}; \quad m \geq 0, \]

is an increasing function of the issuance costs \( p \) and \( f \).

• The volatility of stock returns as a function of the firm’s valuation,

\[ \sigma^*(v) = \sigma \frac{V^*\left((V^*)^{-1}(v)\right)}{v}; \quad V^*(0) \leq v \leq V^*(m_1^*), \]

is an increasing function of the issuance costs \( p \) and \( f \).

\[ \implies \]

• A reduction in issuance costs should reduce the responsiveness of firm’s valuations to changes in their cash reserves.

• A reduction in issuance costs should lead to a fall in the volatility of stock returns.
Value function

\[ V^*(m) = \sup_{I_t, L_t} \mathbb{E}^m \left[ \int_0^{\tau_B} e^{-rt} (dL_t - dI_t) \right] \]

- Cash reserve process \( M \) at the optimum.
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- Optimal issuance policy
  - Firm value jumps from \( V^*(0) \) to \( V^*(m_0^*) \)
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Value function (1)

Road map:

• Write a system of variational inequalities that the value function $V^*$ should satisfy.

• Show that this system has a unique regular solution.

• Establish that this solution is indeed the optimal value function.
Value function (2)

Heuristics

\[ V^*(m) \geq V^*(m - l) + l \]
\[ V^*(m - l) \geq 1 \]
\[ V^*(m) \geq V^*(m + \frac{i}{p} - f) - i \]

\[ V^*(m) \geq \]

\[ \mathbb{E}^m \left[ e^{-r(t \wedge \tau_B)} V^* \left( m + \int_0^{t \wedge \tau_B} \left[ (\mu + (r - \lambda) M_s) ds + \sigma dW_s \right] \right) \right] \]
\[ -rV^*(m) + \mathcal{L}V^*(m) \leq 0 \]

\[ \mathcal{L}u(m) = (\mu + (r - \lambda)m)u'(m) + \frac{\sigma^2}{2} u''(m). \]
Value function (3)

▷ Guess

- Issuance policy

\[
V^*(0) = \max_{i \in [0, \infty)} \left\{ V^* \left( \frac{i}{p} - f \right) - i \right\}^+, \\
V^*(0) = \max_{m \in [-f, \infty)} \left\{ V^*(m) - p(m + f) \right\}^+
\]

- Dividend policy \( m \geq m_1^* \),

\[ V^{*'}(m_1^*) = 1. \]

\( V^* \) is postulated to be twice continuously differentiable over \((0, \infty)\),

\[ V^{**'}(m_1^*) = 0. \]
\textbf{Value function (4)}

▷ Variational system: Find \((V, m_1)\)

\[ V(m) = 0; \quad m < 0, \quad (1) \]

\[ V(0) = \left[ \max_{m \in [-f, \infty)} \{V(m) - p(m + f)\} \right]^+, \quad (2) \]

\[ -rV(m) + \mathcal{L}V(m) = 0; \quad 0 < m < m_1, \quad (3) \]

\[ V(m) = \frac{\mu + (r - \lambda)m_1}{r} + m - m_1; \quad m \geq m_1. \quad (4) \]

▷ Solving the system

Fix \(m_1 > 0\), \(V_{m_1}\) solution to:

\[ -rV_{m_1}(m) + \mathcal{L}V_{m_1}(m) = 0; \quad 0 \leq m \leq m_1, \]

\[ V'_{m_1}(m_1) = 1, \]

\[ V''_{m_1}(m_1) = 0. \]

\(V_{m_1}\) solution to (1)-(4) linearly extended to \([m_1, \infty)\).
Value function (5)

\[
V(0) = \left[ \max_{m \in [-f, \infty)} \{V(m) - p(m + f)\} \right]^+ 
\]

\[\exists! \, \hat{m}_1 \, V_{\hat{m}_1}(0) = 0,\]

(i) If \( \max_{m \in [-f, \infty)} \{V_{\hat{m}_1}(m) - p(m + f)\} = 0 \)

\[V^* = V_{\hat{m}_1}\]

(ii) If \( \max_{m \in [-f, \infty)} \{V_{\hat{m}_1}(m) - p(m + f)\} > 0 \)

\[\forall m_1, \, \exists! \, m_p(m_1) \, s.t \, V'_{m_1}(m_p(m_1)) = p\]

\[V_{m_1}(0) = V_{m_1}(m_p(m_1)) - p[m_p(m_1) + f].\]

\[m_1^*, \, m_p(m_1^*) = m_0^*, \, V^* = V_{m_1^*}\]

\[V^*(m_0^*) - V^*(0) = p(m_0^* + f) = i^*\]