BENIN SUMMER SCHOOL AND CONFERENCE REPORT

Recent Developments in Nonlinear Analysis and Applications

Organizers

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June 28 - July 03, 2010 & July 05-09, 2010

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1 Overview and Outcome of the meeting

On June 28, 2010, Over fifty mathematicians from seventeen different countries gathered in Cotonou, Benin, to take part in a summer school and conference on the Recent Developments in Nonlinear Analysis and Applications. This event was organized for the first time by the "Institut de Mathématiques et de Sciences Physiques (IMSP, Benin)", the department of mathematics of the "Université d'Abomey-Calavi (UAC, Benin)", in collaboration with some african mathematicians from Canada and the USA. The meeting brought together a wide range of mathematicians from three different continents (Africa, Europe and America) and seventeen different nationalities who spent two weeks discussing, working, sharing and learning from one another. The audience included specialists in nonlinear partial differential equations, calculus of variations, kinetic theory, as well as researchers with interest in their applications. Among the participants were also junior faculty and graduate students from several african countries and the USA who enjoyed this rare opportunity interacting with leading researchers in North America and Europe.

The first week of the meeting was devoted to the summer school. During this week (June 28 - July 03, 2010), a series of four mini-courses on the recent developments in nonlinear analysis and its applications were given by the following leading experts in nonlinear analysis:

1. John Ball, on "the Q-tensor theory of liquid crystals".

2. Luis Caffarelli, on "Review of regularity theory for second order equations".

3. Nassif Ghoussoub, on "Mathematical problems arising in Electrostatic micro-electro mechanical systems (MEMS) or on some nonlinear eigenvalue problems"

4. Cédric Villani, on "Landau damping".

Each section of the mini-course lasted one hour and each mini-course covered four sections; this allowed plenty of time for discussions between the lecturers and the participants of the summer school.

The second week (July 05-09, 2010) focused on the conference. A total of fifteen talks were presented. Each talk lasted one hour fifteen minutes, with a 30 minutes break in between. This not only gives the speakers enough time to go in details in their talks, but it also allowed the audience to have a lot of time to interact with the speakers during break times.

All the presentations of the meeting focused on very recent results and methods in nonlinear analysis, and the participants were unanimously impressed with their contents and the quality of the speakers.

Also during the first week, a round table was organized (on June 30, 2010) to discuss about ways to improve the level of mathematics in Africa and to increase the number of african researchers in mathematics. It was then pointed out during this round table by several researchers from Europe and North America that, based on experience in their home countries, this will require several exchanges and interactions with world-leader mathematicians. Therefore, it was suggested among other things:

- to improve communication technology in Africa in order to help african mathematicians access the works of other mathematicians in the world through the web, video conferences, etc.

- to organize periodic scientific meetings in Africa (like this one) where some world-leader mathematicians will be invited to give lectures and to interact with local african mathematicians.

Moreover, during the second week of the meeting, a forum was organized (on July 8, 2010), with the theme "Mathematics, Sciences and Society", and it was chaired by Pr. Nassif Ghoussoub. Few business men from Benin were invited to this forum. The goal was to inform them of the connections made in developed countries between the business world and the academic world concerning research in mathematics, and their impacts on the economical and scientific progress in their countries. The purpose was essentially to encourage business people to get involve in the academic world, and to finance research in mathematics.

Overall, all the participants were very satisfied of the quality and the outcome of the meeting. It is expected that the interactions built during this meeting will foster long term collaboration between the participants of these three continents. This of course will benefit graduate students from all these three continents through some exchange

programs; for example, good mathematics students from Africa could benefit of a fellowship to help them go to Europe or North America to do their graduate studies or to conduct research work under the supervision of a professor in Europe/North America, with the collaboration of another supervisor in Africa.

2 Presentation highlights

2.1 Summer school lectures

Luis Caffarelli opened the summer school with his mini-course on regularity issues for nonlinear problems involving non local diffusions. He mentioned that non local diffusion problems arise in several contexts, in fluid mechanics in turbulence, atmosphere-ocean interaction, flame propagation, in material sciences in planar crack propagation, semi permeable membranes, in optimal control and game theory for random processes involving jump processes, and more recently in image processing. He showed that there may be of variational nature (divergence form), or of fully non linear type. He then introduced few methods to solve regularity issues for these problems, and finally presented few open related problems.

John Ball talked about the Q-tensor theory of liquid crystals. In his lectures, he surveyed what is known about the mathematics of the de Gennes Q-tensor theory for describing nematic liquid crystals. He mentioned that this theory has been little studied by mathematicians despite its popularity with physicists, and it poses many interesting mathematics questions. In particular, he showed the relation of this theory to other theories of liquid crystals, specifically those of Oseen-Frank and Onsager/Maier-Saupe.

Nassif Ghoussoub presented mathematical problems arising in the Micro-Electro-Mechanical Systems (MEMS) and Nano-Electro-Mechanical Systems (NEMS), which are two basic ingredients of contemporary technology that combine electronics with miniature-size mechanical devices. He explained the problems in the engineering point of view, then he presented the mathematical ingredients which are used to answer some of the related mathematical problems. He also presented a lot of open problems.

Finally, Cédric Villani talked about Landau-damping which is a subject which goes back to the forties, when Landau stunned the physical community by predicting collisionless relaxation in plasmas. This subject then became a cornerstone of classical plasma physics. He mentioned that it is also present in other areas of physics such as galactic dynamics, and it has given rise to a number of interpretations since then. He presented the theory that he developed with Clément Mouhot to establish Landau damping in the nonlinear perturbative regime, solving then an open problem posed by Backus in 1960. Some of the insights of their analysis are drawn from the K-A-M theory in dynamical systems, and the interpretation of relaxation by transfer of smoothness from spatial to high-frequency kinetic modes. He also showed the mathematical and physical implications of their results.

2.2 Conference lectures

2.2.1 Elliptic equations

Martial Agueh presented a variational method to prove uniqueness of ground states to some quasilinear elliptic PDEs. His method is based on Optimal transport arguments and relies on a correspondence between the ground states of these PDEs and the equilibrium solutions of Fokker-Planck type equations. This method also allows to identify all the optimal functions of many geometric inequalities, such as the Sobolev inequalities, the logarithmic Sobolev inequalities, and certain Gagliardo-Nirenberg inequalities.

Jean-Pierre Gossez talked about a joint work with J. Fleckinger and F. Thélin on the maximum and antimaximum principles beyond the first eigenvalue for the Dirichlet problem

$$-\Delta u = \mu u \ \text{ in } \ \Omega, \quad u = 0 \ \text{ on } \ \partial\Omega.$$

It is known that the maximum and antimaximum principles give information on the sign of the solution when the parameter μ varies near the first eigenvalue of the corresponding homogeneous problem. In his talk, he introduced an analogue of these two principles when μ varies near a higher eigenvalue. Nodal domains played a central role in his lecture, as well as the Payne conjecture relative to the nodal line of a second eigenfunction in the plane.

Liamidi Leadi talked also about an eigenvalue problem, namely, the principal eigenvalue of the p-Laplacian operator with a potential in an unbounded domain.

Finally Guy Degla discussed about the resolution of a class of singular elliptic equations.

2.2.2 Kinetic equations

Eric Carlen discussed a recent work on the Patlak-Keller-Segel model, jointly done with A. Blanchet and J.A. Carrillo. He showed that this equation has a one parameter family of steady-state solutions with thick tails whose second moment is not bounded. Then he proved that these steady state solutions are stable, and found basins of attraction for them using an entropy functional coming from the critical fast diffusion equation in the plane. Finally, he constructed solutions of Patlak-Keller-Segel equation satisfying an entropy-entropy dissipation inequality for this entropy functional. No estimation of rates of convergence is provided in his talk.

Maria Carvalho presented an L^1- convergence to equilibrium for the weak solution of the homogeneous Boltzmann equation for soft potentials ($-4 \leq \gamma < 0$), with or without angular cutoff. She proved the time-averaged L^1- convergence to equilibrium for all weak solutions whose initial data have finite entropy and finite moments up to order greater than $2 + |\gamma|$. She then discussed the rate of convergence to equilibrium depending on the value of γ.

Irene Gamba talked about the existence and uniqueness of distributional and classical solutions for the Boltzmann transport equations for soft potentials with data near local Maxwellians. She presented convolution estimates of Young's inequality type for

the case of hard potentials and Hardy-Littlewood-Sobolev and Brascamp Lieb type inequality for soft potentials. The main technique used in her talk was the radial average symmetrization combined with classical tools of harmonic analysis.

2.2.3 Numerical methods

Gérard Awanou discussed numerical methods to solve the Monge-Ampère equations. He introduced several finite element methods, discussed their convergence, and finally gave numerical results for smooth solutions and standard cases where the solutions are known not to be smooth.

Olivier Besson presented an existence and uniqueness result for a linear conservation laws subject to initial and final condition. He also showed some numerical simulations of a linear conservation law with a non well-known velocity field. Some applications to cardiac image reconstruction are also presented.

2.2.4 Other lectures on nonlinear analysis and applications

Bernard Dacorogna talked about the pullback equation and Darboux theorem. Precisely, this is the question of existence of a diffeomorphism $\varphi : \mathbb{R}^n \to \mathbb{R}^n$ verifying $\varphi^*(g) = f$ where f and g are closed differential k-forms. He first considered the case $k = 2$, generalizing the celebrated Darboux theorem in two directions: firstly by obtaining optimal regularity in Hölder spaces for the local problem, and secondly by proving global existence and regularity of the solution under some necessary additional hypotheses. Then he discussed the easier case $k = n - 1$, and finally made some comments on the more difficult case, when $3 \leq k \leq n - 2$.

Gilles Gnacadja discussed "persistence" in chemical reaction network theory, i.e., the asymptotic property of positive functions of time that do not approach zero, be it steadily or in a recurring fashion. He began by explaining persistence in the context of chemical reaction networks, and highlighted some of the known results. He then introduced vacuous persistence, which he proposed as a stronger form of persistence to account for the possible absence of some species at initial state, a common situation in biochemistry experiments. He finally discussed some results on vacuous persistence.

Diara Seck talked about sand transport and pollution problems. He used homogenization tools to obtain existence and uniqueness results for the sand transport problem, whose modeling leads to degenerate parabolic equations. For the pollution problem, he used tools of topological optimization to propose a method for the location of pollution in the porous media, then he studied the nonlinear PDE arising in his model.

Other interesting talks on the theory of nonlinear analysis were given by Justin Feuto, Côme Goudjo and Aurélien Goudjo.