Stochastic Models for Convective Momentum Transport

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Examples of convective momentum transport (CMT)

Non-squall convective system:

- CMT decelerates mean wind

(b) $\bar{U}$ (black), $-\langle w' u' \rangle_z$ (color)
(c) $\bar{V}$ (black), $-\langle w' v' \rangle_z$ (color)

Tung and Yanai (2002b)
Examples of convective momentum transport (CMT)

Squall line:

- CMT accelerates $\bar{U}$

(b) $\bar{U}$ (black), $-(\overline{w'u'})_z$ (color)
(c) $\bar{V}$ (black), $-(\overline{w'v'})_z$ (color)

Tung and Yanai (2002b)
Statistics of convective momentum transport (CMT)

Top: \(-\overline{(w'u')}z \frac{\bar{U}}{|\bar{U}|}\)

Bottom: \(-\overline{(w'v')}z \frac{\bar{V}}{|\bar{V}|}\)

Circles: IOP mean
Horizontal lines: standard deviation

- Mean CMT: weak damping (cumulus friction)
- But standard dev. of CMT is huge!
- Examples demonstrate that both acceleration and deceleration can be intense
Motivation for stochastic models for CMT

1. Convective parameterizations in GCMs usually include only cumulus friction:

\[
\partial_t u + \partial_x (u^2) + \partial_z (wu) + \partial_x p = -\partial_z (w'u') \\
\approx -d_c (u - \hat{u})
\]

- Wu et al. (2007) include a deterministic CMT parameterization and improve the mean climatology
- Goal of present work: to develop a simple stochastic CMT model that includes intermittent intense bursts of CMT as in observations

2. GCMs fail to capture realistic variability of tropical convection

- A stochastic parameterization of convection could improve this
Spectral Power of Tropical Precipitation in Observations and GCMs

From Lin et al. (2006)
Stochastic models to capture the intermittent impact of smaller scale events on the larger scales


Stochastic Models for Convective Momentum Transport

Outline

1. Description of simple stochastic model for CMT

2. Test case: column model

3. Test case: convectively coupled wave
3 Convective Regimes with Different CMT

1. **Dry regime.**
   - Weak or no cumulus friction.
   - Favored for dry environments, regardless of shear.

2. **Upright convection regime.**
   - Stronger cumulus friction.
   - Favored for moist, weakly sheared environments.

3. **Squall line regime.**
   - Intense CMT, either upscale or downscale depending on the shear.
   - Favored for moist, sheared environments.
Markov jump process for transitions between regimes

3-state continuous-time Markov jump process

- at each large-scale spatio-temporal location \((x, t)\)
- with transition rates depending on local values of large-scale variables at \((x, t)\)

Denote the discrete, stochastic regime variable by

\[ r_t = 1 \text{ (dry)} \]
\[ r_t = 2 \text{ (conv.)} \]
\[ r_t = 3 \text{ (squall)} \]

\(T_{ij}\): transition rate from regime \(i\) to regime \(j\)

based on observations such as LeMone, Zipser, & Trier (1998)
Transition rates

\[ T_{12} = \frac{1}{\tau_r} \mathcal{H}(Q_d) e^{\beta \Lambda (1-\Lambda)} e^{\beta Q} Q_d \quad \text{dry} \to \text{conv.} \]

\[ T_{13} = 0 \quad \text{dry} \to \text{squall} \]

\[ T_{21} = \frac{1}{\tau_r} e^{\beta \Lambda \Lambda} e^{\beta Q} (Q_{d,ref} - Q_d) \quad \text{conv.} \to \text{dry} \]

\[ T_{23} = \frac{1}{\tau_r} \mathcal{H}(|\Delta U_{low}| - |\Delta U|_{min}) e^{\beta |\Delta U_{low}|} e^{\beta Q} Q_c \quad \text{conv.} \to \text{squall} \]

\[ T_{31} = T_{21} \quad \text{squall} \to \text{dry} \]

\[ T_{32} = \frac{1}{\tau_r} e^{\beta U (|\Delta U|_{ref} - |\Delta U_{low}|)} e^{\beta Q} (Q_{c,ref} - Q_c) \quad \text{squall} \to \text{conv.} \]

- Exponentials capture sensitive dependence on large-scale variables
- \( \tau_r, \beta \): model parameters
- \( Q \): cloud heating
- \( \Lambda \): measures dryness of lower-mid troposphere relative to boundary layer
- \( \Delta U \): vertical wind shear
Different convective regimes have different CMT

\[ F_{CMT} = -\partial_z (\overline{w' \cdot u'}) = \begin{cases} -d_1 (U - \hat{U}) & \text{for } r_t = 1 \\ -d_2 (U - \hat{U}) & \text{for } r_t = 2 \\ F_3 & \text{for } r_t = 3 \end{cases} \]

\[ F_3 = -\partial_z (\overline{w' \cdot u'}) = \kappa [\cos(z) - \cos(3z)]. \]

\[ \kappa = \begin{cases} - \left( \frac{Q_d}{Q_{d,ref}} \right)^2 \frac{\Delta U_{mid}}{\tau_F} & \text{if } \Delta U_{mid} \Delta U_{low} < 0 \\ 0 & \text{if } \Delta U_{mid} \Delta U_{low} > 0 \end{cases} \]

Formulas for \( F_3 \) and \( \kappa \) motivated by observations, CRM simulations, and a simple multi-scale model ...
Formula for $F_3 = -\partial_z(w'u') = \kappa[\cos(z) - \cos(3z)]$

Exactly solvable multi-scale model (Majda and Biello, 2004; Biello and Majda 2005; Majda, 2007)

$$w' = S'_\theta$$
$$u'_x + w'_z = 0$$

Choose $S'_\theta$ to include stratiform heating lagging deep convective heating:

$$S'_\theta = k \cos[kx - \omega t] \sqrt{2} \sin(z) + \alpha k \cos[k(x + x_0) - \omega t] \sqrt{2} \sin(2z)$$

Exact solution: 1st, 2nd mode heating generates CMT in the 1st, 3rd modes

$$\partial_z(w'u') = \frac{3\alpha k}{2} \sin(kx_0)[\cos(z) - \cos(3z)]$$
Formula for $\kappa = \begin{cases} 
- \left( \frac{Q_d}{Q_{d,ref}} \right)^2 \frac{\Delta U_{mid}}{\tau_F} & \text{if } \Delta U_{mid} \Delta U_{low} < 0 \\
0 & \text{if } \Delta U_{mid} \Delta U_{low} > 0 
\end{cases}$

CRM results: Liu and Moncrieff (2001)

Vertical tilts of squall lines, and their CMT, depend on the mid-level shear
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3. Test case: convectively coupled wave
Test case: column model

\[
\frac{\partial u}{\partial t} = F_{CMT} \\
\]

\[
u(z, t) = \sum_{j=1}^{3} u_j(t) \sqrt{2} \cos(jz) \\
\]

Other model variables are imposed

Demonstrates intermittent bursts of CMT

Useful for calibration of model parameters
Outline

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Model for convectively coupled waves

Congestus  Deep convection  Stratiform
Model for convectively coupled waves


\[ \begin{align*}
\frac{\partial u_1}{\partial t} - \frac{\partial \theta_1}{\partial x} &= F^1_{CMT} \\
\frac{\partial u_2}{\partial t} - \frac{\partial \theta_2}{\partial x} &= F^2_{CMT} \\
\frac{\partial u_3}{\partial t} &= F^3_{CMT} \\
\frac{\partial \theta_1}{\partial t} - \frac{\partial u_1}{\partial x} &= H_d + \xi_s H_s + \xi_c H_c - R_1 \\
\frac{\partial \theta_2}{\partial t} - \frac{1}{4} \frac{\partial u_2}{\partial x} &= H_c - H_s - R_2
\end{align*} \]

+ evolution equations for \( \theta_{eb}, q, H_s \)

and formulas for nonlinear interactive source terms

such as convective heating, downdrafts, etc.
Convectively coupled wave simulation

6000-km periodic domain
- to capture a single convectively coupled wave

\[ \Delta x = 50 \text{ km} \]
- representative of a GCM’s grid spacing

Initial conditions:
- small perturbation to uniform radiative–convective equilibrium solution
Wave-mean structure

Average in a reference frame moving with the wave at $-17.5$ m/s

a. Contours of potential temperature

b. Contours of total convective heating

C. $H_d$ (K/day) regime

x (1000 km)
Comparison: with and without stochastic CMT

Stochastic CMT generates a nontrivial mean flow that can interact with the wave

$u_1$: dash-dot

$u_2$: dash

$u_3$: solid

Summary

- A simple stochastic model for CMT was developed and tested
  - 3-state continuous-time Markov jump process, \( r_t \), represents the convective regime at each large-scale spatio-temporal location \((x, t)\) (dry regime, upright convection regime, and squall line regime)
  - Transition rates depend on large-scale resolved variables (cloud heating, wind shear, etc.)
  - CMT from unresolved scales acts on large-scale spatio-temporal location \((x, t)\) in different ways depending on the convective regime at \((x, t)\)

- Test case: column model
  - Intermittent bursts with physically reasonable values
  - Useful for calibrating model parameters

- Test case: convectively coupled wave
  - Stochastic CMT creates nontrivial mean flow that can interact with wave